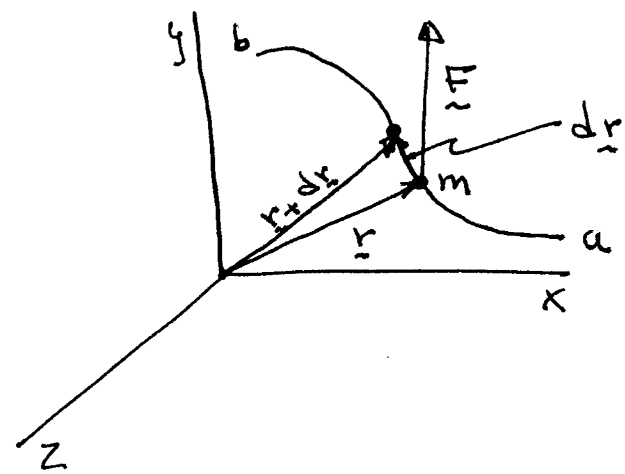
Handout 5 ME 230

Work and Energy

The work done by a Force $\underset{\sim}{F}$ is defined as

$$U = \int_{\substack{c \\ \text{path}}} \underset{\sim}{F} \cdot d\underset{\sim}{r}$$



For a particle

$$\underset{\sim}{V} = \frac{d\underset{\sim}{r}}{dt} \Rightarrow d\underset{\sim}{r} = \underset{\sim}{V} \cdot dt$$

so

$$U = \int_c \underset{\sim}{F} \cdot \underset{\sim}{v}\, dt \qquad (1)$$

if m is constant we know that

$$\underset{\sim}{F} = m \cdot \frac{dv}{dt} \qquad \text{this into (1) gives}$$

$$U = \int_c m \frac{dv}{dt} \cdot v\, dt = m \int_c \underset{\sim}{v} \cdot d\underset{\sim}{v}$$

& since $d(\underset{\sim}{v}^2) = d(v \cdot v) = d\underset{\sim}{v} \cdot \underset{\sim}{v} + \underset{\sim}{v} \cdot d\underset{\sim}{v} = 2(v \cdot d\underset{\sim}{v})$

so $U = \frac{m}{2} \int_c d(v^2) \Rightarrow \boxed{U = \frac{1}{2} m V_b^2 - \frac{1}{2} m V_a^2}$

where the path of integration extends from a to b along C.

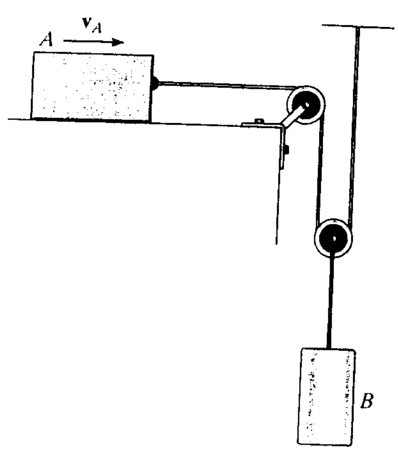
Let $T = \frac{1}{2} m v^2$ T = kinetic energy

For a particle of constant mass:

$$U = \Delta T = T_b - T_a$$

Ex1

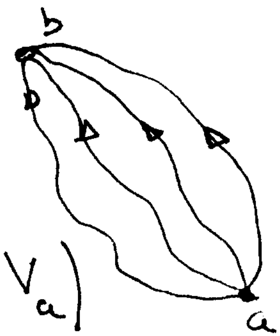
*14–16. The 3-lb block A rests on a surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. Determine the distance the 8-lb cylinder B must descend so that A has a speed of $v_A = 5$ ft/s starting from rest.



Conservative Forces

If the work done by force $\underset{\sim}{F}$ between a and b is independent of the path the force is said to be <u>conservative</u>.

In this case



$$U = \int_c \underset{\sim}{F} \cdot d\underset{\sim}{r} \overset{\nabla}{=} -\int dV = -(V_b - V_a)$$

V is the potential energy.

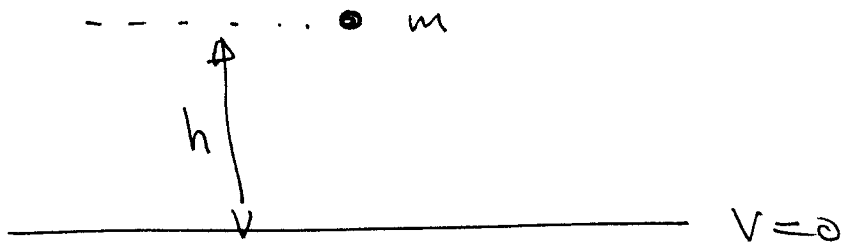
Combining the idea of kinetic and potential energy for a conservative force field:

$$U = T_b - T_a = -(V_b - V_a)$$

or

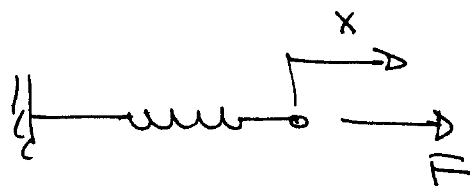
$$\boxed{T_b + V_b = T_a + V_a}$$

Gravity

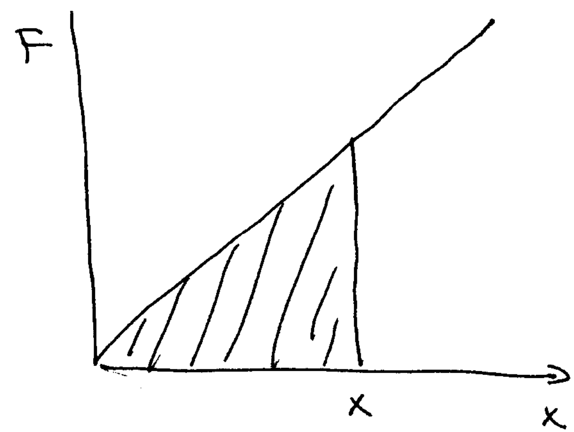


$$V = m \cdot g \cdot h$$

Spring



The area under the curve represents the potential energy



$V = \frac{1}{2} k x^2$ where x is measured from the unstretched length.

Ex2

A spider is suspended from the ceiling on a thread of negligible mass having a length *l*. Supposing that the thread is Linearly elastic with a spring coefficient k, and that l_o is the unstretched length, calculate the total work which the spider will have to expend to climb to the ceiling. Compare this with the work required to climb an inextensible tread of length *l*.

Ex. 3

A small hole is drilled through the earth along a diameter, and a ball of mass m is dropped from rest down the hole. Find the maximum velocity of the ball assuming a fixed, spherical, homogeneous, nonrotating earth.