

Sample Problems

Math 464

One notebook sized page of notes will be allowed on the final. You may write on both sides of the page. Calculators will be allowed. This set of problems only covers material since the midterm. The final will be comprehensive. There will be questions covering the first half and second half of the course.

1. Let $Q(f) = af(\frac{1}{6}) + bf(\frac{1}{2}) + cf(\frac{5}{6})$ be a quadrature formula on $[0, 1]$.
 - a) If Q is to have precision at least 2, what are the values of a, b, c ?
 - b) What is the maximum precision of the resulting quadrature?
2. Let $f(x) = \sin(x)$ and let $T_N(f), S_N(f)$ be the composite trapezoid and Simpson rules on N panels on $[0, 2]$.
 - a) Compute $S_2(f)$ and $T_2(f)$.
 - b) Use the error *bound* to find the smallest N so that $|e_N^S(f)| \leq 10^{-6}$.

3. Let

$$g(x) = \begin{cases} x^3 & \text{if } 0 \leq x \leq 1 \\ a(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1 & \text{if } 1 \leq x \leq 2 \end{cases}$$

- a) Is $g(x)$ a cubic spline on $[0, 2]$? Justify your answer.
 - b) Is $xg(x)$ a cubic spline on $[0, 2]$? Justify your answer.
 - c) What should the value of a be if g is to be a natural cubic spline?
4. a) What points x_0, x_1, x_2, x_3 should be used in $[0, 1]$ to minimize the error bound in polynomial interpolation at four points? Express your answer using the symbols $1, 2, \pm, \sqrt{}$. Hint: Use $\cos x = \sqrt{\frac{1 + \cos 2x}{2}}$.
b) What is the largest value attained by $(x-x_0)(x-x_1)(x-x_2)(x-x_3)$ in $[0, 1]$, where x_0, x_1, x_2, x_3 are the points chosen in part a)?

Be careful in this problem, $[0, 1]$ is not the interval $[-1, 1]$.

5. a) Using your calculator, solve $e^{-x} = x$ by means of Newton's method. Let $x_0 = 0$. Compute x_1, x_2, x_3, x_4, x_5 .

- b) If you were able to compute in infinite precision, for which n would x_n have 100 correct digits?
6. a) Let $p(x) = 2x^4 - 3x^3 + 4x^2 - 2x + 1$. Use *nested multiplication* to evaluate $p(2)$. Show the multiplications.
- b) Use nested multiplication to evaluate $p'(2)$. Show the multiplications. Do not take the derivative of $p(x)$ and then use the polynomial $p'(x)$ to evaluate $p'(2)$.
7. Let $p_3(x)$ be the unique polynomial of degree at most 3 that interpolates $f(x) = 2^{x+2}$ at the points $(x_0, x_1, x_2, x_3) = (-2, -1, 1, 2)$. Notice that 0 is not one of the points.
- a) Form the divided difference table for f at these points and write $p_3(x)$ in its Newton form.
- b) Give the best upper bound for $\|f - p_3\|_\infty$ on $[-2, 2]$.