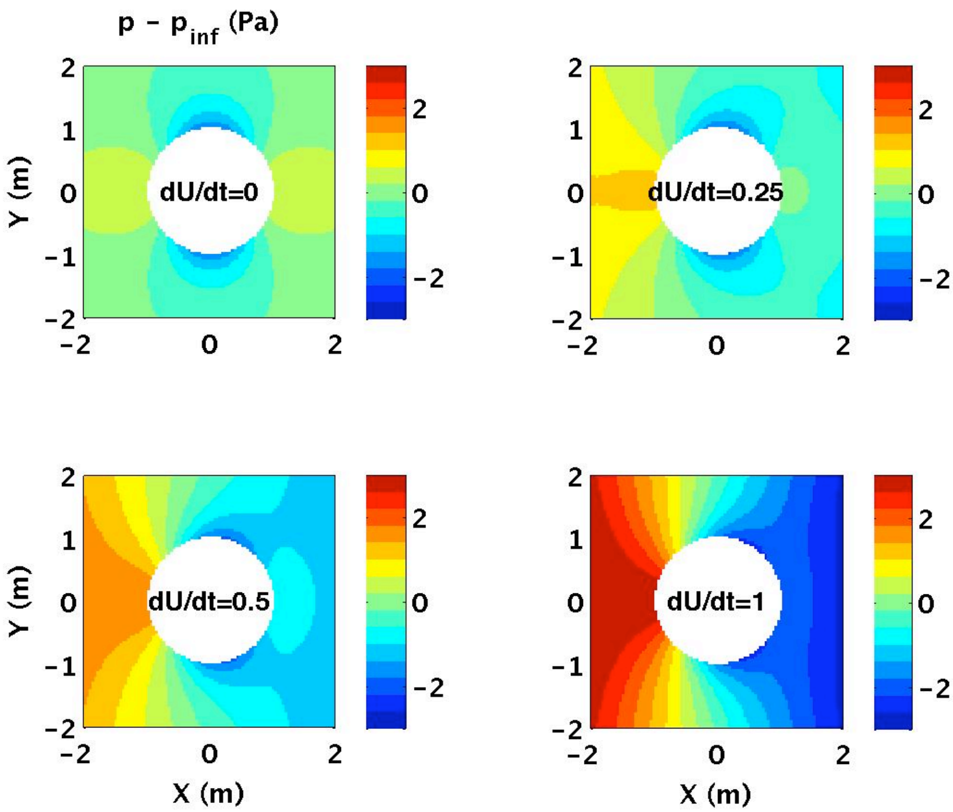
FLUIDS 2009 *12/11/2009*

Notes on Problem 3 of the final exam:

Here is what the pressure field looks like for several different values of the acceleration of the background flow dU/dt.



As the pressure field becomes more asymmetric the force on the cylinder becomes greater. The asymmetry is due to two parts. The first is just due to a large scale horizontal pressure gradient that is required to create dU/dt. Calculating the force due to this part is just like calculating buoyancy. The second part is due to the deformation of this accelerating flow by the cylinder. It can be calculated independently by considering the problem in which the fluid is stationary but the cylinder is accelerating. This is the classic problem of "apparent mass" (see for example the famous old textbook by Lamb) which gives the result that the force per unit z is equal to $\rho \frac{dU}{dt}\left(\pi a^2\right)$. The reason it is called apparent mass is that the size of the force is identical to that which the accelerating cylinder would have if it was filled with fluid and was being accelerated with no fluid outside. But in fact the force is not due to the inertia of the fluid in the cylinder (which may be assumed massless) but rather is due to the pressure forces on the outside that we calculated. In the problem we did for the final we found twice this force, because it happens that the two parts of the pressure disturbance are equal.

Lamb, H. 1930, *Hydrodynamics*, Cambridge University Press, 687 pp.

```
% cylinder_pt.m  12/11/2009  Parker MacCready
%
% this plots the pressure anomaly for potential flow
% around a cylinder including dU/dt

clear

% make axes
xymax = 2;
x = linspace(-xymax,xymax,100);
y = linspace(-xymax,xymax,100);
% note that x and y don't include 0
[X,Y] = meshgrid(x,y);
% and polar coordinates
R = sqrt(X.^2 + Y.^2);
sin_th = Y./R;
cos_th = X./R;

% physical parameters
U = 1; % velocity (m s-1)
Ut_vec = [0 .25 .5 1]; % acceleration (m s-2)
a = 1; % radius of cylinder (m)
rho = 1.2; % density (kg m-3) of air
% other parameters
ar2 = (a./R).^2;
ar4 = (a./R).^4;

% set up figure
close all
pm_fig;
figure
colormap(jet(15));

for ii = 1:4

    Ut = Ut_vec(ii);

    pp = 0.5*rho*U*U*(2*ar2.*(cos_th.^2 - sin_th.^2) - ar4);
    pp_unsteady = pp - rho*Ut*(R + a^2./R).*cos_th;
    pp_unsteady(R<=1) = NaN;

    subplot(2,2,ii)
    pcolor(X,Y,pp_unsteady);
    shading flat
    caxis([-3 3]);
    colorbar
    if ii==3 | ii==4; xlabel('X (m)'); end;
    if ii==1 | ii==3; ylabel('Y (m)'); end
    if ii==1; title('p - p_{inf} (Pa) '); end;
    axis equal
    axis tight
    text(0,0,['dU/dt=',num2str(Ut)],'horizontalalignment','c');

end
```

```
function [] = pm_fig(fs1)
% pm_fig.m
% does (hopefully) nice things to figures

if nargin==0; fs1 = 14; end; % fontsize
set(0,'defaultaxesfontsize',fs1);
set(0,'defaulttextfontsize',fs1);
set(0,'defaultaxesfontweight','bold');
set(0,'defaulttextfontweight','bold');
set(0,'defaultaxesfontname','Ariel');
```