Long-term isohaline salt balance in an estuary

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Abstract

The salinity budget in a 3-D numerical model of an estuary is analyzed using isohaline surfaces to define the volume of integration. Such surfaces move with the currents, and as a result of turbulent mixing. The isohaline analysis allows us to clarify the processes which create and destroy water in different salinity classes in the estuary. We find that the estuary naturally divides into three salinity classes, with different mechanisms maintaining the volume in each, when averaged over a time long enough to have a steady mean salinity field (two spring-neap cycles in our model). In the low salinity region, where isohalines never leave the estuary, there is an advective–diffusive balance. The river flow tends to increase the volume of low salinity water, while isohaline “drift” tend to decrease it. The “drift” is the non-advective motion of isohalines, caused by turbulent diffusion. In the high salinity region, direct injection of ocean water at the mouth brings in salt, balanced mainly by this drift. There the mean drift of isohalines is toward higher salinity, whereas in the low salinity region the drift is toward lower salinity. Correspondingly, the drift tends to increase the volume of water in the mid-salinity range, consistent with turbulent creation of mixed water out of ocean and river endpoints. This increase is balanced by the direct ejection of that water out the mouth of the estuary, due to restratification during neap tides. The magnitude of this ejection depends upon the location of the estuary mouth, which is somewhat arbitrary. The ability of the estuary to permanently export mixed water at the mouth is a key factor in the overall salinity structure. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Estuarine salinity structure reflects the competing influences of river and oceanic waters, as they are advected by tidal and other currents, and mixed by turbulence. In an effort to understand why a given part of an estuary has an observed salinity, researchers construct salinity budgets. The simplest of these use a small number of “boxes” to define the estuary, and assume a steady, tidally averaged salinity field. The boxes are arranged along the length of the estuary, and often there are two boxes in depth, reflecting the fact that we can usually describe salinity structure by a simple increase of salinity along-channel, with some stratification. Then using salinity and river flow data (Dyer, 1997; Cokelet et al., 1991) one may
diagnose the time-mean advection and turbulent diffusion of salinity through the sides of the boxes. This powerful method has been extended to include time-varying salinity and river flow by Hagy et al. (2000). The benefit of the diagnostic method is that it is readily applied to real estuary data, and that the results may then be used to determine residence times. The drawback is that by simplifying the estuarine salinity field into stationary boxes the actual physical processes which shape the salinity structure are lost or obscured.

In this paper we formulate an alternate diagnostic salinity budget for an estuary, one which uses moving isohaline surfaces to define the volumes of integration. This work extends that of MacCready and Geyer (2001) to include long time scales (several spring-neap periods), and the case in which isohalines are ejected from the ocean end. The budget is worked out in Section 2. The numerical model and its basic fields are presented in Section 3. The isohaline analysis of the model results is presented in Section 4. A simple thought model is presented in Section 5 to help make sense of the model results.

2. Isohaline budget formulation

Consider the conservation of volume in a volume, $V$ (sketched in Fig. 1) which is bounded in part by the bottom topography and free surface in an estuary. At the river end we assume zero salinity, although there may still be tidal currents, and the volume boundary is a stationary vertical cross-section with area $A_R$. At the ocean end the bounding surface is an isohaline surface, with salinity $s_I$ and area $A_I$, which is, in general, curved and moving. If the surface extends beyond the mouth of the estuary into the ocean, we truncate the volume using another vertical, stationary cross-section, with area $A_O$. The time rate of change of the volume is given by

$$\frac{\partial V}{\partial t} = - \int_{A_R} u_R \, dA - \int_{A_I} u_I \, dA - \int_{A_I} (u \cdot \mathbf{n} - u_I) \, dA,$$  \hspace{1cm} (2.1)

where $u_R$ and $u_O$ are the fluid velocities through the river and ocean ends (positive out of the volume). The 3-D fluid velocity vector is $\mathbf{u}$, and $\mathbf{n}$ is the unit vector outward normal to volume $V$. The outward-normal velocity of the isohaline surface is defined as $u_I$. If there is no mixing then $u_I$ is the same as the normal fluid velocity, $\mathbf{u} \cdot \mathbf{n}$, in which case the last term in Eq. (2.1) vanishes. Kundu (1990, pp. 74–76) gives a derivation of the mathematical basis for Eq. (2.1). Our volume is an example of Kundu’s “General Case”, as it is neither a fixed volume nor a material volume. The rate of change of the volume integral of salinity is given by a similar expression:

$$\frac{\partial}{\partial t} \int_V s \, dV = - \int_{A_O} s u_O \, dA - s_I \int_{A_I} (u \cdot \mathbf{n} - u_I) \, dA$$

$$- \int_{A_I} \mathbf{F} \cdot \mathbf{n} \, dA.$$  \hspace{1cm} (2.2)

where $s$ is salinity, and $\mathbf{F}$ is the turbulent salinity flux vector. In the Boussinesq approximation, salt flux is the same as salinity flux multiplied by the mean fluid density. Using Eq. (2.1) to estimate the term $(\mathbf{u} \cdot \mathbf{n} - u_I)$ in Eq. (2.2), we find

$$\frac{\partial}{\partial t} \int_V s \, dV = s_I \left( \frac{\partial V}{\partial t} + \int_{A_R} u_R \, dA + \int_{A_O} u_O \, dA \right)$$

$$- \int_{A_I} \mathbf{F} \cdot \mathbf{n} \, dA - \int_{A_O} s u_O \, dA.$$  \hspace{1cm} (2.3)

Physically, the terms in Eq. (2.3) represent:

I. The rate of change of the volume integral of salinity.
II. The apparent flux of salinity into $V$ due to the “drift” of isohalines. The terms in II are equivalent to $-s_1 \int_{A_1} (\mathbf{u} \cdot \hat{n} - u_0) \, dA$, which is $-s_1$ times the outward-normal volume transport through the isohaline.

III. Area integral of the turbulent salinity flux into the volume.

IV. Injection/ejection of salinity by advection through the ocean end of the volume, if it exists.

Consider two simple cases. If there is no turbulent mixing then terms II and III in Eq. (2.3) are zero, and the time rate of change of net salinity (I) is due only to ejection out of the ocean end (IV). If, on the other hand, the salinity field and river flow are steady, and the isohaline is completely within the estuary, then the balance is entirely between terms II and III

$$s_1 Q_R = -\int_{A_1} \mathbf{F} \cdot \hat{n} \, dA,$$

where $Q_R \equiv -\int_{A_R} u_R \, dA$ is the river volume flux, positive into the estuary. The “drift” term is the l.h.s. of Eq. (2.4). This is an advective–diffusive balance. In the case of steady, one-dimensional flow, the area integrals may be dropped, and the terms in Eq. (2.4) written simply as $us = K \partial s / \partial x$, where $K$ is a diffusivity.

3. Numerical model setup

We will evaluate the budget using results from the Regional Ocean Modeling System (ROMS) model (Haidvogel et al., 2000) applied to an idealized estuary. The model bathymetry is a straight channel, 15 m deep in the thalweg, 150 km long, and 1 km wide. The configuration is based loosely on the Hudson River estuary. The channel shoals to 12 m deep at the edges, and has a 5 m Gaussian sill across it at mid-channel. This sill proved to be a useful way to achieve a stable estuarine system with this model. It has the effect of an internal hydraulic control, keeping the salt intrusion from penetrating past the open boundary on the river end. The model flow is forced with a semidiurnal tidal current, with 12 h period, modulated at fortnightly (14 day) period. The forcing is achieved in the model using a specified, oscillatory volume transport through both ends. A phase shift between the transports at the two ends is applied, creating a progressive tidal wave. There is also a mean volume flux down-estuary due to the river, with strength $Q_R = 372 \, \text{m}^3 \, \text{s}^{-1}$. The along-channel salinity gradient at the open boundaries is specified to be a function of the salinity at either location. The open boundary salinity gradient goes to zero as the salinity approaches an oceanic value (as near the mouth) or as it approaches zero (as at the river end). The model setup is described further in MacCready and Geyer (2001). The model is run for several spring-neap cycles, and eventually falls into a pattern in which the salinity field almost exactly repeats itself every two cycles. In Fig. 2, we plot the salinity (a) and velocity (b) at certain locations, over days 153–181 (28 days) of the simulation. The salinity stratification near the mouth varies from well mixed to highly stratified, approximately in phase with the tidal range. During well-mixed conditions the estuary is slowly getting fresher, which is consistent with reduced up-estuary salt flux during that time. Clearly, the adjustment time during the well-mixed phase is longer than the duration of spring tide. It is also apparent in Fig. 2a that the first neap tide is smaller, in the sense of causing less restratification, than the second neap tide. During neap tide the surface velocity near the mouth (Fig. 2b) shows a distinct ebb dominance, consistent with a net ejection of surface water, due to the gravitational circulation.

The field of turbulent mixing is also strongly controlled by the stage and strength of the tide. In general the vertical eddy diffusivity is strong throughout the water column (decreasing near the top and bottom) during both ebb (Fig. 3a) and flood (Fig. 4a). The vertical turbulent salinity flux (Figs. 3b and 4b) is a function of both the eddy diffusivity and the salinity stratification, and as a result it is much more localized in space than the eddy diffusivity alone. At max ebb, Fig. 3, the combination of stronger flows and the bunching of isohalines near the mid-channel sill causes most of the salt flux to occur at the sill. By scaling, and by the design of the numerical model, turbulent
salinity flux is dominated by its vertical component. Hence the turbulent salt flux across an isohaline surface is calculated in each grid cell (where the isohaline exists) as the cell area times the vertical turbulent salinity flux interpolated to the depth of the given isohaline. This procedure may lead to small errors where the isohaline is near vertical, but we find below that the global salinity budget closes to within reasonable approximation.

When we calculate the area integral of turbulent flux through a single isohaline ($s_I = 10$, Fig. 3c) we see that the net turbulent flux (term III in Eq. (2.3)) is strongly peaked, generally being greatest at max ebb (Fig. 3c and d). This is consistent with flux due to enhanced shear and stratification at mid-depth where the gravitational circulation is augmented by the shear of the tidal flow. In this case the turbulent flux also appears to be hydraulically controlled, localized in the lee of the sill. There is often a smaller peak at max flood, and during the second neap-to-spring transition (days 172–177), the flux (Fig. 4b) is actually greatest 1 h before max flood (Fig. 4c and d).

The pattern of mixing on flood is consistent with bottom boundary layer turbulence impinging on the base of the stratified region. It is likely that the greater stratification present at the time of Fig. 4 relative to Fig. 3 accounts for the decreased importance of ebb mixing.

The net turbulent salinity flux (2.3, term III) is also a function of the horizontal area of the given isohaline, which increases markedly during neap tides (Fig. 5a), and even extends out the mouth of the estuary part of the time (Fig. 5b). However, the maximum diahaline turbulent salt flux through $s_I = 10$ still occurs during spring tides (Fig. 5c and d) especially during the latter half of spring tide, e.g. days 165–168.

4. Differential isohaline budget

We have seen how one term in the isohaline salinity budget evolves over the tidal and spring-neap cycles. Next, we look at the complete budget for a single isohaline. Also in this section we
develop a “differential” budget, treating the volume between two isohalines. Beginning with the complete single-isohaline budget, all terms in Eq. (2.3) are plotted versus time in Fig. 6 for the isohaline $s_I = 10$. The terms have been averaged over one tidal period. The budget does not close exactly, because of errors inherent to interpolation to the isohaline, and because we use fields saved only every hour instead of every time step. Nonetheless the budget closure is reasonable, as seen by the “Error” curve in Fig. 6, which is calculated as $I - (II + III + IV)$ in Eq. (2.3), again time averaged over one tidal period. Over the two spring-neap cycles the volume-integrated salinity behind $s_I = 10$ increases and decreases, averaging to zero net change. Turbulent flux always adds to the net salinity, because it is always a down-gradient flux. This flux is greater during spring tides, by about a factor of two. The ejection term causes a loss of net salinity during both neaps, particularly the second, when the isohaline extends past the mouth. The drift term is dominated by a large positive spike lasting several days (days 168–171). Examination of individual sections during
this period shows that this is a time when, first, there is a large volume of water with salinity > 10, then mixing “peels down” isohalines through this volume. In general, this rapid isohaline “drift” may occur when a region of strong salinity gradient is next to a region of weak salinity gradient.

We may now use the isohaline budget to explore the way in which the complete estuarine salinity structure is maintained over time. First we compute the isohaline budgets for the whole range of isohalines in the estuary using Eq. (2.3). We do this for $s = 1-27$ in steps of 1. Then for each of these budgets we form the time average of each budget term, averaging over the two spring-neap cycles. Over this time the rate of change of net salinity, term I in Eq. (2.3), averages to near zero. We next take the difference between the time-averaged budget results of neighboring isohalines. The result is that we find a time-averaged budget
for the integral of salinity between any two isohalines. We will call this the differential budget. Since the salinity between any two isohalines only has a small range, we are then essentially looking at the budget of what maintains the volume (or lack of volume) between any two isohalines. The volume in a given salinity range will be called \( \Delta V \):

Mathematically we may write the differential budget as

\[
\frac{\partial}{\partial S_1} \left( \frac{\partial}{\partial t} \int_V s \, dV \right) = \left\{ \left\langle \frac{\partial \Delta V}{\partial t} \right\rangle - Q_R + \frac{\partial}{\partial S_1} \left\langle \int_{A_0} u_0 \, dA \right\rangle \right\} \\
- \frac{\partial}{\partial S_1} \left\langle \int_{A_1} F \cdot \mathbf{n} \, dA \right\rangle - \frac{\partial}{\partial S_1} \left\langle \int_{A_0} s u_0 \, dA \right\rangle.
\]

(4.1)

where the time-average over 28 days is denoted by angle brackets \( \langle \rangle \). As in Eq. (2.3), the terms may be interpreted physically as: “time rate of change = drift + turbulent flux–ejection”. However, now these terms apply to the differential volume between isohalines instead of the full volume \( V \). The differential budget is plotted in Fig. 7. We may understand the differential budget (Fig. 7) in a fairly simple way. First, the time-derivative terms in Eq. (4.1) are near zero. Next, up to \( s = 6 \), there is essentially no mean ejection to the ocean, meaning that isohalines fresher than that never leave the estuary. In that region, the differential budget reduces to two terms, an advective–diffusive balance

\[
- \left\langle Q_R \right\rangle = \frac{\partial}{\partial S_1} \left\langle \int_{A_1} F \cdot \mathbf{n} \, dA \right\rangle.
\]

(4.2)

We mark the value of \( - \left\langle Q_R \right\rangle \) with a star in Fig. 7, and it is apparent that the advective–diffusive balance Eq. (4.2) does indeed hold, approximately, for \( s \leq 6 \). This result was predicted in MacCready and Geyer (2001), although we did not have a steady salinity balance to test the concept at the time that paper was written.

The middle-salinity range, \( s = 6–21 \), shows a balance between ejection to the ocean removing salinity (and hence volume) from each differential volume, and drift replacing that volume. At higher salinity that relationship still holds, but the sign is changed, and now injection from the ocean adds to the differential volumes. In addition, the turbulent flux term removes some salinity from all the differential volumes with higher salinity.

5. Thought model of the differential budget

To understand the entire structure of the differential budget, consider a simple thought problem. Start with a volume which has three equal sub-volumes, one with freshwater, one with oceanic salinity, and one an equal mixture of the two (Fig. 8a). We will keep track of the various salt flux terms in the differential isohaline budget for each of the three salinity classes. In a simplified estuarine fortnightly cycle we first allow turbulent mixing to homogenize the entire volume (Fig. 8b),
analogous to spring tides. It is only during the mixing phase that the drift term is active. During the mixing phase turbulent mixing added \((V/3)15\) net salinity to the fresh sub-volume, but the drift of the bounding isohaline removed the same amount (the fresh volume had zero net salinity to start, and it still has zero). Mixing did not add any net salinity to the middle-salinity volume during the mixing phase, because salt was diffusing out the fresh end as fast as it was diffusing in through the salty end. However the drift term did triple the size of the mid-salinity volume, adding \(2(V/3)15\). The salty sub-volume lost \((V/3)15\) net salinity to turbulent mixing, and it also lost that same amount to the drift of its bounding isohaline (adding these two losses we see that it now has zero net salinity, consistent with the fact that it has zero volume). Now, we turn off the mixing, and, in the exchange phase (analogous to neap tides), Fig. 8c and d, force the estuary to return to its initial state by adding fresh water (river flow), and salty water (equivalent to the lower half of a gravitational exchange flow), and remove intermediate salinity water (equivalent to the upper half of a gravitational exchange flow, or our ejection term). During this exchange phase the fresh end gains zero net salinity, because the incoming water is fresh. The intermediate salinity volume loses two-thirds of its volume, for a net salinity loss of \(2(V/3)15\). The salty sub-volume is replenished by injection, for a net salinity gain of \((V/3)30\). Adding up the fluxes from the mixing and exchange phases, we find that the time-averaged picture, sketched in Fig. 9, looks remarkably similar to the differential budget in Fig. 7. There are also unexplained differences. In particular, the magnitudes of the drift and balancing injection terms near the mouth are much larger in the numerical model than in the thought model.

Fig. 6. Terms in the isohaline budget (2.3) for \(s_l = 10\), vs. time over two spring-neap cycles. Terms have been averaged over one 12 h tidal period. Positive fluxes denotes increasing the net salinity within the volume, as turbulent diffusion (III) always does, while negative fluxes denotes a loss, as is true of the ejection term (IV) during both neap tide restratifications.
Fig. 7. Terms in the time-averaged differential salinity budget, plotted vs. salinity. Points are plotted mid-way between the two neighboring isohalines which define each differential isohaline volume. The star at the left is at (minus) the time-averaged river volume flux.

Fig. 8. Thought model for a simple isohaline budget. The volume undergoes a fortnightly cycle of mixing followed by restratification.

Fig. 9. Time-integrated, differential isohaline budget for the thought problem in Fig. 8. Many general features of this budget match those in the full differential budget of Fig. 7, but there are also differences.
6. Conclusions

We have developed in this paper a mathematical framework for the salt budget in an estuary, using an isohaline surface as the seaward end of the bounding volume for the budget. Three processes were found to change the net salinity within the volume. Turbulent diffusion always adds net salinity, because it is down-gradient. However, turbulent diffusion also allows for non-advective motion of the isohaline surface, because the salinity field itself may be altered by the mixing. This non-advective motion is referred to as the isohaline drift. The salt flux due to drift may be larger or smaller than the turbulent salt flux, and is not constrained in sign. The third process affecting the salt budget, neglected in the original isohaline analysis of MacCready and Geyer (2001), occurs when part of the bounding isohaline leaves the estuary. The isohaline salt balance was calculated for time-dependent results from a 3-D numerical model of an idealized estuary. The estuary was forced with steady river flow, but the tidal amplitude varied over a two-week period, giving rise to periods of more or less stratification. Ejection was a dominant process during neap tides. After several spring-neap cycles, the model developed a quasi-steady state in which its salinity structure would very nearly repeat itself every four weeks (two spring-neap cycles). This four-week period was then used as an averaging period for evaluating terms in the isohaline budget, because the salinity field was unchanged between start and finish.

Differentiating the salt budget with respect to the salinity of the bounding isohaline, we developed a budget for the evolution of net salinity between two neighboring isohalines. Effectively this is also a budget for the “differential volume” between isohalines. The differential budget helps us interpret the long-term processes maintaining the salinity structure of the estuary. The most important aspects of the differential budget may be summarized as follows. Turbulent flux divergence tends to decrease the volume of high salinity water and increase that of low salinity. This is consistent with down gradient salt flux. The mean drift of isohalines tends to increase the volume of mid-salinity water. This is an expression of the creation of mid-salinity water by turbulent salt flux. In compensation, the mean drift tends to decrease the volumes of high and low salinity waters. Finally, the creation of mid-salinity water by the mean drift (preferentially during spring tides) is balanced by the direct ejection of that water out the mouth of the estuary (during neaps). The location of the mouth of the estuary is, in practice, somewhat arbitrary. The choice of this location will affect the magnitude of the ejection term, increasing it as the mouth is moved landward.

This whole process was simplified at the end of the paper to a conceptual model using a tank of stratified water. Following the fortnightly progression, the tank is first completely mixed (spring tides), and then is restratified (neap) by the addition of both low salinity water (river flow) and high salinity water. To maintain volume, middle salinity water is removed. The exchange of mid-salinity water for high salinity water is analogous to the effects of the gravitational circulation during neap tides in our model estuary.

The value of the differential isohaline analysis is that it allows us to understand the physical processes which maintain the salinity structure of an estuary. The most important of these is the creation of mid-salinity water by turbulent mixing, and its removal by ejection. A curious feature of the differential isohaline analysis is that all of the creation of mid-salinity water is ascribed to the drift term, instead of the turbulent flux term. The reason for this is that the differential budget concerns flux divergence, and the turbulent flux divergence naturally passes through zero near mid-salinity. However, the fact that our budget follows isohalines allows the drift term to express the resulting growth in volume of the mid-salinity region.

What is striking about the results of our differential budget is that they ascribe much of the mean salinity structure to the drift-ejection balance in mid-salinity waters. Because the analysis follows isohalines, it naturally moves with the salinity structure as it changes through the fortnightly cycle of mixing and restratification. If we performed a similar analysis on a steady-state estuary solution, such as Hansen and Rattray (1965), we would find that the balance was almost entirely due to mixing balancing drift, a balance.
we only find near the head of our estuary where
isohalines never leave the system. The implication
of our study is that the mean salinity structure of
an estuary with a significant fortnightly cycle may
critically rely on processes such as the ejection
of mid-salinity water during neaps. A watermass-
based analysis, such as our differential isohaline
budget, is a useful tool for quantifying such
processes, particularly in numerical models.

While our isohaline analysis may be readily
applied to numerical models, it would be much
more difficult to use with observational data from
a real estuary. Philosophically, our budget has
some aspects in common with the data which
would be collected following a patch of dye. One
could imagine that a dye cloud, released in
freshwater, would slowly increase in mean salinity
due to turbulent mixing. This is analogous to
the growth of our mid-salinity waters. Eventually,
the dye would leave the estuary, by ejection to the
ocean. Our analysis highlights the obvious point
that this removal would occur only once the
salinity of the dye was as great as that of the
isohalines being ejected at the mouth.

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