**Math 124B** **Practice Midterm 2** **Winter 2003**

Your Name

Your Signature

Student ID #

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Section (circle one) AA AB AC BA BB BC

| Problem | Total Points | Score |
|---|---|---|
| 1 | 9 | |
| 2 | 8 | |
| 3 | 10 | |
| 4 | 7 | |
| 5 | 10 | |
| 6 | 6 | |
| Total | 50 | |

- This exam is closed book. You may use one side of one $8\frac{1}{2} \times 11$ sheet of handwritten notes.
- Do not share notes.
- Graphical and symbolic calculators are not allowed.
- In order to receive credit, you must show and **explain** your work. Ways to explain:
  - Write down computations/formulae used on your exam paper; don't just do them in your head or punch them into calculator.
  - Mark (boldly) points placed on a graph and/or make a short chart/table of values computed to draw a graph.
  - Make notes about your reasoning process for how you got your answer. A test is about you showing me what you know. Don't leave it up to me to guess what you know.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you use a trial and error (or guess and check) method when an algebraic method is available, you will not receive full credit.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (9 points) Compute the following derivatives.

(a) (3 points) $\frac{\ln(x)}{1-x^2}$

(b) (3 points) $t^{\sin(t)}$

(c) (3 points) $\tan^{-1}(e^x)$

(d) (3 points) $\sin(ax+b)$ where $a$ and $b$ are constants.

2 (8 points) Suppose that $y(x)$ is defined implicitly by the equation

$$y^2 \sin(x) + \cos(y) = 0$$

(a) Find a formula for $\frac{dy}{dx}$ in terms of $x$ and $y$ and use it to find the equation of the tangent line to the curve at $(0, \frac{\pi}{2})$.

(b) Use the tangent line to approximate the value of $y$ for $x = .2$.

3 (10 points) An experimental vehicle's height is given by the quadratic function:

$$h(t) = at^2 + bt + c$$

where $a$,$b$,and $c$, are constants.

At $t = 10$ seconds, the vehicle's altimeter shows that its height is 1500 meters. Its speedometer shows that the instantaneous velocity at the same time is 100 meters/second and the accelerometer shows that it is accelerating at 10 meters/sec$^2$ (also at $t = 10$ seconds). Find the constants $a$,$b$, and $c$, and write the exact formula for $h(t)$.

4 (7 points) Suppose that the volume of a baloon full of gas (special gas) at time t is given by the formula

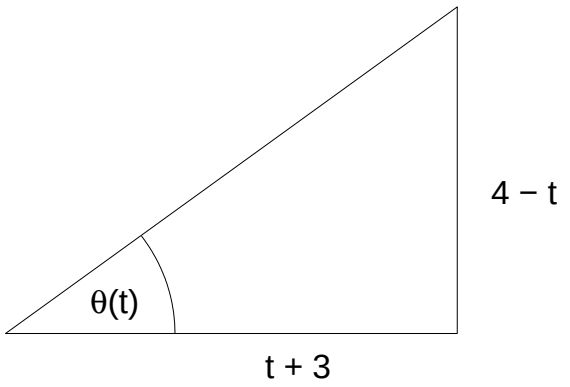
$$V(t) = V_0(1 + \mathrm{e}^{-at})$$

The pressure is related to the volume by

$$P(V) = KV^{\frac{2}{3}}$$

Both $K$ and $a$ are constants. Find the rate of change of the pressure as a function of time.

5 (10 points) The function $\theta(t)$ is the angle at the vertex of a right triangle whose sides are varying with time. The adjacent side has length $A(t) = t + 3$ and the opposite has length $O(t) = 4 - t$. See the picture below:




Find a formula for $\frac{d\theta}{dt}$ as a function of $t$ only (i.e. no $\theta$'s, just $t$'s in the final formula). In addition, your final formula should contain no trigonometric or inverse trigonometric functions.