

Introduction to Fluid Dynamics

Midterm Exam, 11/2/2007, due at the start of class 11/9/2007

Open book (Kundu and Cohen only), open notes, work alone

1. Recall the steady, incompressible, 2D flow from Problem Set #2, defined by

$$(u, v) = D(x, -y)$$

where D is a constant with units s^{-1} . The vertical velocity is zero, assume $\rho = \rho_0 = \text{const.}$, and that there is no gravity.

A[5]. What is the vorticity field for this flow?

B[5]. What is the magnitude of the viscous term in the momentum equations?

C[5]. Justify the fact that the Bernoulli function is constant everywhere in this flow, and not just on a streamline (assume zero viscosity).

D[10]. Use Bernoulli to determine the pressure field $p(x, y)$ to within a constant. Sketch your answer along with some streamlines and velocity vectors.

E[5]. Explain in words why the change in *magnitude* of the velocity along a streamline (and hence following a fluid parcel) makes sense in relation to the pressure field.

F[5]. Explain in words why the change in *direction* of the velocity vector along a streamline (and hence following a fluid parcel) makes sense in relation to the pressure field.

G[5]. Comment on the relative magnitudes of the cross-path and along-path pressure gradients at a point of maximum path curvature (anywhere on $y = \pm x$).

2. Consider a cup (a right circular cylinder with an open top) that is partially filled with water. The cup is floating in a large lake of density $\rho = \rho_0$, whereas the fluid in the cup is a bit denser, having density $\rho_{cup} = \rho_0 + \Delta\rho$.

A[10]. Derive an expression for the height of the water in the cup relative to that in the lake (HINT: the water surface level in the cup should be deeper than that of the lake, and the cup is tall enough above the water inside so that it does not sink).

B[10]. If we drill a small hole in the bottom of the cup, which way will the pressure gradient cause water to flow through it? Justify your answer mathematically.

C[10]. If we drill a small hold in the side of the cup, which way will the pressure gradient cause water to flow through it? Justify your answer mathematically.

3. Consider "Plane Poiseuille Flow" as described in Kundu and Cohen 9.4 (and Fig. 9.4d).

A[10]. Describe the momentum balance of a fluid parcel in words. Give your answer for a parcel at the center (midway between the two plates) and for one near one of the plates.

B[10]. Why is the Bernoulli function not conserved along a streamline?

C[10]. What terms are important in the mechanical energy balance equation (KE) for a fluid parcel? Give your answer just for a parcel at the center (midway between the two plates).