FLUIDS 2009

Midterm Exam 10/30/2009, due at the start of class Friday 11/6/2009

Open book (you may use any reference), open notes, no discussion with other students

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1. Recall the 2D flow from the last problem set, defined by $(u,v)=(Ax,-Ay)$.

1.i. Show that there is no effect of viscosity on the fluid momentum, even if the fluid is viscous. [5]

1.ii. Given your previous knowledge of the pressure field, show that the Bernoulli function is constant everywhere, and not just on a streamline. [5]

1.iii. Show that your result of constant Bernoulli function is consistent with all the assumptions used in class to derive the Bernoulli function. [5]

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2. For plane Poiseuille Flow (KC 9.4 and class notes 10/30/2009) where
$u=-\left(\frac{1}{2}\frac{p_x b^2}{\mu}\right)\left[1-\left(\frac{z}{b}\right)^2\right]$ and the plates are located at $z=-b$ and $z=b$.

2.i. Please sketch all the forces acting on the faces of a small, cubical fluid parcel located at: (a) the midpoint between the two plates, and (b) close to the top plate. [5]

2.ii. Take a path integral of the momentum equation on a line along the direction of fluid flow (in the same way we did when deriving the Bernoulli function) to find a mathematical expression for the total change in Bernoulli function along the path. This is often called the "head loss." [5]

2.iii. Find the expression for the dissipation ε [W kg^{-1}] as a function of z. [5]

2.iv. If the fluid is water, the plates are separated by 2 mm, and the maximum velocity is 10 cm s^{-1}, how long would it take this dissipation rate to raise the temperature of the water near the wall by 1 degree K? You may assume that the fluid is at constant pressure, and hence use C_p. [10]

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3. Consider a progressive shallow water wave whose surface height has the form $\eta=\eta_0\cos(kx-\omega t)$. The horizontal velocity is given by $u=u_0\cos(kx-\omega t)$ where $u_0=(\eta_0/H)c$, and the phase speed is $c=\omega/k=\sqrt{gH}$. Since the flow is unsteady we can't make use of the Bernoulli function (according to the assumptions in class). However, by looking at the solution in a frame of reference moving with the phase speed

c, it becomes steady. Please show that in this frame of reference the Bernoulli function is approximately conserved along the free surface. You may neglect the contribution of the vertical velocity to the total speed, because it is negligibly small compared to the horizontal velocity. Also, you may drop terms that are higher order in the "small" quantity η_0/H. [5]

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4. You are squirting water at a wall from a hose. The speed of water leaving the end of the hose is 2 m s^{-1}, and the cross-sectional area of the hose end is 4 cm^2. Estimate the force on the wall using a volume integral of the momentum equation. [10]

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5. How big of a helium balloon would you need to lift a young boy (mass = 20 kg) off the ground? [5]

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6. Take a question you have about any fluid mechanics problem (e.g. one related to your research) and use it to construct a homework problem. Clearly there is no "right" answer to this one, but these qualities are desirable:

(a) use concepts from the class or reading so far

(b) are stated clearly enough for your fellow students to solve

[EXTRA CREDIT]