Introduction to Fluid Dynamics

Problem Set #4, handed out 11/14/2007, ***due at the start of class 11/21/2007***

1. Consider (again) "Plane Poiseuille Flow" as described in Kundu and Cohen 9.4 (and Fig. 9.4d).

A[20]. Integrate the kinetic energy equation (here given as kinetic energy per unit volume, in "Eulerian" form)

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho\mathbf{u}\bullet\mathbf{u}\right)=\nabla\bullet\left[-\mathbf{u}\bullet\left(\frac{1}{2}\rho\mathbf{u}\bullet\mathbf{u}\right)-\mathbf{u}p+\nu\nabla\left(\frac{1}{2}\rho\mathbf{u}\bullet\mathbf{u}\right)\right]+p(\nabla\bullet\mathbf{u})-\rho g w-\mu\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}$$

over a rectangular volume that goes from the bottom plate to the top plate, as done in class 11/14/2007 for Couette flow. Which of the 7 terms above contributes to the energy balance. How is this different from Couette flow?

B[20]. Shift your frame of reference so that you are moving to the right with the average flow speed, $\overline{u}$ (that is the average of u over the distance between the plates). This is called a "Galilean Transform" and the equations of motion are unchanged by such a transformation. Note however that the velocity you observe, call it u' is given by $u'=u-\overline{u}$, where u is the x-velocity in the original frame of reference. Note also that now the solid boundaries are moving. Repeat your volume integral of the kinetic energy equation. Now which of the 7 terms are important? This is an example of how the "story" told by the energy equation may be different depending on the frame of reference.