



For questions 3-4 let  $B$  be the coefficient matrix for the system of equations:

$$\begin{aligned}x_1 + 2x_2 + x_3 - 3x_4 &= 0 \\3x_1 + 6x_2 + x_3 - 9x_4 &= 0 \\-2x_1 - 4x_2 + 2x_3 + 6x_4 &= 0\end{aligned}$$

3.i. Find **reduced echelon** matrix which is row equivalent to  $B$ .

3.ii. Find all  $\mathbf{x}$  in  $\mathbf{R}^4$  which solve  $B\mathbf{x} = \boldsymbol{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .

4.i. Let  $B^T$  be the **transpose** of the matrix defined at the top of the previous page. Compute **rank**( $B^T$ ).

4.ii. Is the *nullspace* of  $B$  **orthogonal** (i.e. perpendicular) to the *range* of  $B^T$ ? Why or why not?

5. Let  $C = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 1 & 6 \\ 0 & 0 & 2 \end{bmatrix}$ . Find  $C^{-1}$ , the inverse of  $C$ .

6. Let  $D = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$ . Find two independent eigenvectors for  $D$ .

7. Suppose  $E$  is a  $(2 \times 2)$  matrix with  $E \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ , and  $E \begin{bmatrix} 1 \\ -1 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ . Compute  $E^6 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ .

8. Let  $P(\mathbf{v})$  be the perpendicular projection of  $\mathbf{v}$  onto the line through  $\theta$  and  $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ . Find the matrix  $F$  with  $F\mathbf{v} = P(\mathbf{v})$  for all  $\mathbf{v}$  in  $\mathbf{R}^2$ .

9. Find the linear combination of  $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  and  $\mathbf{w}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  which is **closest** to  $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ .

10. Find the linear polynomial  $y = p(t) = a + bt$  which interpolates the data:

t	1	2
y	4	3