Math 464 Midterm

October 29, 1999

Name:____

One notebook sized page of notes is allowed on the test. Scientific calculators in which no formulas or text has been stored are allowed.

1. (15 pts) You are given only the following information:

$$\sin .202 = .2006, \ \sin .200 = .1987$$
$$\sin .001 = .001, \ \cos .201 = .9799$$

Using only this information, compute $\sin .202 - \sin .200$, in two different ways. Which computation do you think is more accurate? Why? You should *not* consult the cosine or sine function on your calculator for this problem.

2. (25 pts) Consider the continued fraction

$$s = \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}}$$

(a) Show that s is the fixed point of some function g.

(b) Find s. (Express s as a rational combination of integers and square roots of integers, not as a decimal.)

(c) Show that $|g'(x)| < \dfrac{1}{4}$ for all $x \geq 0$.

(d) Let x_n be the fraction with n 2's. Show that

$$|x_{n+1} - s| < \frac{|x_1 - s|}{4^n}$$

and hence the sequence x_n converges to s.

3. (40 pts)

(a) Find the $A = LU$ factorization of

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

by using Gaussian elimination. L should have 1's on the main diagonal. Write out the matrices L, U explicitly.

(b) Use part a) to solve:

$$Ax = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

(c) Using the LU factorization, compute $\det(A)$.

(d) Either using Cramer's rule or Gaussian elimination, compute A^{-1}.

(e) Compute $\kappa_\infty(A)$, the condition number of A in the infinity norm.

(f) Suppose the computed solution to the equation in part (b) is $x_c = [-.5, 2, .25]^T$. Use $\kappa_\infty(A)$ to find upper and lower bounds on $\dfrac{||x_c - x_t||_\infty}{||x_t||_\infty}$.

You may use your calculator on part (e). The other parts should be done by hand and your answers should be rational numbers.

4. (20 pts)

   (a) Can Jacobi or Gauss-Seidel be used to solve the equation in part (b) of problem 3?

   (b) Let

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}$$

   Compute M_G, the Gauss-Seidel matrix of A.

   (c) Compute $||M_G||_\infty$ and $||M_G||_1$. Does the Gauss-Seidel iteration for A converge?