Introduction to Fluid Dynamics
Problem Set 2, 10/12/2007, Due at the start of class 10/19/2007

1. Consider 2-dimensional flow in the x-y plane where the velocity is given by

$$(u,v) = D(x,-y)$$

where D is a constant with units s^{-1}.

a.[5] Sketch the velocity field. Is this field divergent?
b.[5] Find the mathematical expression for a streamline in the form $y = f(x)$.
c.[10] Find the expression for a parcel path as a function of time for a parcel that passes through $\left({x_0}^L, {y_0}^L\right)$ at $t = 0$. This will be equations for $x^L(t)$ and $y^L(t)$.
d.[5] Find the expressions for the u and v velocity for this same parcel, again as functions of time.
e.[10] Say you have dye along the line $y = x$ at $t = 0$. What happens to this line over time? Find the expression for the evolution of the slope of the line as a function of time. Does the distance between two parcels on the line change over time (*i.e.* does the line "stretch")?

2. Consider a 2-dimensional flow in the x-y plane where the velocity is given by

$$(u,v) = \left[U, V\cos(\omega t)\right]$$

where U and V are constants with units of velocity, and ω is a constant with units [rad s^{-1}].

a.[5] What is the equation for a parcel path for a parcel that is at the origin at $t = 0$?
b.[10] Develop the mathematical expression for the "streakline" that would result from a continuous injection of dye at the origin. Hint: the solution is not steady, but at any time it must pass through the origin. Sketch your answer at a couple of different times.