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MATH 308 Winter 1992 FINAL EXAM

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Closed book exam, except one 8×11 sheet of notes. There are 10 questions, each worth 20 points. To receive *full credit* you must **show all your work and give reasons.**

For problems 1 and 2 let
$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$
.

1. Find the **characteristic polynomial** of A.

2. Find a basis for \mathbb{R}^3 consisting of eigenvectors for A.

For questions 3-4 let *B* be the coefficient matrix for the system of equations:

$$x_1 + 2x_2 + x_3 - 3x_4 = 0$$

$$3x_1 + 6x_2 + x_3 - 9x_4 = 0$$

$$-2x_1 - 4x_2 + 2x_3 + 6x_4 = 0$$

3.i. Find **reduced echelon** matrix which is row equivalent to *B*.

3.ii. Find all \mathbf{x} in \mathbf{R}^4 which solve $B\mathbf{x} = \mathbf{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

4.i. Let B^T be the **transpose** of the matrix defined at the top of the previous page. Compute $rank(B^T)$.

4.ii. Is the *nullspace* of B **orthogonal** (i.e. perpendicular) to the *range* of B^T ? Why or why not?

5. Let $C = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 1 & 6 \\ 0 & 0 & 2 \end{bmatrix}$. Find C^{-1} , the inverse of C.

6. Let $D = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$. Find two independent eigenvectors for D.

7. Suppose E is a (2×2) matrix with $E\begin{bmatrix} 1\\1\end{bmatrix} = 2\begin{bmatrix} 1\\-1\end{bmatrix}$, and $E\begin{bmatrix} 1\\-1\end{bmatrix} = -2\begin{bmatrix} 1\\1\end{bmatrix}$. Compute $E^{6}\begin{bmatrix} 1\\0\end{bmatrix}$.

8. Let $P(\mathbf{v})$ be the perpendicular projection of \mathbf{v} onto the line through θ and $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Find the matrix F with $F\mathbf{v} = P(\mathbf{v})$ for all \mathbf{v} in \mathbf{R}^2 .

9. Find the linear combination of $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{w}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ which is **closest** to $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

10. Find the linear polynomial y = p(t) = a + bt which interpolates the data:

t	1	2
У	4	3