NAME **SAMPLE**

MATH 308 — SAMPLE FIRST EXAM — April 13, 1990

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Closed book exam, except one 8×11 sheet of notes may be consulted. There are 8 questions, with a total of 100 points. **To** *receive full credit* **you must show all your work and give reasons. You may point to and use the results (work) for any problem in solving (explaining) any later problem.**

Let $A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 3 & 6 & 4 & 2 \\ 0 & 0 & 4 & -3 \end{bmatrix}$ and let $\mathbf{a}_i$ denote the i'th column of A. Each of problems 1-3 concern these objects.

1.(10 pts.) If A is the **coefficient** matrix of a linear system, how many *unknowns* does that system have? (Explain.)
If A is the **coefficient** matrix of a linear system,

2.(15 pts.) Find a **reduced echelon** matrix which is row equivalent to A.

3.(10 pts.) If $B = [\,\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\,]$, how many solutions does the **homogeneous** matrix equation $B\mathbf{x} = \theta$ have? (Why?) (You may, of course, refer to your solution to #2 if that can help explain.)

4.(15 pts.) Suppose a square matrix J satisfies $J^2 = 4J$, but $J \neq 4I$. Explain why J **must be singular**.

5.(10 pts.) Suppose the (100×2) matrix $V = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix}$ satisfies $V\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \theta$.

Explain why { $\mathbf{u}_1$, $\mathbf{u}_2$ } is **dependent**, when $\mathbf{u}_1 = \mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{u}_2 = -\mathbf{v}_1 + \mathbf{v}_2$.

6.(15 pts.) Let $F = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 1 & 6 \\ 0 & 0 & 2 \end{bmatrix}$. Find F^{-1}, the inverse of F.

7.(10 pts.) Let $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, and $N = \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix}$.

a) Which product MN or NM is defined? (Explain.)

b) Calculate the entry in the 2^{nd} row and 1^{st} column of the product defined in a).

8.(15 pts.) Suppose a matrix C is changed by the following elementary row operations, $R_2 - 2R_1$, $R_3 + R_1$, $R_3 + 3R_2$ into the matrix F as represented in the following diagram:

$$C \xRightarrow{R_2-2R_1} D \xRightarrow{R_3+R_1} E \xRightarrow{R_3+3R_2} F = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find C and find all $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ with $C\mathbf{x} = \theta$.

Note for Spring 1995: Our exam may also cover some of the application areas covered in Johnson/Reiss/Arnold (1.5 and 1.9).