This part of your final project consists of specific problems relating mostly to the material in chapter 3. Each group should prepare solutions to the following 3 problems. As usual, the write up should give **solutions** not just answers.

DUE: On the last day of class - June 6, 2003. (Requests for an extension to the day of final will be granted, but you must request it on your Part I submission.)

Each group will submit one copy of its (revised) final report on **part 1** of the project **and** one copy of its **solutions to these problems**.

You may certainly consult the text, all handouts and your class notes when solving these problems. **Do not consult** any other texts or other resources originating outside of the Math 308 contexts, whether animate or inanimate.

Contact me (543-1148 or 324-3801) if you have any questions.

Each member of your group *should* **bring a copy of your groups Part II solutions to the final exam**; References to it will save time and writing in solving some of the final exam problems. Note this copy is in addition to the pages of notes allowed, while working on the final.

Credit for the project will place 50 points on this part and 50 points on the first part. With each problem or numbered investigation receiving approximately equal weight. Altogether the project will contribute 10% to your final average, as indicated on the initial information sheet.

Each of questions 1-3 below relates to the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$.

1. Find the characteristic polynomial for A (in factored form if possible). (If you provide a factorized polynomial be sure your solution shows how that factorization was obtained.)

2.(a) Show that $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector for A above.

(b) Find $\mathbf{v}_2$, $\mathbf{v}_3$ such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an **orthogonal basis** for $\mathbf{R}^3$ and each $\mathbf{v}_i$ is an eigenvector for A.

3. Find functions $u(t)$, $v(t)$ and $w(t)$ which solve the initial value problem

$$\begin{aligned} u'(t) &= u(t) + v(t) - w(t), & u(0) &= 1 \\ v'(t) &= u(t) + v(t) + w(t), & v(0) &= -3 \\ w'(t) &= -u(t) + v(t) + w(t), & w(0) &= 2 \end{aligned}$$

whose coefficient matrix is A. (The ’ in $u'(t)$ represents the derivative of $u(t)$.) Your solution should use facts from this course; material from differential equations is not required. Specifically the discussion in pp. 347-349 of JRA shows how the material of this course leads to a solution to this problem.

4. Calculate the dimension of the vector space of all solutions to

$$\begin{aligned} x_1 - 3x_2 + 4x_3 + 2x_4 - x_5 &= 0 \\ 2x_1 - 6x_2 + 6x_3 + 4x_4 \phantom{{}-2x_5} &= 0 \\ x_1 - 3x_2 + 5x_3 + 2x_4 - 2x_5 &= 0 \end{aligned}$$

Be sure to explain how you know your answer is the dimension requested. An explanation based on the definition of *dimension* is preferred.