Principle of Impulse and Momentum

$$\sum \underset{\sim}{F} = \frac{d}{dt}(m \cdot \underset{\sim}{v})$$

If we integrate this we get:

$$\int_{t_1}^{t_2} \sum \underset{\sim}{F}\, dt = mv\Big|_{t=t_2} - mv\Big|_{t=t_1} = mV_2 - mV_1$$

if mass is constant.

$$\boxed{\underbrace{\int_{t_1}^{t_2} \sum \underset{\sim}{F}\, dt}_{\text{Linear Impulse}} = m\underset{\sim}{v}_2 - m\underset{\sim}{v}_1}$$

or we can write

$$\Delta t \cdot \underset{\sim}{F}_{average} = m\underset{\sim}{v}_2 - m\underset{\sim}{v}_1$$

where Δt is the time period over which the force acts.

Conservation of Linear Momentum

If we have no external forces:

$$\frac{d}{dt}(mv) = 0 \quad \Rightarrow$$

$$\boxed{mv = \text{constant}}$$

For two objects:

$m_A \rightarrow \quad \leftarrow m_B$

$\boxed{m_A \cdot \underset{\sim}{v_A} + m_B v_B = \text{constant}}$ ← if there are <u>no</u> external forces

Impacts

$$\underbrace{m_A \underset{\sim}{v_A} + m_B v_B}_{\text{Before impact}} = \underbrace{m_A \underset{\sim}{v_A'} + m_B v_B'}_{\text{After Impact}} = (m_A + m_B) \cdot v_{cm}$$

Velocity of center of mass is ~~un~~ not affected by the impact.

$$v_{cm} = \frac{m_A v_A + m_B v_B}{m_A + m_B} \quad \text{or} \quad v_{cm} = \frac{m_A v_A' + m_B v_B'}{m_A + m_B}$$

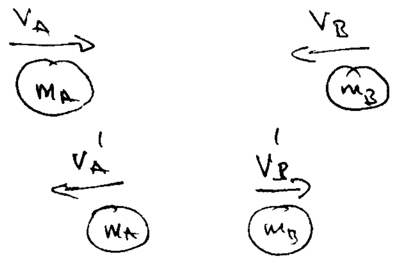
<u>Perfectly Plastic Impact</u>

$v_A' = v_B' = v$ the objects stick together.

$$v = \frac{m_A v_A + m_B v_B}{m_A + m_B}$$

Energy is <u>not</u> conserved but Linear momentum is.

Direct Central Impact



before

after

coefficient of restitution

$$\boxed{e = \frac{v_B' - v_A'}{v_A - v_B}}$$

, e relates the relative velocities before and after impact.

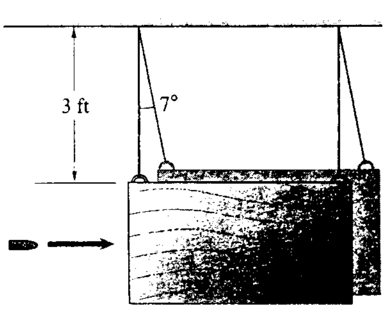
Energy is conserved during impact if $e=1$

$e=1$ perfectly elastic

$e=0$ perfectly plastic

Linear momentum is conserved for any e.

A 1-oz bullet moving horizontally hits a suspended 100-lb block of wood and becomes embedded in it. If you measure the angle through which the wires supporting the block swing as a result of the impact and determine it to be 7°, what was the bullet's velocity?



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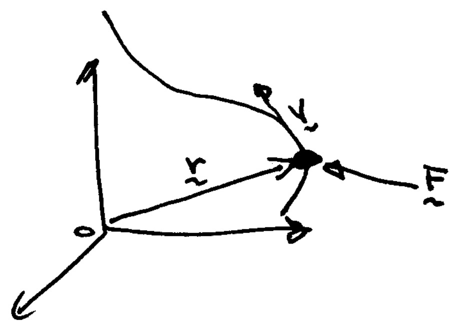
Angular Momentum (moment of the linear momentum)

Angular momentum $= \boxed{H_o = \underset{\sim}{r} \times m\underset{\sim}{v}}$



Moment due to $\underset{\sim}{F}$ w.r.t. o is

$M = \underset{\sim}{r} \times \underset{\sim}{F}$ but $F = \frac{d}{dt}(mv)$ so

$M = \underset{\sim}{r} \times \frac{d}{dt}(m\underset{\sim}{v}) = \frac{d}{dt}(\underset{\sim}{r} \times m\underset{\sim}{v})$

[since $\frac{d}{dt}(\underset{\sim}{r} \times m\underset{\sim}{v}) = \cancel{\left(\frac{d\underset{\sim}{r}}{dt} \times m\underset{\sim}{v}\right)}^{\to 0} + \underset{\sim}{r} \times \frac{d}{dt}(mv) = \underset{\sim}{r} \times \frac{d}{dt}(m\underset{\sim}{v})$]

Now call $\underset{\sim}{r} \times m\underset{\sim}{v} = H_o$ – angular momentum

so

$$M = \frac{d}{dt}(H_o) \quad \text{where } H_o = \underset{\sim}{r} \times m\underset{\sim}{v}$$

Integrating over time

$$\boxed{\int_{t_1}^{t_2} M\,dt = \int_{t_1}^{t_2} (\underset{\sim}{r} \times \underset{\sim}{F})\,dt = H_o\Big|_{t=t_2} - H_o\Big|_{t=t_1}}$$

For no moment $(\underset{\sim}{r} \times \underset{\sim}{F} = 0)$ we get

$$\boxed{H_o = \text{constant}}$$

conservation of angular momentum

The bar rotates *in the horizontal plane* about a smooth pin at the origin. The 2-kg sleeve C slides on the smooth bar, and the mass of the bar is negligible in comparison to that of the sleeve. The spring constant $k = 40$ N/m, and the unstretched length of the spring is 0.8 m. At $t = 0$, the angular velocity of the bar is $\omega_0 = 6$ rad/s, $r = 0.2$ m, and the radial velocity of the sleeve is $v_r = 0$.

What is the angular velocity of the bar when the spring is unstretched?