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Lecture and Assignment Schedule

Spring 2004
MATH 308

Lecture	Topics	Problems	Due Date	Optional extras

Mar 29	JRA 1.1-2
	• Systems of Linear Equations
	  ○ Solve by Gauss (forward) elimination of variables moving from left to right, top to bottom and back elimination, from right to left, bottom to top.
	  ○ Represent by (augmented) matrices
	• Equivalence of systems and row equivalence of matrices:
	  ○ Changes to equations or rows which don't change the solutions of the associated system
	  ○ Swap, scale, add multiple of one to another.
	• Echelon Form matrices: What the associated matrix looks like after elimination
		• Read the supplement and errata for the text before working the problems.
		• Complete the web based student information form before 9AM Fri Apr. 2
		• 1.1, 1(Why? Refer to (1),p.2), 5(Why?), 11, 12, 23, 27, 35, 38
		• 1.2, 37, 39,
			Mar. 31	• 1.1, 13, 19, 24, 30, 31, 33, 39
				• 1.2, 36, 39
				• Read Appendix A1-5 and start using Matlab.

Mar. 31	More on JRA 1.2 • Reduced Echelon Form(RREF) and Gauss Elimination • Solving a system whose associated matrix is in RREF • Systems with no solutions: inconsistent systems of linear equations. • General solution to linear system involves *unconstrained* and *constrained* variables.	• 1.2, 1, 5, 15, 21, 23, 31, 33, 41, 50, 53, 56(In 15, 21 "**explain why**" not "state that", i.e. give a solution not just an answer)	Apr. 2	• 1.2, 11, 12, Transform 18 into RREF, 29, 43, 45, 51, 54 • A2.1,A3.1,A5.1,2 Note a Matlab "diary" print out of A3.1 would constitute a solution to 1.2, 30, 31

Apr. 2	• Consistent Systems of Linear Equations.
	  ○ Structure of (RR)EF for augmented matrix of system with solutions
	  ○ Fewer non-zero rows r in (RR)EF than variables n in equations
	  ○ Fewer equations than unknowns means either inconsistent or infinitely many solutions
	  ○ Homogeneous equations, RHS all 0, are consistent
		• 1.3, (In the question for Exercises 1-4 replace "independent" by "unconstrained" in both occurances) 1, 3, 5, 8, 9, 12, 13, 15, 19, 21(Why?), 23(Why?), 25, 32	Apr. 5	• 1.3, 7, 11, 16, 27, 28,

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Apr. 5	JRA 1.4
	• NetworksApplications:
	  ○ Electrical Networks
	  ○ Traffic flow
		• 1.4, 1(See errata for error in book answer), 6, 9,	Apr. 7	• 1.4, 2, 5, 10

	◦ **Quiz 1 on 1.1-1.2**			
Apr. 7	JRA 1.5 • Matrix Operations ◦ Sums and scalar multiples ◦ Vector form of general solution ◦ R^n : A vector space ◦ Matrix times vector $A\mathbf{x}$, just like coefficients times variables in equations ◦ Matrix times matrix $AB = A[\mathbf{B}_1 ... \mathbf{B}_n] = [A\mathbf{B}_1 .... A\mathbf{B}_n]$ where $\mathbf{B}_i$ are columns of B	• 1.5, 1(c,d), 5, 13, 17, 43, 44, 45, 47, 49 (In 13,17 "**show**" not "state") • 1.5, 9, 21, 31, 52, 55, 56, 57(Also decide which of the two calculations - P(Px) and (PP)x requires more work/multiplications), 60, 63	Apr. 9	• 1.5, 35, 53, 58,59, 61, 67, 66, 68, 70, 71
Apr. 9	JRA 1.6 • Properties of Matrix Operations ◦ Grouping and order of terms do not matter for matrix addition ◦ Grouping does not matter for matrix multiplication ◦ Order **matters** for matrix m**ultiplication** ◦ Transposes and symmetry ◦ Powers ◦ $(AB)^T = B^T A^T$ ◦ Identity $I=[\mathbf{e}_1 ... \mathbf{e}_n]$ ◦ Can you cancel: When does $AB=AC$ yield $B=C$? (Think about unique solutions!) ◦ Size (norm) of vectors	• 1.6, 3, 7, 15, 27, 31, 33, 35, 48, 57	• Apr. 12	• 1.6, 1, 11, 13, 21, 24,28, 30, 32, 41, 50, 60, 62(b)

Lecture	**Topics**	**Problems**	**Due Date**	**Optional extras**
Apr. 12	JRA 1.7. • Linear Independence and Non-singular matrices ◦ Linear combinations ◦ Zero vectors ◦ Linear independence ◦ Non-singular square matrices ◦ Unit vectors ◦ Recognizing dependent sets ◦ p vectors in m space, $p>m$, are dependent	• 1.7, 1, 2, 9, 17, 24, 27, 35, 47, 50,	Apr. 16	• 1.7, 6, 9, 18, 22, 25, 53, 55, 58
Apr. 14	JRA 1.8 Applications: ◦ Data fitting ◦ Numerical Integration	• 1.8, 6, 12, 19	Apr. 16	• 1.8, 1, 7, 8, 9, 10, 11, 13, 25, 27 31, 34
Apr. 16	JRA 1.9 Matrix inverses • Non-singular matrices and unique solutions. Theorem 13 in 1.7	• 1.9, 3, 7, 11, 19, 22, 38, 41, 54, 58, 68, 70, 73(Add the word "singular" just before "matrix" in 73)	Apr. 23	• 1.9, 1, 6, 17, 25, 27, 33, 50, 52, 67, 72, • Supplemental and conceptual Exercises.

	- Definition - Calculating inverses - Uses of inverses - Existence of inverses - Properties (Thm. 17) including $(AB)^{-1}=B^{-1}A^{-1}$ (While important, omit Ill-conditioned matrices)			Matlab exercise #1, p. 108 extends 1.5.57.

Lecture	Topics	Problems	Due Date	Optional extras
Apr. 19	Quiz on 1.3-1.8 JRA 3.1 and 3.2 - Geometric vectors in R^2 and R^3 and their algebraic properties: - Addition - Scalar multiplication - Subsets defined by - Geometric properties - Linear or non-linear equations Intro to 3.2 - Algebraic properties of *n*-tuples of numbers	3.1, 5, 7, 8, 19, 23, 25, 27	Apr. 23	
Apr. 21	JRA 3.2 - Vector space properties of R^n - Zero vector - Sums - Scalar multiples - Order of summing doen't matter - Grouping of summands doesn't matter - Subspaces: Subsets which contain the zero vector and all sums and scalar multiples of vectors in the set.	3.2, 1, 7, 9, 11, 15, 18, 19, 28, 30, 32 ("union", $U \cup V$, means "all in U or V")	Apr. 23	
Apr. 23	JRA 3.3 - Examples of Subspaces - Span of a subset - all linear combinations of vectors in subset. Smallest subspace containing subset - Null space of a matrix A - all x which solve homogeneous equation Ax=0 - Range of a matrix A: all y for which the equation Ax=y has some solution.	3.3, 15, 19, 21(a,b,c), 25, 35, 40	Apr 30	

Lecture	Topics	Problems	Due Date	Optional extras
Apr. 26	More on JRA 3.3, Subspace examples. Column space of a matrix A - Sp({columns of A}) = *R*(A)	3.3, 33, 37, 47, 50, 51, 52, 53	Apr 30	

	• Row space of a matrix A -- the span of the rows of A. ◦ Row equivalent matrices A and B have the same row space.			
Apr 28	JRA 3.4 Bases for Subspaces • Spanning set for a subspace W: Sp(S)=W • Minimal spanning set: leave out any vector and it no longer spans • Basis for W is a linearly independent spanning set. • Standard basis for R^n: $\mathbf{e}_1, ..., \mathbf{e}_n$, $\mathbf{e}_i$ has 1 in i'th row. • Coordinates with respect to a basis	3.4, 1, 9(b,c), 11, 23(a), , 28, 31(Alternate hint: Use Theorem 18, 1.9), 36, 37	Apr 30	3.4, 7, 9(a,d), 38, 39
Apr 30	JRA 3.5 Dimension: Number of vectors in a basis. • Any set of p+1 vectors in a subspace spanned by p vectors is linearly dependent • Any two bases for the same space have the same number of vectors. • rank(A)=dim($R(A)$), nullity(A) = dim($N(A)$) • rank(A)+nullity(A)=#columns of A	3.5, 5, 9, 13, 17, 23, 27(a), 31 34, 35, 38,	May 7	3.5, 1, 3, 7, 18,27(b), 29

Lecture	Topics	Problems	Due Date	Optional extras
May 3	Quiz on 1.9,3.1-3.4 Continuation of 3.5	Previous assignment		
May 5	JRA 3.6 Orthogonal bases. • Define orthogonal (perpendicular) $\mathbf{x}^T\mathbf{y} = 0$ • Set of mutually orthogonal non-zero vectors is independent. • Orthogonal bases: ◦ Finding coordinates is easy - take dot product, i.e. to solve $\mathbf{b}=x_1\mathbf{u}_1 + ... + x_p\,\mathbf{u}_p$ just use orthogonality to get $x_i = \mathbf{u}_i^T\mathbf{b}/\mathbf{u}_i^T\mathbf{u}_i$ ◦ Construction by orthogonalization (based on orthogonal projection)	3.6, 2, 5, 10, 13, 20, 21, 22, 23, 28	May 7	3.6, 1, 8, 9, 14, 15, 24-26
May 7	JRA 3.7: Linear Transformations • Function T:V-->W with ◦ $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ ◦ $T(a\mathbf{u}) = a\,T(\mathbf{u})$ • Main example: Multiplication by a matrix A. $T_A(\mathbf{x}) = A\mathbf{x}$ • Geometric examples: Orthogonal projection on a subspace W with an orthogonal basis $\{\mathbf{u}_1,\mathbf{u}_2,...\mathbf{u}_p\}$,	3.7, 2, 6, 7, 8, 10, 18, 19, 20(Hint:First find the matrix A of T by solving the equation $A[\mathbf{v}_1\mathbf{v}_2]=[\mathbf{u}_1\mathbf{u}_2]$), 29, 34, 45(b), 46(b)	May 14	3.7, 1, 3, 4,5, 11, 12, 15, 17, 23, 25, 33

	$T(\mathbf{b})=x_1\mathbf{u}_1+...+x_p\mathbf{u}_p$ with $x_i = \mathbf{u}_i^T\mathbf{b}/\mathbf{u}_i^T\mathbf{u}_i$. Also: Rotations and reflections. • Matrix of a lin. trasformation $T:\boldsymbol{R}^n \text{-->} \boldsymbol{R}^m$, $[T]=[T(\mathbf{e}_1)...T(\mathbf{e}_n)]$, i.e. column *i* of [T] is $T(\mathbf{e}_i)$	

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May 10	JRA 3.8: Least-Squares Solutions to inconsistent systems. • Given a matrix A and a vector b, the a least squares solution to $A\mathbf{x} = \mathbf{b}$ is any $\mathbf{x}^*$ for which $\|\|A\mathbf{x}^*-\mathbf{b}\|\|$ < or = $\|\|A\mathbf{x}-\mathbf{b}\|\|$ for all $\mathbf{x}$. • Geometrically: Drop a perpendicular from **b** to the $R(A)$, then the coefficients of the linear combination of the columns of A at the base of the perpendicular give the least squares solution • x* solves the equation $A^TA\mathbf{x}=A^T\mathbf{b}$ (or more geometrically, $A^T(A\mathbf{x}-\mathbf{b})=0$) as these equations express that each column of A is perpendicular to $A\mathbf{x}-\mathbf{b}$ • Examples and applications: low degree polynomial fits to lots of data, linear fits to more than 2 data points, quadratic fits to more than 3 data points	3.8, 1, 3, 7, 9, 11, 12	May 14	3.8, 17,
May 12	JRA 3.9 Theory and pratice of least squares. How and why the middle two bullets from May 10 work.	3.9, 1, 3, 8, 11, 16,	Do but not to hand in. Solutions available May 14	Ch. 3 Supplement. and Conceptual exercises 3.9, Matlab Exercises, p.270, 1
May 14	The Eigenvalue Problem - JRA 4.1 • *Eigenvalue* for A: scalar λ with $A\mathbf{x}= \lambda\mathbf{x}$ for some vector $\mathbf{x}\neq\boldsymbol{\theta}$ • *Eigenvector* for A: $\mathbf{x}\neq\boldsymbol{\theta}$ such that $A\mathbf{x}= \lambda\mathbf{x}$ i.e. Directions **x** such that multiplication by A is just a "stretching" (multiplication) by a scalar λ.	4.1, 1, 3, 7, 15, 17, 19	May 21	4.1, 18,

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May 17	Quiz on 3.4-3.9 JRA 4.2	4.2, 2, 7, 9, 17, 24, 25, 27, 29, 34	May 21	4.2, 11, 15, 26, 28, 30
May 19	4.3	4.3, 3, 7, 9, 13, 16, 19, 23, 26	May 21	4.3, 14, 17, 25

May 21	4.4	4.4, 5, 7, 11, 15, 18, 24, 27	May 26	4.4, 26, 28, 29
Hint: A characteristic polynomial $\pm t^n + a_{n-1} t^{n-1} + ... + a_0$ with integer coefficients can only have an integer root p when b is an integer divisor of a_0, the constant term. Use this to find roots when the degree is 3 or 4 by examining the factors of a_0 and using long division or synthetic division to check if $t \pm p$ divides the polynomial.				
Lecture	**Topics**	**Problems**	**Due Date**	**Optional extras**
May 24	4.5	4.5, 1, 4, 11, 13, 14, 23 , 24, 28, 29	Do but do not hand in. Solutions available May 26	
May 26	4.6	4.6, 21, 23, 25, 29, 37, 39,	Do, but do not hand in. Solutions available May 27	4.6, 1, 5, 7, 13, 17
May 28	4.7 • **Project part I(Preliminary) due** • **Quiz on 4.1-4.6**	4.7, 1, 3, 4, 10, 14, 15, 25, 26, 27	Do not hand in. Sol on Jun 1	4.7, 38-41
Lecture	**Topics**	**Problems**	**Due Date**	**Optional extras**
May 31 -- HOLIDAY				
June 2	4.8	4.8, 1, 3, 7, 9, 15	Sol on June 3	4.8, 23-26
June 4	Review • **Final quiz on 4.6-7** • **Final project due**	Final Project -Part I and II	Part II may be postponed until the final	
June 7-11 is exam week. Final EXAM covering Chapters 1,3&4 will be held in our classroom on the DATE and TIME listed in the Official UW Exam Schedule				

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