Fluids 2009

Problem Set #2, assigned 10/12/2009, due 10/19/2009

1. Gradient of a scalar field and divergence of a vector field: Say you have a 2D pressure field given by $p(x,y) = A\cos(kx)\cos(ly)$, where A, k, and l are constants.

[4] 1.i. Sketch the vector field $-\nabla p$. Include a scale arrow for the vectors, and be sure to specify the units of any expressions.

[3] 1.ii. What is the divergence of this field?

[2] 1.iii. If the vector field was the velocity would the flow be incompressible?

2. The material derivative: Assume the temperature $T(x,y,z,t)$ is conserved following a fluid parcel.

[2] 2.i. What is the equation for the evolution of T?

If the initial temperature field is given by $T(x,z,t=0) = \frac{T_0}{L}x + \frac{T_0}{H}z$ and the velocity field is given by $u = (u,v,w) = \left(\frac{U}{H}z, 0, 0\right)\ldots$

[1] 2.ii. What is the rate of change of temperature following a fluid parcel?

[3] 2.iii. What is the rate of change of temperature at $z = H/2$?

[4] 2.iv. What is the gradient of the temperature field as a function of time and space?

3. Buoyancy: Say you have a spar buoy of vertical length H (like in the lab we did) that is at rest at the interface of a two layer fluid. The lower layer has density $\rho = 1000$ kg m^{-3} and the upper layer has density $\rho = 999$ kg m^{-3}.

[4] 3.i. What is the solution for vertical oscillations if the buoy is always somewhere on the interface? Make sure to simplify your expression for the frequency as much as possible.

[3] 3.ii. What is the frequency if the oscillations (and the long axis of the buoy) are along an angle θ relative to the vertical?

[4] 3.iii. What is the solution if H is negligibly small compared with the amplitude of the oscillations (meaning that the buoy spends most of its time in either the lower or the upper layer)?