Math 324 A&B **Final Review** **Winter 2002(Summer 2003 also)**

(a) All of the review for Chapter 16, concept check, true-false and problems.

(b) Here's a "sample" exam. (Probably too long for the summer 1 hour period. But you must be able to work these types of problems).

1. This problem is based on problem 48 from section 16.7 of the text.

   A torus is the (donut-shaped) surface obtained by rotating a circle about an axis that does not intersect the circle. Let S be the torus given by

   $$\mathbf{r}(\theta, \varphi) = \Big\langle (2 + \cos\varphi)\cos\theta,\ (2 + \cos\varphi)\sin\theta,\ \sin\varphi \Big\rangle.$$

   where $0 \leq \theta \leq 2\pi$ and $0 \leq \varphi \leq 2\pi$. The surface area of S is given by

   $$\int\!\!\int_S dS.$$

   Compute the surface area of the torus.

2. Let S consist of those points (x, y, z) of the unit sphere such that $y \geq 0$ and $z \geq 0$.

   (a) Draw a picture of S.

   (b) Define and draw a unit normal field on S.

   (c) Give parametric equations for ∂S. Be sure to use the Stokes's orientation. Also, draw arrows on your picture indicating the Stokes's orientation on the boundary.

3. Consider the vector field

   $$\mathbf{F}(x, y, z) = \Big\langle yze^{xyz} + 1,\ xze^{xyz} + 2,\ xye^{xyz} + 3 \Big\rangle.$$

   (a) Is $\mathbf{F}$ conservative? If so, find a potential function. If not, explain why not.

   (b) Compute the integral

   $$\int_C \mathbf{F} \cdot d\mathbf{r}$$

   where C is the oriented curve parameterized by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle, 0 \leq t \leq 1$.

4. Let S denote the portion of $x = 1 - y^2 - z^2$ such that $x \geq 0$. Let S be oriented by the unit normal field

   $$\mathbf{n}(x, y, z) = \frac{\langle -1, -2y, -2z \rangle}{\sqrt{1 + 4y^2 + 4z^2}}.$$

   compute

   $$\int\!\!\int_S \operatorname{curl}\langle x, -z, y \rangle \cdot d\mathbf{S}.$$

5. Let C be the square with vertices $(1,1)$, $(-1,1)$, $(-1,-1)$, and $(1,-1)$ oriented in that order. Compute

$$\int_C \langle x\ln(x^2+y^2), -y\ln(x^2+y^2)\rangle \cdot d\mathbf{r}.$$

6. (Chap 16 review problem 36) Let $\mathbf{x} = \langle x, y, z\rangle$. Compute the outward flux of

$$\mathbf{F}(\mathbf{x}) = \mathbf{x}/|\mathbf{x}|^3$$

through the ellipsoid $4x^2 + 9y^2 + 6z^2 = 36$. (The answer is NOT 0.)