

Notes on Lab 1: The Spar Buoy

Assume the density of the water is 1025 kg m^{-3} and the diameter of the spar is $1''=2.54 \text{ cm}$.

Write out the force balance of the buoy. The force down is just due to gravity. The force up is due to the difference in pressure acting on the top of the buoy, which is $p_{ATM}A$ pushing down, and that on the bottom of the buoy, which is

$p_{ATM}A + \rho gHA = p_{ATM}A + \rho gV$ pushing up. Thus the total force due to pressure is ρgV .

$$0 = \rho gV - mg$$

$$\Rightarrow m = \rho V$$

where m is the mass of the buoy which has volume V below the waterline at rest, cross-sectional area A and length H , again below the waterline at rest.

Write an equation for the oscillations... define $\eta(t)$ as the deviation of vertical position away from rest

$$F = ma$$

$$\rho gA(H - \eta) - mg = m\eta''$$

$$\rho gAH - \rho gA\eta - mg = m\eta''$$

then making use of $m = \rho V$ we can rewrite this as

$$\frac{\rho gV}{\rho V} - \frac{\rho gA\eta}{\rho V} - g = \eta''$$

$$\eta'' + \frac{g}{H}\eta = 0$$

and this has solution

$$\eta = \eta_0 \cos\left(\sqrt{\frac{g}{H}} t\right)$$

so for a mean submerged depth of 2 m, the frequency of oscillation is

$$\text{frequency} = \sqrt{\frac{g}{H}} = \sqrt{\frac{9.8 \text{ m s}^{-2}}{2 \text{ m}}} = 2.2 \text{ radians s}^{-1}$$

and so the period is $T = (2\pi \text{ rad}) / (2.2 \text{ rad s}^{-1}) \approx 3 \text{ seconds}$.