Scores by question							Total

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MATH 308 **SAMPLE Second Exam** February 26, 1992

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Closed book exam, except one 8×11 sheet of notes may be consulted. There are 7 questions, each worth 15 points. To *receive full credit* you must show all your work and give reasons. You may use your answer to any question to help answer any later question, if it is relevant.

Here is a sequence of row operations which change a matrix A into a reduced row echelon form.

$$A = \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 2 & 4 & 5 & 6 & 8 \\ 3 & 6 & 6 & 9 & 9 \end{bmatrix} \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 0 & 0 & -3 & 0 & -6 \\ 0 & 0 & -6 & 0 & -12 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 0 & 0 & -3 & 0 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{-1}{3} R_2} \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 - 4R_2} \begin{bmatrix} 1 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Use A and the information above to help answer questions 1-4.

1. Compute **nullity**(A). (As always your written work must "justify" your answer.)

2. Is $\bar{\mathbf{v}}$ = [3 6 2 9 -1] in the **row space** of A? Why?

Here's the information on A repeated from page 1.

$$A = \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 2 & 4 & 5 & 6 & 8 \\ 3 & 6 & 6 & 9 & 9 \end{bmatrix} \overset{====>}{\substack{R_2 - 2R_1 \\ R_3 - 3R_1}} \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 0 & 0 & -3 & 0 & -6 \\ 0 & 0 & -6 & 0 & -12 \end{bmatrix} \underset{R_3 - 2R_2}{====>} \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 0 & 0 & -3 & 0 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underset{\frac{-1}{3} R_2}{====>} \begin{bmatrix} 1 & 2 & 4 & 3 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \overset{R_1 - 4R_2}{====>} \begin{bmatrix} 1 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Either write $A_5 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ as a linear combination of the other columns of A or explain why that is impossible.

4. Find an **orthogonal basis** for $R(A)$, the *range* of A.

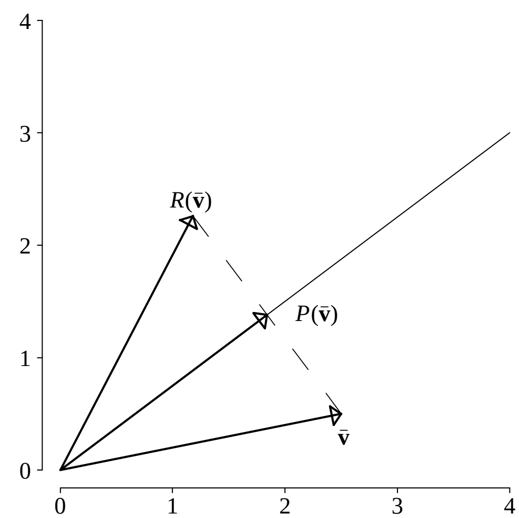
5. In both parts of this question, $T: \mathbf{R}^2 \to \mathbf{R}^2$ is a linear map with $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

a) Find $T(\bar{\mathbf{e}}_1)$.

b) Find the matrix for T, i.e. find a matrix B with $T(\bar{\mathbf{x}}) = B\bar{\mathbf{x}}$ for all $\bar{\mathbf{x}}$ in $\mathbf{R}^2$.

6. Suppose C is a (10×6) matrix with nullity(C) = 6. Can $C\bar{\mathbf{x}} = \bar{\mathbf{b}}$ be solved for *all* $\bar{\mathbf{b}}$. (As always - explain your answer.)

7. Let $R:\mathbf{R}^2 \rightarrow \mathbf{R}^2$ be symmetric or perpendicular **reflection** in the line l through θ and $\bar{\mathbf{u}} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$. (For any $\bar{\mathbf{v}}$, its' image, R($\bar{\mathbf{v}}$) is such that P($\bar{\mathbf{v}}$) is the mid-point of the line segment from $\bar{\mathbf{v}}$ to R($\bar{\mathbf{v}}$), where P is the perpendicular *projection* onto l.)



a) Use the mid-point property to find a,b with R($\bar{\mathbf{v}}$) = aP($\bar{\mathbf{v}}$) + $b\bar{\mathbf{v}}$.

b) Compute R($\begin{bmatrix} 3 \\ 0 \end{bmatrix}$). (You should use the formula for P($\bar{\mathbf{v}}$) involving dot-products and part 7a.)