3 Orthogonal Polynomials – due Monday, February 28

Write a program to compute

1. The orthogonal polynomials with respect to a discrete weighted inner product on a data set of x_i.
2. The expansion coefficients of a least squares polynomial approximation to a given function based on these polynomials and the data values $y_i = f(x_i)$.
3. The values at a given number of equally spaced points of any least squares polynomial based on these orthogonal polynomials.

The inputs to the overall program should be n, w, x, y, m, d where n is the number of data points, w is an n dimensional vector of weights, the data points are $(x_i, y_i),\ i = 1, \ldots, n,\ m+1$ is the number of evaluation points and d is the maximum degree of the set of orthogonal polynomials. The subprogram for (1) should generate polynomials $q_j(x), j = 0, \ldots, d$ which are orthogonal with respect to the inner product $< f, g >= \sum_1^n w_i f(x_i) g(x_i)$. These polynomials should be calculated according to the three term recurrence relation described on pages 268-270. The subprogram (2) should compute the “Fourier” coefficients $c_k =< y, q_k > / < q_k, q_k >$ of the “function” y. And the subprogram (3) should calculate the values of the approximation $p(x) = \sum_0^d c_k q_k(x)$ at $m+1$ equally spaced points in the interval $[x_1, x_n]$.

Run your program with the following data and plot the approximation p.

$n = 11,\ m = 100,$
$w = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1),$
$x = (-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5),$
$y = (3, 2, 3, 5, 3, 4, 3, 2, 2, 3, 2),$
$d = 2, 5, 10,$ (one run for each value of d)

Also run and plot the approximations when $y = 1/(1+x^2)$. (The $d = 10$ case is a polynomial interpolant (Why?) and should remind you of one of the polynomial interpolants plotted last quarter.)