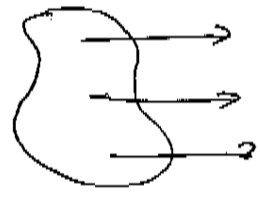
Chapter 18 Handout 11

Rigid Body in Planar Motion

$$T = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$ General plane motion

Translation only:

$$T = \frac{1}{2} m v^2$$

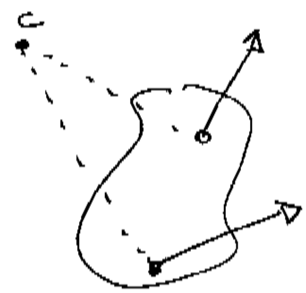


Fixed axis rotation:

$$T = \frac{1}{2} I_o \omega^2$$

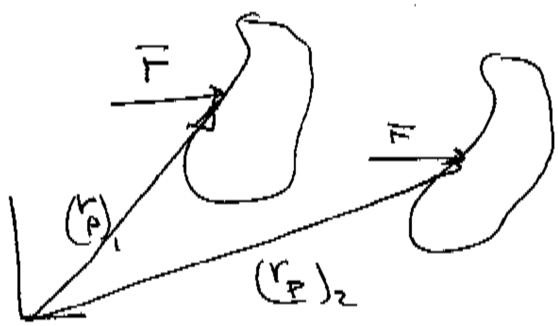
Instantaneous center of rotation

$$T = \frac{1}{2} I_c \omega^2$$



Work and Potential Energy

$$U = \int_{(r_P)_1}^{(r_P)_2} \underset{\sim}{F} \cdot d\underset{\sim}{r}$$



$$U = \int_{\theta_1}^{\theta_2} M \cdot d\theta$$

If all forces and moments are conservative then

$$T + V = \text{constant}.$$

Impulse-momentum equations

Linear momentum: $L = m \cdot v_G$

$$\int_{t_1}^{t_2} \Sigma F \, dt = m v_2 - m v_1$$

center of mass velocity

$m v_{cm}$ = constant if no external forces

Angular Momentum:

About center of mass

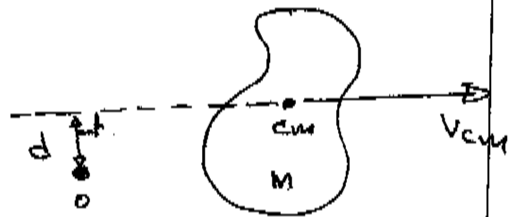
$$H_{cm} = I_{cm} \cdot \omega$$

$$\Sigma M_{cm} = \dot{H}_{cm} = I_{cm} \cdot \alpha$$

$$\boxed{\int_{t_1}^{t_2} \Sigma M_{cm} = H_{cm_2} - H_{cm_1} = I_{cm} (\omega_2 - \omega_1)}$$

About any point:

$$H_o = \underbrace{I_{cm} \cdot \omega}_{H_{cm}} + m V_{cm} \cdot d$$



[or $\quad H_o = I_{cm} \cdot \omega + (\underset{\sim}{r} \times m \underset{\sim}{v}_{cm}) \cdot \underset{\sim}{k}$]

$\underset{\sim}{r}$ is the position vector from o to cm.

$$\int_{t_1}^{t_2} \Sigma M_o \, dt = H_{o_2} - H_{o_1} = I_{cm} (\omega_2 - \omega_1) + m \left[(V_{cm} \cdot d)_2 - (V_{cm} \cdot d)_1 \right]$$

For no moments we have $\boxed{\Delta H_o = 0}$

ie: $I_{cm}(\omega_2 - \omega_1) + m \left[(V_{cm} \cdot d)_2 - (V_{cm} \cdot d)_1 \right] = 0$