

Fluids 2009

Problem Set #3 Assigned Tuesday 10/20/2009, due Wednesday 10/28/2009

1. Consider a steady flow field where the velocity is given by $(u, v, w) = (Ax, -Ay, 0)$. A is a constant with units s^{-1} . In all that follows of problem 1 you can ignore the w velocity, treating the flow as 2-dimensional in the x, y plane.

1.i. Show that the flow field is incompressible. [2 points]

1.ii. The flow streamlines are defined (Kundu & Cohen 3.4) by the relation $\frac{dY}{dx} = \frac{v}{u}$

where $Y(x)$ is the equation for the y -position of a given streamline as a function of x .

We are using the notation of capitalizing Y to make it clear that this is y on a streamline, not the independent variable y . Thus for our flow field the differential equation for a

streamline would be given by $\frac{dY}{dx} = \frac{-Y}{x}$. Find the general solution for the shape of a

streamline. Hint: guess a form of the solution $Y = Cx^n$ where C is a constant (different values of C would define different streamlines, and n is a constant to be determined. [3]

1.iii. Sketch the x, y field of velocity vectors and some streamlines. Be careful to indicate where the velocity is relatively bigger or smaller. [5]

1.iv. The equations for conservation of momentum in this system are given by

$$\frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x}, \quad \frac{Dv}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y}$$

where ρ is constant. Determine, to within a constant, the pressure field $p(x, y)$ that is required for the momentum equations to be satisfied. Sketch contours of this pressure field. [10]

1.v. Even though the flow field is steady, a given fluid parcel still accelerates and decelerates as it moves along its path. These accelerations can be associated both with changes of speed and with changes of direction. Sketch these acceleration vectors for two places on a given streamline: one place where the streamline intersects the line $y=x$, and one place where the streamline intersects the line $y = x / 10$. Describe why the parcel acceleration vectors at these two locations are consistent with the pressure field. [10]

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2. Say you have a standing shallow water wave in a box of horizontal (x) length 100 m and in which the resting depth is 10 m. Assume no flow in the y -direction.

2.i. What are the periods of the two lowest frequency solutions? Sketch their patterns of surface height as a function of x . [5]

2.ii. If the maximum amplitude of surface height oscillation is 1 m please find the ratios of the scales of the terms we neglected in the MASS (neglected ηu_x and $\eta_x u$) and X-MOM (neglected uu_x) equations to those we kept. For this exercise you may assume that the scale of a quantity is given by its maximum amplitude anywhere, and at any time, in the solution fields. Give your answers for both the frequencies you computed in 2.i. [10]

2.iii. What is the amplitude of horizontal displacement experienced by a fluid parcel? How does the size of this compare with the intrinsic horizontal scale of the solution fields, which is k^{-1} for a field that varies in x like $\cos(kx)$. Give your answers for both the frequencies you computed in 2.i. [5]

NOTE: the term "amplitude" for an oscillatory quantity means half the total variation. So, for $\eta = \eta_0 \cos(\omega t)$ the amplitude would be η_0 .