

Fluids 2009

Problem Set #1 SOLUTIONS, assigned 10/5/2009, due 10/12/2009 at the start of class

1.i. The force due to pressure could be drawn as vectors of equal length pushing in on each face of the cubes (same for both).

1.ii. The forces on the shared face are pointing in opposite directions because the "outward normal" for cube (a) is in the direction opposite to that of cube (b).

2.i. The height of the free surface in the tube can be calculated by assuming that the pressure at depth $z=-H$ must be the same in both the vat and in the tube in order for the interface between the two fluids to be at rest. This can be written mathematically as

$$p_{VAT}(z = -H) = p_{TUBE}(z = -H)$$

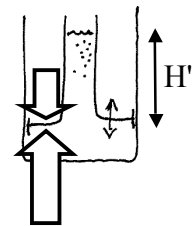
$$p_{ATM} + \rho_0 g H = p_{ATM} + (\rho_0 - \Delta\rho) g (H + \eta)$$

$$\Rightarrow \eta = \frac{\Delta\rho}{\rho_0 - \Delta\rho} H$$

2.ii. At A water will, of course, flow out of the tube. At C the water won't move either direction because, as reasoned in 2.i, the pressure inside and outside the tube must be the same there. At point B we would have to calculate the pressure inside and outside the tube for an arbitrary z -position, and then make use of the fact that we now know η to figure out the exact pressure difference at any depth. However it is simpler to just note that at a point just below the water surface the water will certainly flow out of the tube because of the excess pressure due to η .

3.i. The pressures at A and B are the same, both given by $p(z = -H) = p_{ATM} + \rho_0 g H$. This is clear for point B where we can just integrate vertically as done in class. For point A a strict vertical integral would encounter the difficulty that we don't know the pressure on the upper vat wall above A. One way around this is to argue that since the fluid between A and B is not accelerating then $\partial p / \partial x = 0$ between them and so they must be at the same pressure.

3.ii. The upper part of the vat would be pushed up. To see this, consider the forces on either side of the horizontal wall of the upper part. On the top of this surface the pressure is p_{ATM} , whereas on the underside of this surface the pressure is $p_{ATM} + \rho_0 g H'$ which is clearly bigger.



4. For scales given in this m-file:

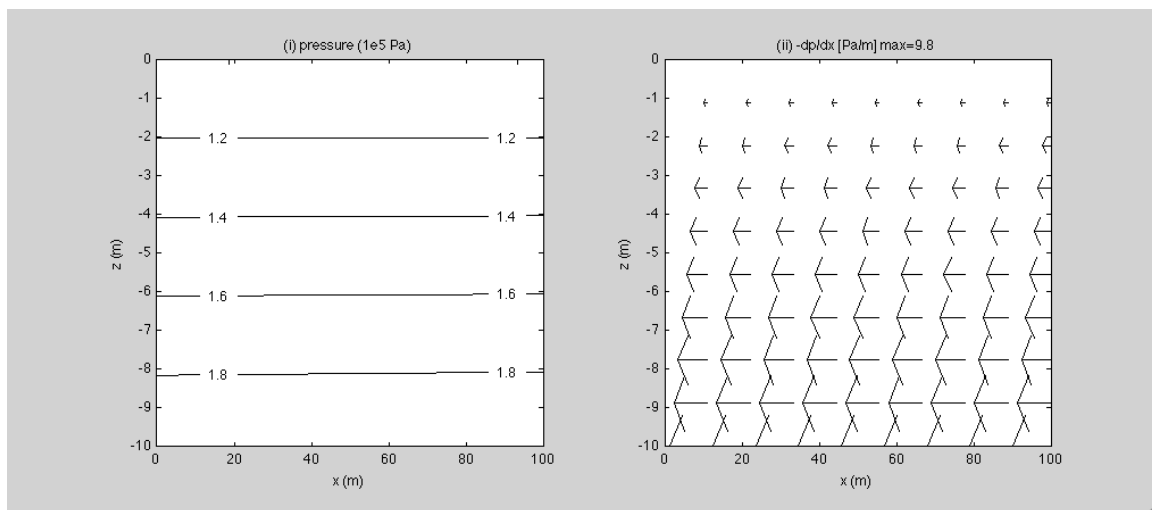
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% PS1_4.m 10/6/2009 Parker MacCready
% This plots the initial pressure field and the the x-force pattern due to
% it for problem 4 of PS 1 (Fluids 2009).
clear
% make spatial coordinates
x = linspace(0,100,10); % [m]
z = linspace(-10,0,10); % [m]
% make them into matrices
[X,Z] = meshgrid(x,z);
rho0 = 1e3; % density [kg m-3]
Drho = X/10; % density anomaly [kg m-3] this is a linear increase of
% density in the x-direction that has a total change of 10 kg m-3 over the
% length of the tank
rho = rho0 + Drho;
p_atm = 1e5; % atmospheric pressure [Pa]
g = 9.8; % gravity [m s-2]
p = p_atm - g*rho.*Z;

figure
subplot(121)
[cc,hh] = contour(X,Z,p/1e5,[1:.2:2],'-k');
clabel(cc,hh)
axis square
xlabel('x (m)');
ylabel('z (m)');
title('(i) pressure (1e5 Pa)')
% calculate minus the pressure gradient
dx = x(2) - x(1); dz = z(2) - z(1);
[dpdx,dpdz] = gradient(p,dx,dz);
subplot(122)
quiver(X,Z,-dpdx,0*dpdx,0,'-k');
axis([min(x) max(x) min(z) max(z)]);
axis square
xlabel('x (m)');
ylabel('z (m)');
title(['(ii) -dp/dx [Pa/m] max=',num2str(max(max(dpdx)))]);

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4.i. The pressure field is mainly increasing with depth, and also increasing a tiny bit to the right, as shown below.



4.ii. The x-force per unit volume is given by $-\partial p/\partial x$. You could contour this, or you could represent it as a vector field as I have done in the plot above.

4.iii. Initially the fluid will accelerate as shown in the right hand panel of the figure above. So there is no motion at the surface, and an acceleration to the left which is greater at greater depth. This will soon pile up water on the left hand side, leading to an increase in the surface height there. This can create a pressure gradient pushing water at all depths *to the right*.

4.iv. The final state without mixing has flat isopycnals. The original horizontal stratification will have slumped down. This involves deeper water moving to the left (forced by the original pressure gradient) and surface water moving to the right (forced by the ensuing tilt of the free surface). There is also some vertical motion required to "close" this circulation, with water on the left moving up and that on the right moving down.