Power Controlled Minimum Frame Length Scheduling in TDMA Wireless Networks with Sectored Antennas

Arindam K. Das, Robert J. Marks, Payman Arabshahi, Andrew Gray

Abstract— **We consider the problem of power controlled minimum frame length scheduling for TDMA wireless networks. Given a set of one-hop transmission requests, our objective is to schedule them in a minimum number of time slots, so that each slot schedule is free of self-interferences and meets desired SINR constraints. Additionally, the transmit power vector corresponding to each slot schedule should be minimal. We consider two different versions of the problem, a per-slot version and a per-frame version, and develop mixed integer linear programming models which can be used for solving the problems optimally. In addition, we propose a heuristic algorithm for the per-slot version.**

I. Introduction

In this paper, we consider the problem of power controlled adaptive frame length scheduling in TDMA wireless networks. To the best of our knowledge, the issue of joint scheduling and power control was first addressed by Tamer and Ephremides in [1, 2]. Given a set of one-hop transmission requests constituting a request list, they suggest a two-phase algorithm which essentially decouples the scheduling and power control objectives. The scheduling objective, which is used to remove "self-interferences" from the request list, is achieved by executing a centralized algorithm at the scheduler. After successful execution of this phase, the authors show that the power control problem in TDMA or hybrid TDMA/CDMA ad hoc networks is similar to the power control problem in cellular systems. Consequently, algorithms developed for the latter can be used for ad hoc networks.

In [1, 2], the authors assume that the frame length comprises a fixed number of time slots, which is determined heuristically. Our work, on the other hand, focusses on the adaptive frame length case wherein, given a request list, the objective is to schedule them in a minimum number of time slots, so that each slot schedule is free of self-interferences and meets desired SINR constraints. Additionally, the transmit power vector corresponding to each slot schedule should be minimal. Ideally, the optimization should be carried out on a per-frame basis. However, this approach requires an excessive number of variables prohibiting optimal offline analysis even for moderately sized request lists. Consequently, for the most part, we focus on a (sub-optimal) per-slot optimization approach which is much more tractable than a per-frame approach. Another aspect of our work, which differs from [1, 2], is our consideration of sectored antennas as opposed to omnidirectional antennas.

The minimum frame length scheduling objective in our work has also been researched in the context of link scheduling in spatial-TDMA (STDMA) networks [3, 4]. The work in [3] provides a comparison of graph-based versus SINR based link scheduling policies while [4] proposes optimal link and node scheduling algorithms using mixed integer linear programming (MILP) techniques. However, transmitter power is assumed to be fixed in these papers. Our work can therefore be seen as an extension of the link scheduling problem with variable transmitter powers. A closely related work is [5] which discusses a heuristic algorithm based on graph coloring.

The aspect of power control, on the other hand, has been researched extensively in the context of channelized and CDMA based cellular systems [6-17]. Several centralized as well as distributed algorithms have been suggested for the power control problem. While initial research focussed on linear first-order methods [7-10,12], faster second order linear methods [13] as well as nonlinear methods [14] have recently been suggested in the literature. Experimental results in [14] show that their nonlinear algorithm converges faster than the first order linear methods discussed in [10,12]. Besides the issue of algorithm design for fast distributed power control, there is a considerable body of research on convergence properties of distributed algorithms, of particular significance being the work by Huang and Yates [16].

The rest of this paper is organized as follows. In Section II, we outline our network model and assumptions. In Section III, we discuss MILP models for optimal solution of the per-frame and per-slot versions of the power controlled minimum frame length scheduling problem. We also include a numerical example which illustrates that a per-slot approach is not guaranteed to use a minimum number of time slots, as a per-frame approach would. A heuristic algorithm for the per-slot version is explained in Section IV. Finally, Section V summarizes our conclusions.

II. Network Model, Assumptions and Definitions

In this section, we outline our network model and assumptions. Some of these assumptions are similar to those made in [1, 2].

1. We consider a fixed N-node wireless network in a two-

A.K. Das is with Department of Electrical Engineering, University of Washington, Box 352500, Seattle, WA 98195.
e-mail: arindam@ee.washington.edu.

R.J. Marks is with School of Engineering and Computer Science, Baylor University, 1311 S. 5th Street, Waco, TX 76798-7356.
e-mail: Robert_Marks@Baylor.edu.

P. Arabshahi and A. Gray are with Jet Propulsion Laboratory, 4800 Oak Grove Drive, MS 238-343, Pasadena, CA 91109.
e-mail: {payman, gray}@arcadia.jpl.nasa.gov

dimensional plane. The nodes could be organized as a flat architecture or as a hierarchy of clusters. In either case, we assume the presence of a scheduler which is responsible for scheduling a set of transmission requests from different nodes. Nodes which wish to transmit in a particular frame send their requests to the scheduler which attempts to accommodate the requests using as few time slots as possible, meeting certain scheduling and signal-to-interference-noise-ratio (SINR) constraints. We will refer to the set of requests to be accommodated during a particular frame as the *request list*, denoted by $\mathcal{R} \stackrel{\triangle}{=} \{m \rightarrow n\}$, where m is the transmitting node and n is the receiving node. The element $\mathcal{R}_i$ refers to the ith transmission request in $\mathcal{R}$. Also, the notations $\mathcal{R}_i^{tx}$ and $\mathcal{R}_i^{rx}$ are used to refer to the transmitter and the receiver corresponding to the ith request. The minimum number of time slots required to accommodate all transmission requests in $\mathcal{R}$, subject to certain constraints discussed subsequently, constitutes a *frame*.
2. We assume that all transmission requests in the request list are of equal priority.
3. The data rate for all slots is fixed.
4. We assume that the data packets generated by the nodes are of identical length. Time is divided into equal sized slots, with each slot duration being equal to the packet transmission time plus an adequate "guard band".
5. We assume that the request list comprises of one-hop transmission requests only. Extending this work to the case where the request list may comprise of multi-hop transmission requests is a subject of future research.
6. We assume that each node is aware of the location of its neighbors by means of location-discovery schemes. This information is required for the receivers to feed back the SINR measurements to their transmitters. Additionally, we assume the existence of a separate *contention-free* control channel which the receivers use to feed back their SINR measurements.
7. Each node has the following scheduling constraints:

- It cannot transmit and receive in the same time slot. For example, the requests $i \rightarrow j$ and $j \rightarrow k$ cannot be scheduled in the same time slot.
- It cannot receive from more than one transmitter in the same time slot. For example, the requests $i \rightarrow j$ and $k \rightarrow j$ cannot be scheduled in the same time slot.
- It cannot transmit to more than one receiver in the same time slot. For example, the requests $i \rightarrow j$ and $i \rightarrow k$ cannot be scheduled in the same time slot.

Collectively, we refer to the above constraints as *primary constraints*. Any slot schedule which satisfies these constraints is referred to as a *primary feasible schedule* or a *conflict free schedule*. In graph theoretic terms, primary feasible schedules constitute matchings on a directed graph representation of the request list.
8. All nodes are assumed to have identical S-sector antennas. The number of sectors, S, is related to the beamwidth, θ (in degrees), as follows:

$$S = 360/\theta \tag{1}$$

Each sector is assumed to span the angular region $[(s-1)360/\theta, (s)360/\theta]$ in the 2-D plane, where $1 \leq s \leq S$ is the sector number. Note that $\theta = 360$ ($\Rightarrow S = 1$) corresponds to an omnidirectional antenna.
9. The efficiency of the antennas is assumed to be 100%; $i.e.$, all the input power is assumed to be converted to radiated power. With this assumption, the signal power at the intended receiver j, due to a transmission from node i at a power level of P_t Watts, is given by:

$$P_r = P_t K_t K_r G_{ij} \tag{2}$$

where K_t and K_r are the gains of the transmit and receive antennas and G_{ij} is the link gain from node i to node j. Strictly speaking, the received power is proportional to the quantity on the right hand side of (2) [21]. In the context of this paper, however, we assume w.l.o.g that the proportionality constant is equal to 1 and therefore (2) holds to equality.

Letting α ($2 \leq \alpha \leq 4$) be the channel loss exponent, d_{ij} the Euclidean distance in meters between nodes i and j and f_{ij} the flat fading component, the link gain G_{ij} is modelled as:

$$G_{ij} = f_{ij}\, d_{ij}^{-\alpha} \tag{3}$$

Note that even if the distance-dependent term in (3) is known precisely, the fading component is modelled as a non-negative random variable[1], which implies that it is impossible to have a perfect knowledge of the G_{ij} factors *a priori*.
10. We assume that both transmission and reception is directional. Consequently, the parameters K_t and K_r refer to the mainlobe gains of the transmit and receive antennas.
11. Typically, the mainlobe gain in sectored antennas is specified in units of dBi, or, with respect to an isotropic antenna. For example, a mainlobe gain of 12dBi implies that $K_t/K_{ti} = 16$, where K_{ti} is the gain of an isotropic antenna in transmit mode.

Additionally, the sidelobe and backlobe gains are generally specified with respect to the mainlobe gain, known as the front-to-back (F/B) ratio. For example, a F/B ratio of 20dB implies that the back and sidelobe gain is $1/100^{th}$ of the mainlobe gain.

In this paper, we make the following simplifying assumptions:

- The F/B ratio is exact and holds uniformly across the entire side/backlobe. It should be noted that, in practice, manufacturers may quote a minimum F/B ratio or specify it at a particular angle of the backlobe structure. In either case, there may be angular regions where signal attenuation may deviate from the specified F/B ratio.
- For any node, whether in transmit mode or in receive mode, the reference isotropic antenna has unity gain; $i.e.$, $K_{ti} = K_{ri} = 1$.
- For any node, whether in transmit mode or in receive mode, the mainlobe gain is proportional to antenna directivity. Coupled with our previous assumptions on antenna efficiency and reference isotropic antenna gain, this implies: $K_t = S \times K_{ti} = S$ and $K_r = S \times K_{ri} = S$, where S, the number of sectors, is related to sectoral beamwidth as in (1). In dB (or dBi) units, the mainlobe gain is therefore:

$$K_t = K_r = 10\log_{10} S \tag{4}$$

[1] Several probability distributions have been suggested for the fading parameter, $e.g.$, Nakagami, Rayleigh or Ricean.

Using the relation $K_t = K_r = S$, we simplify eqn. (2) as:

$$P_r = P_t S^2 G_{ij} \quad (5)$$

12. In [2], the authors assume an interference model whereby each transmitter can potentially cause interference at any other receiver, irrespective of the distance between them. This is in contrast with the interference model recognized in IEEE 802.11, for instance, which assumes a circular transmission range for each transmitter, beyond which interference is ignored. The authors in [2] argue that their interference model is more realistic since the aggregate interference from far-away transmitters may be significant enough to disrupt an ongoing transmission.

The interference model we assume is similar in spirit to that in [2]. However, because of different mainlobe and sidelobe gains in sectored antennas, the interference power at any receiver is also a function of the relative location of the transmitter w.r.t the receiver and *vice versa*. We illustrate with an example.

Consider the various transmission scenarios in Figure 1. Assume that the F/B ratio of all nodes is 20dB. In Figure 1(a), nodes i and k are located in the same sector with respect to node l. Also, nodes j and l are located in the same sector with respect to node i. Consequently, the mainlobe of l is directed towards k and i, and the mainlobe of i is directed towards j and l. The interference power at node l is therefore equal to: $P_{ij}S^2G_{il}$ where G_{il} is the link gain from node i to node l. However, in Figure 1(b), while the mainlobe of node i is still directed towards j and l, node l's backlobe is now directed towards i. Since an F/B ratio of 20dB corresponds to a backlobe gain of $1/100^{th}$ the mainlobe gain, the interference power at node l in this case is equal to: $P_{ij}(\frac{S^2}{100})G_{il}$. Finally, in Figure 1(c), the backlobe of node i is directed towards l and *vice versa*. Consequently, the interference power at node l is equal to: $P_{ij}(\frac{S^2}{10000})G_{il}$. In all three scenarios, the signal power at node l due to the transmission $k \to l$ is equal to $P_{kl}S^2G_{kl}$.

In general, given a transmission pair $i \to j$ and $k \to l$, the interference power at node l is equal to $P_{ij}S^2\delta_{il}G_{il}$, where δ_{il} is the *sectoring gain factor* at node l. Let θ_{mn} denote the sector location (1, 2, $\cdots$, S) of node n w.r.t node m. The parameter δ_{il} is then given by:

$$\delta_{il} = \begin{cases} 1, & \text{if } \theta_{ij} = \theta_{il} \text{ and } \theta_{lk} = \theta_{li} \\ \left(10^{0.1\times F/B}\right)^{-2}, & \text{if } \theta_{ij} \neq \theta_{il} \text{ and } \theta_{lk} \neq \theta_{li} \\ \left(10^{0.1\times F/B}\right)^{-1}, & \text{otherwise} \end{cases} \quad (6)$$

where F/B is the front-to-back ratio in dB. It should be noted that we have adopted a double subscript notation for δ_{il} for simplicity. Each δ parameter is dependent on a pair of transmissions and therefore δ_{il} should be interpreted as $\delta_{(i,j)(k,l)}$.

13. Following our above discussion, given a transmission pair $i \to j$ and $k \to l$, the SINR at node l, γ_{kl}, can be written as:

$$\gamma_{kl} = \frac{P_{kl}S^2G_{kl}}{P_{ij}S^2\delta_{il}G_{il} + \eta_{kl}} \quad (7)$$

where η_{kl} (> 0) is the thermal noise at receiver l. A double-subscript notation is used for γ and η to ensure a direct correspondence with the set of transmission requests in $\mathcal{R}$.

Dividing the numerator and the denominator of the right-hand-side of (7) by S^2, we obtain:

$$\gamma_{kl} = \frac{P_{kl}G_{kl}}{P_{ij}\delta_{il}G_{il} + (\eta_{kl}/S^2)} \quad (8)$$

14. Corresponding to every transmission request $i \to j$ in $\mathcal{R}$, we assume that there is a target SINR, denoted by $\hat{\gamma}_{ij}$, which must be met at receiver j.

$$\gamma_{ij} \geq \hat{\gamma}_{ij};\ \forall (i \to j) \in \mathcal{R} \quad (9)$$

We will refer to these constraints as the *secondary constraints*.

15. All nodes are equipped with limited capacity batteries. Furthermore, we assume that there is a constraint on the maximum power level (denoted by P^{max}) which a node can use for transmission and that this parameter is identical for all nodes. This maximum power level is adequate to meet SINR constraints for all transmission requests in the absence of any interference.

$$P^{max} \geq \frac{\hat{\gamma}_{ij}\eta_{ij}}{S^2G_{ij}};\ \forall (i \to j) \in \mathcal{R} \quad (10)$$

16. Any slot schedule which meets the primary and secondary constraints, and for which there exists a transmit power vector which is strictly positive and upper bounded by P^{max} ($i.e.$, all elements of the transmit power vector are less than or equal to P^{max}), is referred to as a *feasible schedule*. A feasible slot schedule which accommodates the maximum number of transmission requests is known as the *maximal feasible schedule*.

17. We mentioned in item 1 above that the responsibility of the scheduler is to accommodate the transmission requests using as few time slots as possible, meeting the primary and secondary constraints. Ideally, this optimization should be carried out on a per-frame basis. An alternative to the per-frame optimization approach is to adopt a sequential slot-by-slot optimization approach which ensures that each successive slot is maximally packed. Even for offline analysis, a sequential approach is eminently more solvable than a per-frame approach, as shown in the next section. We should emphasize, however, that a sequential approach is not guaranteed to use the minimum number of slots as a per-frame approach would. Furthermore, the transmit power vectors corresponding to a per-frame optimization model and a sequential per-slot optimization model need not be identical.

The (sub-optimal) heuristic algorithm we discuss in this paper is based on sequential slot-by-slot decision making. Broadly speaking, this involves (a) identifying the maximal feasible schedule and (b) computing the transmit power vector corresponding to the maximal feasible schedule. While both these functions can be executed in a centralized manner at the scheduler, we concentrate on the case where only identification of the maximal schedule is done at the scheduler but the corresponding power vector computation is executed in a distributed fashion by the transmitters. We assume that the scheduler has knowledge of the link gain factors (3), the sectoring gain factors[2] (6), the noise vector (η_{ij}'s), the target SINR's ($\hat{\gamma}_{ij}$'s), the number of sectors (S) and the maximum power level P^{max}.

[2]Since δ_{ij} is dependent on the relative angular locations of the transmitters and the receivers, we do not envision our approach to be feasible in a mobile network.

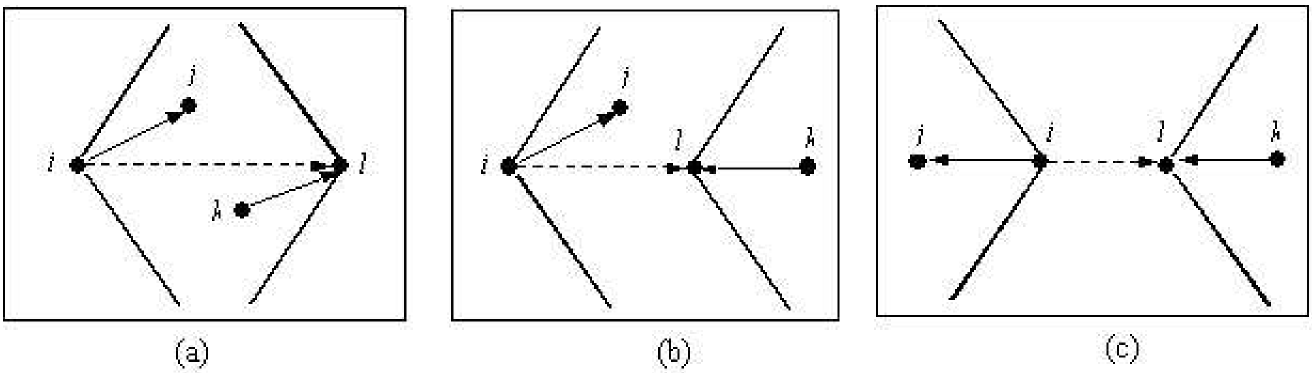

Fig. 1. Illustrating the effect of sectoring on interference power. Let $G_{il} = s_{il} d_{il}^{-\alpha}$ be the link gain from node i to node l. Assume that the F/B ratio of all nodes is 20dB. (a) Consider the effect of the transmission $i \rightarrow j$, at a power level P_{ij}, on receiver l. Note that nodes i and k are located in the same sector with respect to node l. Also, nodes j and l are located in the same sector with respect to node i. In this scenario, the mainlobe of l is directed towards k and i, and the mainlobe of i is directed towards j and l. Consequently, the interference power at node l is equal to: $P_{ij} S^2 G_{il}$. (b) In this figure, while the mainlobe of node i is still directed towards j and l, node l's backlobe is directed towards i. Since an F/B ratio of 20dB corresponds to a backlobe gain of $1/100^{th}$ the mainlobe gain, the interference power at node l in this case is equal to: $P_{ij}\left(\frac{S^2}{100}\right) G_{il}$. (c) In this figure, the backlobe of node i is directed towards l and *vice versa*. Consequently, the interference power at node l is equal to: $P_{ij}\left(\frac{S^2}{10000}\right) G_{il}$.

III. Mathematical Model

In this section, we develop mixed integer linear programming (MILP) models for solving the power controlled minimum frame length scheduling problem. The optimization approaches we discuss in this section assume perfect knowledge of the link gain and sectoring gain parameters and is therefore intended to be used for offline studies to benchmark the performance of practical, heuristic algorithms.

The first model we propose is for per-slot optimization which is solved iteratively until all transmission requests have been allocated one time slot. This model is subsequently extended for the per-frame case. We conclude this section with a numerical example which shows that slot optimization, as opposed to frame-optimization, may not be optimal in terms of the number of time slots used.

A. Per-slot optimization model

We first introduce the notation $\mathcal{R}(s)$ denoting the request list at the start of iteration s. We also define a set of nonnegative continuous power variables $\{P_{ij} : \forall (i \rightarrow j) \in \mathcal{R}(s)\}$, upper bounded by the parameter P^{max}, and a set of binary variables $\{X_{ij} : \forall (i \rightarrow j) \in \mathcal{R}(s)\}$ such that $X_{ij} = 1$ if the transmission $i \rightarrow j$ is scheduled in the current slot s and 0 otherwise. The variable definitions are therefore:

$$P_{ij} \geq 0; \qquad \forall (i \rightarrow j) \in \mathcal{R}(s) \qquad (11)$$

$$P_{ij} \leq P^{max}; \quad \forall (i \rightarrow j) \in \mathcal{R}(s) \qquad (12)$$

$$X_{ij} \in \{0, 1\}; \quad \forall (i \rightarrow j) \in \mathcal{R}(s) \qquad (13)$$

The objective function for slot s is:

$$\text{maximize: } \sum_{(i,j)} X_{ij} : \ (i \rightarrow j) \in \mathcal{R}(s) \qquad (14)$$

Next, we model the primary constraints listed in item (5), Section II. We denote the set of transmitting nodes in $\mathcal{R}(s)$ by $\mathcal{N}^{tx}$ and the set of receiving nodes in $\mathcal{R}(s)$ by $\mathcal{N}^{rx}$. Let $\mathcal{N}$ be the set of all nodes in $\mathcal{R}(s)$; $i.e., \mathcal{N} = \{\mathcal{N}^{tx} \cup \mathcal{N}^{rx}\}$.

1) A node cannot receive from more than one transmitter in the same time slot. In graph theoretic terms, this is equivalent to the statement "the indegree[3] of any node in the slot schedule must be less than or equal to 1", which is modelled as:

$$\sum_{k} X_{ki} \leq 1 : \forall i \in \mathcal{N}^{rx}, \ (k \rightarrow i) \in \mathcal{R}(s) \qquad (15)$$

2) A node cannot transmit to more than one receiver in the same time slot. In graph theoretic terms, this is equivalent to the statement "the outdegree[4] of any node in the slot schedule must be less than or equal to 1", which is modelled as:

$$\sum_{j} X_{ij} \leq 1 : \ \forall i \in \mathcal{N}^{tx}, \ (i \rightarrow j) \in \mathcal{R}(s) \qquad (16)$$

3) A node cannot transmit and receive in the same time slot. Coupled with the previous two constraints, this condition is equivalent to the statement "the degree[5] of any node in the slot schedule must be less than or equal to 1". This is modelled as:

$$\sum_{j} X_{ij} + \sum_{k} X_{ki} \leq 1 : \forall i \in \mathcal{N} \qquad (17)$$

Although constraints (15) and (16) are subsumed by the aggregated constraint (17), experimental results suggest that using all three sets of constraints (or either (15) or (16) along with (17)) usually results in a considerable speed-up of the solution time[6].

We now turn our attention to the SINR constraints or the secondary constraints (item (12), Section II. By a straightforward extension of (7), the SINR constraint corresponding to the transmission request $i \rightarrow j$ can be written as:

$$\frac{P_{ij} G_{ij}}{\sum_{(k,l)} P_{kl} \delta_{kj} G_{kj} + (\eta_{ij}/S^2)} \geq \hat{\gamma}_{ij} \qquad (18)$$

[3]The indegree of a node is defined as the number of links incident to the node.

[4]The outdegree of a node is defined as the number of links directed away from the node.

[5]The degree of a node is defined as the sum of its indegree and outdegree.

[6]This phenomenon is certainly not atypical in integer programs.

where $(i \to j) \neq (k \to l) \in \mathcal{R}(s)$[7]. Recall from item (10), Section II, that the parameter δ_{kj} reflects the sectoring gain at node j due to the interfering transmission $k \to l$ when it is receiving from node i. The restriction $(i \to j) \neq (k \to l)$ is valid since constraints (15) - (17) will force the optimal slot schedule to be conflict free.

Note, however, that (18) is really a conditional constraint which ought to be applied only if $i \to j$ is allocated slot s. Otherwise, this constraint should be ignored. In other words, the r.h.s of (18) should be set to 0 if $i \to j$ is not allocated slot s. In order to express (18) as a conditional, we first rewrite it as follows:

$$\frac{P_{ij}G_{ij}}{\sum_{(k,l)\neq(i,j)} P_{kl}\delta_{kj}G_{kj} + (\eta_{ij}/S^2)} \geq \hat{\gamma}_{ij}$$
$$\Rightarrow P_{ij} \geq \frac{\hat{\gamma}_{ij}\eta_{ij}}{S^2 G_{ij}} + \hat{\gamma}_{ij} \sum_{(k,l)\neq(i,j)} \frac{P_{kl}\delta_{kj}G_{kj}}{G_{ij}} \tag{19}$$

Since $P_{ij} \leq P^{max}$, for all $i \to j$, the maximum value of the r.h.s of (19) is given by:

$$\frac{\hat{\gamma}_{ij}\eta_{ij}}{S^2 G_{ij}} + \sum_{(k,l)\neq(i,j)} \frac{\hat{\gamma}_{ij} P^{max}\delta_{kj}G_{kj}}{G_{ij}} \tag{20}$$

Note that all the parameters involved in (20) are assumed to be known and therefore (20) can be solved explicitly.

Next, we define a set of constants $\{M_{ij} : \forall (i \to j) \in \mathcal{R}(s)\}$ such that (21) is satisfied:

$$M_{ij} < -\left[\frac{\hat{\gamma}_{ij}\eta_{ij}}{S^2 G_{ij}} + \hat{\gamma}_{ij} \sum_{(k,l)\neq(i,j)} \frac{P^{max}\delta_{kj}G_{kj}}{G_{ij}}\right] \tag{21}$$

Then, equation (22) correctly conditionalizes constraint (18).

$$P_{ij} - \left[\frac{\hat{\gamma}_{ij}\eta_{ij}}{S^2 G_{ij}} + \hat{\gamma}_{ij} \sum_{(k,l)\neq(i,j)} \frac{P_{kl}\delta_{kj}G_{kj}}{G_{ij}}\right] + M_{ij}X_{ij} \geq M_{ij} \tag{22}$$

To see why, let us first consider the case when $i \to j$ is allocated the current slot $\Rightarrow X_{ij} = 1$. It is easy to verify that, in this case, equation (22) reduces to the desired SINR constraint (19). If $i \to j$ is not allocated the current slot $\Rightarrow X_{ij} = 0$, equation (22) reduces to the redundant ($i.e.$, superseded by the non-negativity constraints on P_{ij} variables) constraint:

$$P_{ij} \geq M_{ij} + \left[\frac{\hat{\gamma}_{ij}\eta_{ij}}{S^2 G_{ij}} + \hat{\gamma}_{ij} \sum_{(k,l)\neq(i,j)} \frac{P_{kl}\delta_{kj}G_{kj}}{G_{ij}}\right]$$

since the expression on the r.h.s of the inequality is strictly negative by (21).

To summarize, solving the objective function (14), subject to constraints (11) - (13), (15) - (17) and (22) yields the maximal feasible schedule for slot s. Each iteration of the sequential MILP involves $|\mathcal{R}(s)|$ binary variables and $|\mathcal{R}(s)|$ continuous variables, where $|\mathcal{R}(s)|$ is the number of requests in $\mathcal{R}(s)$.

[7]The notation $(i \to j) \neq (k \to l)$ implies $i \neq j \neq k \neq l$.

If all requests in $\mathcal{R}(s)$ have been allocated to slot s, the iterative method terminates. Otherwise, we compute $\mathcal{R}(s+1)$ as:

$$\mathcal{R}(s+1) \leftarrow \mathcal{R}(s) \setminus \mathcal{RA}(s) \tag{23}$$

where $\mathcal{RA}(s)$ is the set of requests which have been allocated slot s and solve the optimization problem again on $\mathcal{R}(s+1)$. Figure 2 provides a flow diagram of the sequential MILP optimization algorithm.

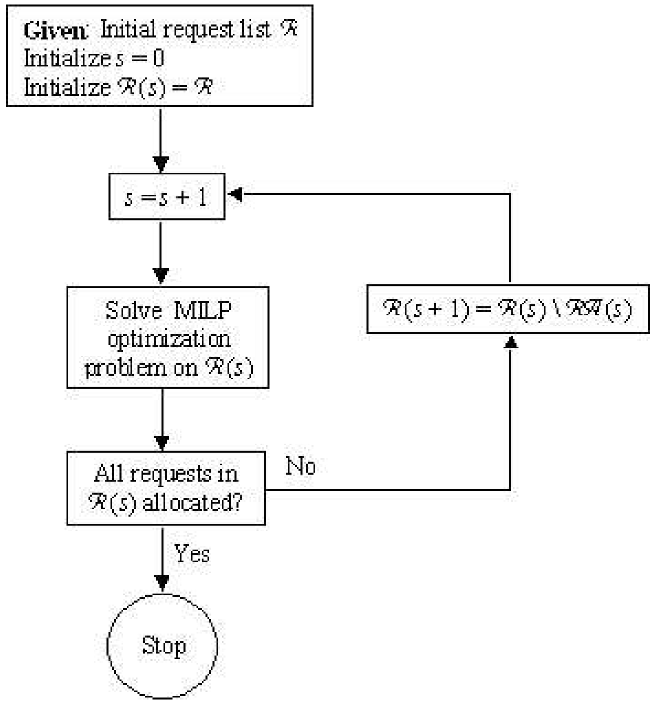

Fig. 2. Flow diagram of the sequential MILP optimization algorithm. The notation $\mathcal{R}(s)$ denotes the request list at the start of iteration s and $\mathcal{RA}(s)$ denotes the set of requests in $\mathcal{R}(s)$ which have been allocated to slot s.

It is clear from the nature of the iterative algorithm that the initial slots would be more heavily packed than the latter slots. Later, in Section IV, we concretize the notion of "heavy packing" in terms of the spectral radius of a coefficient matrix. For now, it suffices to say that heavily packed slots may exhibit extreme sensitivity to even minor perturbations of the coefficient matrix, which is to be expected in view of the dependence of link gain factors on a random fading component. In fact, the optimization problem with a slightly perturbed coefficient matrix may not even remain feasible, particularly with respect to the limiting constraints on P_{ij} variables (12). In that event, even if a slot is optimally packed, it might be prudent to remove a transmission which exhibits the greatest sensitivity to perturbations and reschedule it for a latter slot.

Before concluding this section, we note that the objective in (14) is a function of the scheduling variables $\{X_{ij}\}$ only, the power variables appearing as constraint satisfying auxiliary variables within the MILP model. It might so happen that there is more than one schedule which allocates the same number of requests (though not identical) to the current slot but with different power vectors. For example, suppose that one could choose between two schedules $\mathcal{S}_1$ and $\mathcal{S}_2$, where $|\mathcal{S}_1| = |\mathcal{S}_2|$. Let the power vectors corresponding to these schedules be $\vec{P}(\mathcal{S}_1)$ and $\vec{P}(\mathcal{S}_2)$, such that $\sum_{i\in\mathcal{S}_1} P_i(\mathcal{S}_1) < \sum_{i\in\mathcal{S}_2} P_i(\mathcal{S}_2)$. In this case, an objective function involving only the scheduling variables will have no incentive in choosing the most power efficient schedule which is $\mathcal{S}_1$.

Even if an unique optimal schedule exists, in the absence of any power cost in the objective function, the integer program may not return the optimum power vector; $i.e.$, the power values may not satisfy equations (18) to strict equality. Worse still, requests which have not been allocated may have a positive power cost since it does not affect the value of the objective function. In offline analysis, these issues may not be as critical as the one explained in the previous paragraph since one can always solve the linear system of equations (18) to equality to obtain the optimum power vector corresponding to the slot schedule.

To force the optimization model to choose the *maximally packed* schedule with *minimum power cost*, we can use the following modified objective function which is a convex combination of the scheduling variables and the power variables:

$$\text{maximize: } (1-\mu)\sum_{(i,j)} X_{ij} - \mu\sum_{(i,j)} P_{ij} \tag{24}$$

where $0 \le \mu \le 1$ and the summations in (24) are taken over all $(i \to j) \in \mathcal{R}(s)$. Note that the power term in (24) has been negated to conform with the maximization of the scheduling variables.

As with all weighted objectives, care must be taken in choosing a proper value of μ which result in the desired solution. Viewing the first term in (24) as a reward and the second term as a penalty, it is easy to see that a transmission request will be allocated to the current slot only if the associated reward, $1-\mu$, is greater than the maximum possible penalty, which is equal to μP^{max} since $P_{ij} \le P^{max}, \forall(i \to j)$. A proper choice of μ should therefore satisfy:

$$1-\mu > \mu P^{max} \;\Rightarrow\; \mu < \frac{1}{1+P^{max}} \tag{25}$$

B. Discrete power levels

In practice, transmitters usually do not have continuous power control but instead have to choose from a discrete set of power levels. The MILP model we discussed above can be easily modified to handle the discrete power case.

Let $\{Y_k : 1 \le k \le K\}$ be the set of ordered discrete power levels for each node such that $P^{max} = Y_K$. Corresponding to each power variable P_{ij}, define a set of K binary variables $\{\beta_{ijk} : 1 \le k \le K\}$ such that:

$$\sum_k \beta_{ijk} = 1; \forall(i \to j) \in \mathcal{R}(s) \tag{26}$$

The power variable P_{ij} can then be expressed as:

$$P_{ij} = \sum_k \beta_{ijk} Y_k; \forall(i \to j) \in \mathcal{R}(s) \tag{27}$$

It is obvious that no bounds on the P_{ij} variables (see equations 11 and 12) are needed. All other constraints hold for the discrete power case. Each iteration of the sequential MILP now involves $(K+1)\cdot|\mathcal{R}(s)|$ binary variables and $|\mathcal{R}(s)|$ continuous variables.

C. Per-frame optimization model

The per slot optimization model we discussed in Section III-A can be extended straightforwardly to the per-frame case. Let $\{B_t : 1 \le t \le T^{max}\}$ be a set of binary variables such that $B_t = 1$ if slot t is occupied and 0 otherwise. The maximum number of allowable slots is denoted by T^{max} which is usually obtained heuristically[8]. In the absence of any known upper bound on the number of slots, one can set T^{max} equal to the number of requests in the request list, $i.e.$, $T^{max} = |\mathcal{R}|$. As we will soon see, this results in an integer program with $O(|\mathcal{R}|^2)$ binary variables. Even for moderately sized request lists, $e.g.$ $|\mathcal{R}| \approx 50$, solving an integer program with 2500 binary variables might require an unacceptably high solution time even with state-of-the-art commercial ILP solvers. The per-slot ILP model, on the other hand, requires about $O(|\mathcal{R}|)$ binary variables in the worst case (the initial slots). Since integer linear programming is of exponential time complexity, the time required to solve $|\mathcal{R}|$ integer programs, each with $O(|\mathcal{R}|)$ binary variables, is usually considerably smaller than the time required to solve one integer program with $O(|\mathcal{R}|^2)$ binary variables.

In a per-frame model, since a request in $\mathcal{R}$ can be scheduled in any of T^{max} possible time slots, the X_{ij} and P_{ij} variables need to be defined per slot. Our set of variables are therefore: $\{X_{ijt} : \forall(i \to j) \in \mathcal{R}, 1 \le t \le T^{max}\}$ and $\{P_{ijt} : \forall(i \to j) \in \mathcal{R}, 1 \le t \le T^{max}\}$. We also let $\mathcal{N}^{tx}$ be the set of transmitting nodes in $\mathcal{R}$, $\mathcal{N}^{rx}$ the set of receiving nodes in $\mathcal{R}$ and $\mathcal{N} = \{\mathcal{N}^{tx} \cup \mathcal{N}^{rx}\}$ the set of all nodes in $\mathcal{R}$.

The objective function in the per-frame model is a convex combination of the total number of slots used and the sum of the transmit powers, which should be jointly minimized as shown in (29). As in (24), the parameter μ should be chosen carefully so that the "maximal packing" objective (the first term in (29)) takes precedence over the "power minimization" objective which is the second term in (29). Note that these two objectives are contradictory; a frame schedule which packs all transmission requests in one slot will have a much higher power cost than a schedule which allocates one request per slot. In the latter case, all slots are interference-free and the transmit power cost is determined essentially by the link gain factor and receiver thermal noise. A proper choice of μ is obtained by considering the extreme case when all requests can be packed in one slot. In order to dissuade the optimization algorithm from using any additional slot, it should be ensured that the cost of using even one additional slot, $1-\mu$, is greater than the maximum possible savings in power cost. The following inequality can be used for a conservative estimate of the parameter μ:

$$1-\mu > \mu|\mathcal{R}|P^{max} \;\Rightarrow\; \mu < \frac{1}{1+|\mathcal{R}|\cdot P^{max}} \tag{28}$$

[8]For example, by a sequential application of the procedure discussed in Section IV.

$$\text{minimize: } (1-\mu)\sum_{t}^{T^{max}} B_t + \mu \sum_{t}^{T^{max}} \sum_{(i,j)\in\mathcal{R}} P_{ijt} \tag{29}$$

subject to

$$B_t \in \{0,1\}; \quad 1 \le t \le T^{max} \tag{30}$$

$$0 \le P_{ijt} \le P^{max}; \quad \forall (i \to j) \in \mathcal{R}, \forall t \tag{31}$$

$$X_{ijt} \in \{0,1\}; \quad \forall (i \to j) \in \mathcal{R}, \forall t \tag{32}$$

$$\sum_{t}^{T^{max}} X_{ijt} = 1; \forall (i \to j) \in \mathcal{R} \tag{33}$$

$$|\mathcal{R}|B_t - \sum_{(i,j)\in\mathcal{R}} X_{ijt} \ge 0; \ \forall t \tag{34}$$

$$\sum_{k} X_{kit} \le 1; \forall i \in \mathcal{N}^{rx}, (k \to i) \in \mathcal{R}, \forall t \tag{35}$$

$$\sum_{j} X_{ijt} \le 1; \forall i \in \mathcal{N}^{tx}, (i \to j) \in \mathcal{R}, \forall t \tag{36}$$

$$\sum_{k} X_{kit} + \sum_{j} X_{ijt} \le 1; \forall i \in \mathcal{N}, \ \forall t \tag{37}$$

$$P_{ijt} - \left[\frac{\hat{\gamma}_{ij}\eta_{ij}}{S^2 G_{ij}} + \hat{\gamma}_{ij} \sum_{(k,l)\neq(i,j)} \frac{P_{klt}\delta_{kj}G_{kj}}{G_{ij}}\right] + M_{ij}X_{ijt} \ge M_{ij}; \ \forall (i \to j) \in \mathcal{R}, \ \forall t \tag{38}$$

Fig. 3. MILP model for per-frame optimization.

The rest of the model is shown in Figure 3. Equations (30) - (32) are variable definitions. Other constraints are interpreted as follows:

- *Constraint* (33): This constraint ensures that each request in $\mathcal{R}$ is allocated one time slot.
- *Constraint* (34): This constraint forces B_t to be equal to 1 if at least one request is allocated to slot t. Strictly speaking, the coefficient of each variable B_t can be set equal to the cardinality of the maximum matching of $\mathcal{R}$, which we denote by $|\mathcal{MR}|$. Any slot schedule which has more than $|\mathcal{MR}|$ transmissions must have at least one conflicting pair of transmissions and is therefore infeasible.
- *Constraints* (35) - (37): These constraints ensure that all slot assignments are conflict free and are analogous to (15) - (17), but expressed for each possible slot. Similar to the per-slot model, equations (35) and (36) are subsumed by (37), but using one of them with (37) may speed up the solution time.
- *Constraint* (38): This conditional constraint on SINR requirements is analogous to (22). Note that the variables in (38) are slot-indexed, but the constants, including the M_{ij}'s, are not.

D. Numerical Example: Per-frame vs. Per-slot

In this section, we illustrate with an example the difference between a per-frame optimization approach versus a per-slot optimization approach. Consider the simple 9-node network in Figure 4, the directed lines representing the transmission requests. The nodes are located in a 50m $\times$ 50m grid. We assume the following parameter choices for all requests: $f_{ij} = 1$, $\eta_{ij} = 0.001$ and $\hat{\gamma}_{ij} = 2$. Also, let $\alpha = 2$, $P^{max} = 10$ and $S = 1$.

Figures 4(a) and 4(b) show the optimal slot schedules obtained using the per-frame and per-slot optimization models, for $\mu = 0.001$. The models were solved using the commercially available MILP solver *LINDO* [22]. The numbers above the directed edges represent the slots to which the requests have been assigned. While the per-frame model uses 3 slots to accommodate all requests, the per-slot model uses 4 slots. A comparison of the slot schedules reveals that an additional slot is needed by the per-slot model because of its selection of requests for the first time slot. Specifically, while the per-frame version allocates $3 \to 5$ and $9 \to 1$ to the same slot, the per-slot version allocates $3 \to 5$ and $7 \to 4$ to slot 1 since $7 \to 4$ requires a smaller transmit power support than $9 \to 1$.

From the aspect of transmit power cost, however, the total power cost for the per-slot model (6.675 units) is much smaller than the total transmit power cost of the per-frame model (11.915 units). This is expected since lightly packed slots will, in general, incur smaller transmit power costs than slots which are more heavily packed.

IV. Heuristic Approach

In this section, we explain a sub-optimal heuristic method for obtaining a maximal feasible slot schedule. The method utilizes some properties of non-negative and M(inkowski) matrices, which we briefly review below. After a maximal feasible slot schedule has been identified, the corresponding transmit power vector can be computed in a distributed fashion. As mentioned in Section I, this problem has been researched extensively in the context of cellular power control and we exclude it from the scope of this section. We would like to refer the reader to the bibliography at the end of this paper, in particular [13] and [14], which deal with distributed second order power control methods with very fast convergence properties.

First, we establish some notations.

A. Notation

$\mathbf{A}$	$\mathbf{A}$ is a matrix
$\mathbf{A}'$	transpose of $\mathbf{A}$
$\vec{X}$	$\vec{X}$ is a column vector
$\vec{e}$	unit column vector
$\|\cdot\|_p$	p-norm of a vector or matrix
$\mathbf{A} \ge 0$	each element of $\mathbf{A}$ is non-negative.
$\mathbf{A} > 0$	$\mathbf{A} \ge 0$ and at least one element of $\mathbf{A}$ is positive.
$\mathbf{A} \gg 0$	each element of $\mathbf{A}$ is positive.
$\sigma(\mathbf{A})$	set of all eigenvalues (*spectrum*) of $\mathbf{A}$.
$\rho(\mathbf{A})$	spectral radius of $\mathbf{A}$.
UB$(\rho(\mathbf{A}))$	upper bound on the spectral radius of $\mathbf{A}$.
$\mathcal{R}^{n\times n}$	set of all $n \times n$ real matrices.
$\mathcal{Z}^{n\times n}$	set of all matrices $\mathbf{A} \in \mathcal{R}^{n\times n}$ with $\mathbf{A}_{ij} \le 0, \forall i \neq j$

In addition, we will use the notation $|\cdot|$ to denote either the absolute value of a number or the cardinality of a set. The usage will be clear from the type of the argument.

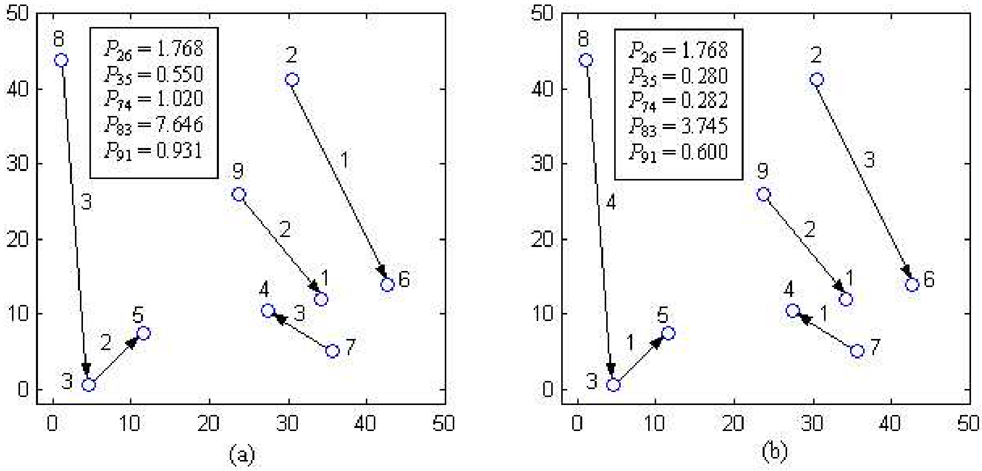

Fig. 4. Slot schedules obtained using (a) the per-frame model and (b) the per-slot model. $\mu = 0.001$ was used for both models. The per-frame model uses 3 slots to accommodate all requests but the per-slot model uses 4 slots. The numbers above the directed edges represent the slots to which the requests have been assigned. Because the per-slot model uses one more time slot, its total transmit power cost (6.675 units) is smaller than the total transmit power cost of the per-frame model (11.915 units).

B. Some definitions and theorems

Definition 1: The ∞-norm (maximum absolute row sum norm) of any matrix $\mathbf{A} \in \mathcal{R}^{n\times n}$ is defined as:

$$\| \mathbf{A} \|_\infty = max_i \sum_j |\mathbf{A}_{ij}| \tag{39}$$

Definition 2: If $\{\lambda_1, \cdots \lambda_n\}$ is the set of eigenvalues of an $n \times n$ square matrix $\mathbf{A}$, the spectral radius of $\mathbf{A}$ is defined as:

$$\rho(\mathbf{A}) = max_i|\lambda_i| \tag{40}$$

Definition 3: The *directed graph* of an $n \times n$ matrix $\mathbf{A}$, $G(\mathbf{A})$, consists of n vertices, $\{V_1, V_2, \cdots V_n\}$, where an edge leads from V_i to V_j if and only if $\mathbf{A}_{ij} \neq 0$.

Definition 4: Any matrix $\mathbf{A}$ of the form:

$$\mathbf{A} = s\mathbf{I} - \mathbf{B}, \ \ s > 0, \ \ \mathbf{B} \geq 0 \tag{41}$$

where $\mathbf{I}$ is the identity matrix and for which $s \geq \rho(\mathbf{B})$ is called an M-$matrix$. Note that $\mathbf{A}$ is characterized by non-positive off-diagonal elements and non-negative diagonal elements.

Definition 5: An M-matrix $\mathbf{A}$ is *non-singular* if $s > \rho(\mathbf{B})$. If $s = \rho(\mathbf{B})$, the matrix $\mathbf{A}$ is *singular*.

We now provide a compilation of several theorems pertaining mostly to positive and non-negative matrices. These theorems and, in most cases, their proofs can be found in the references cited.

Theorem 1: If $\| \cdot \|$ is any matrix norm and if $\mathbf{A} \in \mathcal{R}^{n\times n}$ (see Sections 5.6.8 and 5.6.9 of [19]), then:

$$\rho(\mathbf{A}) \ \leq \| \mathbf{A} \| \tag{42}$$

Theorem 2: A non-negative matrix $\mathbf{A} \in \mathcal{R}^{n\times n}$ is *irreducible* if and only if, for every (i, j), there exists a natural number q such that $\mathbf{A}_{ij}^q > 0$ (see Chapter 2, Section 2 of [18]).

In graph theoretic terms, the above condition is equivalent to the statement "the matrix $\mathbf{A}$ is irreducible if and only if the directed graph corresponding to $\mathbf{A}$ (see Definition 3) is strongly connected".

Clearly, while not all non-negative matrices are not necessarily irreducible, all positive matrices are irreducible. The class of positive matrices is therefore contained within the class of nonnegative irreducible matrices.

Theorem 3: For any non-negative and irreducible $\mathbf{A} \in \mathcal{R}^{n\times n}$, the spectral radius of $\mathbf{A}$ is bounded by (see Section 8.1.22 of [19]):

$$\min_i \sum_j \mathbf{A}_{ij} \leq r \leq \max_i \sum_j \mathbf{A}_{ij} \tag{43}$$

$$\min_i \sum_j \mathbf{A}_{ji} \leq r \leq \max_i \sum_j \mathbf{A}_{ji} \tag{44}$$

Moreover, the inequalities in (43) and (44) are strict unless the upper and lower bounds are equal. We will refer to these bounds as the Frobenius bounds. Note that the upper bounds in (43) and (44) are equal to the ∞ and 1-norms since $\mathbf{A}$ is non-negative.

Theorem 4: Let $\mathbf{B} \in \mathcal{R}^{n\times n}$ with $\mathbf{B} \geq 0$. If $\mathbf{A} = s\mathbf{I} - \mathbf{B}$ where $s > 0$, then $\mathbf{A}$ is non-singular and $\mathbf{A}^{-1} \geq 0$ if and only if $s > \rho(\mathbf{B})$. Moreover, $\mathbf{A}$ is *inverse-positive*, $i.e.$, $\mathbf{A}^{-1} \gg 0$, if and only if $s > \rho(\mathbf{B})$ and $\mathbf{B}$ is irreducible. (for a proof of this theorem, see Section 3.11, pp. 145 of [18])

Theorem 5: Let $\mathbf{A} \in \mathcal{R}^{n\times n}$, $n \geq 2$ be a non-singular M-matrix. Then each principal sub-matrix of $\mathbf{A}$ is inverse-positive. (see Section 2.4, pp. 140 of [18] for a proof)

C. The algorithm

The material in this section essentially answers the question "Given a *conflict-free* request list, what is the maximum number of requests that can be accommodated in the current slot, such that all SINR constraints (18) are satisfied and a feasible transmit power vector exists"? A power vector is considered feasible if all elements of the vector are positive and upper bounded by P^{max}. Towards that end, we propose a couple of conditions which are sufficient to guarantee that a feasible power vector exists. We emphasize that the algorithm we discuss cannot handle the primary constraints (see item 6, Section II) and therefore these should first be removed by executing a maximum matching algorithm on the request list $\mathcal{R}$[9]. Let $\mathcal{MR}$ denote the maximum matching of $\mathcal{R}$.

We first focus on the set of inequalities (18) which we rewrite as follows:

$$\frac{P_{ij}G_{ij}}{\sum_{(k,l)\neq(i,j)} P_{kl}\delta_{kj}G_{kj} + (\eta_{ij}/S^2)} \geq \hat{\gamma}_{ij};\ \forall (i \to j) \in \mathcal{MR}$$
$$\Rightarrow (G_{ij}/\hat{\gamma}_{ij})P_{ij} \ \geq \sum_{(k,l)\neq(i,j)} P_{kl}\delta_{kj}G_{kj} + (\eta_{ij}/S^2) \quad (45)$$

Let $\mathbf{D}$ be a $|\mathcal{MR}| \times |\mathcal{MR}|$ diagonal matrix such that its $(i,k)th$ element is given by:

$$\mathbf{D}_{ik} = \begin{cases} G_{\mathcal{MR}_i^{tx},\mathcal{MR}_i^{rx}}/\hat{\gamma}_{\mathcal{MR}_i^{tx},\mathcal{MR}_i^{rx}} & i = k \\ 0 & \text{otherwise} \end{cases} \quad (46)$$

Recall that $\mathcal{MR}_i^{tx}$ and $\mathcal{MR}_i^{rx}$ denote the transmitting and receiving nodes corresponding to the ith request in $\mathcal{MR}$. All diagonal elements of $\mathbf{D}$ are smaller than 1 since all link gain factors are smaller than 1 and all target SINR's are greater than 1 (or, > 0 dB).

Also, let $\mathbf{B}$ be a $|\mathcal{MR}| \times |\mathcal{MR}|$ matrix whose $(i,k)th$ is given by:

$$\mathbf{B}_{ik} = \begin{cases} \delta_{\mathcal{MR}_k^{tx},\mathcal{MR}_i^{rx}} G_{\mathcal{MR}_k^{tx},\mathcal{MR}_i^{rx}} & i \neq k \\ 0 & \text{otherwise} \end{cases} \quad (47)$$

Following our assumption that every transmitter causes interference at all other receivers, irrespective of the distance between them (see item 12, Section II), and because of the positivity of all δ_{ij}'s and G_{ij}'s, it is easy to see that the $\mathbf{B}$ is strictly positive ($\mathbf{B} \gg 0$).

As an example, suppose $\mathcal{MR} = \{(3 \to 4), (1 \to 2), (5 \to 6)\}$. The matrices $\mathbf{D}$ and $\mathbf{B}$ corresponding to $\mathcal{MR}$ are:

$$\mathbf{D} = \begin{bmatrix} G_{34}/\hat{\gamma}_{34} & 0 & 0 \\ 0 & G_{12}/\hat{\gamma}_{12} & 0 \\ 0 & 0 & G_{56}/\hat{\gamma}_{56} \end{bmatrix} \quad (48)$$

$$\mathbf{B} = \begin{bmatrix} 0 & \delta_{14}G_{14} & \delta_{54}G_{54} \\ \delta_{32}G_{32} & 0 & \delta_{52}G_{52} \\ \delta_{36}G_{36} & \delta_{16}G_{16} & 0 \end{bmatrix} \quad (49)$$

Finally, we define the vector $\vec{\zeta}$, the ith element of which is given by η_{ij}/S^2.

$$\vec{\zeta} \overset{\triangle}{=} \left\{ \frac{\eta_{ij}}{S^2};\ \forall (i \to j) \in \mathcal{MR} \right\} \quad (50)$$

Note that $\vec{\zeta} \gg 0$ since $\vec{\eta} = \{\eta_{ij}; \forall (i \to j) \in \mathcal{MR}\} \gg 0$ by assumption.

With the above definitions in place, equation (45) can be written in matrix form as follows:

$$(\mathbf{D} - \mathbf{B})\vec{P} \geq \vec{\zeta} \quad (51)$$

where $\vec{P}$ is the unknown transmit power vector. Since all diagonal elements of $\mathbf{D}$ are in the interval $(0,1)$, we can rewrite it as:

[9]For computing the maximum matching, a request list can be interpreted as a directed graph with edges defined by the elements of $\mathcal{R}$. All edges in the digraph can be assigned an unit weight.

$$\mathbf{D} = \mathbf{I} - \tilde{\mathbf{D}} \quad (52)$$

where $\mathbf{I}$ is the identity matrix and $\tilde{\mathbf{D}}$ is a diagonal matrix whose $(i,i)th$ element is equal to $1 - \mathbf{D}_{ii}$. Defining

$$\tilde{\mathbf{B}} = \tilde{\mathbf{D}} + \mathbf{B} = \mathbf{I} - \mathbf{D} + \mathbf{B} \ \cdots \ (\text{using eqn. } 52) \quad (53)$$

we can rewrite (51) as follows:

$$(\mathbf{I} - \tilde{\mathbf{B}})\vec{P} \geq \vec{\zeta} \quad (54)$$

The minimum power solution of (54) is therefore given by:

$$\vec{P}^{opt} = (\mathbf{I} - \tilde{\mathbf{B}})^{-1}\vec{\zeta} \quad (55)$$

if the inverse exists.

It is easy to see that the matrix $\tilde{\mathbf{B}}$ is irreducible (see Theorem 2, Section IV-B) since $\tilde{\mathbf{B}} \gg 0$. Also, $\mathbf{A} \overset{\triangle}{=} (\mathbf{I} - \tilde{\mathbf{B}})$ is an M-matrix (41), with $s = 1$. By Theorem 4 in Section IV-B, therefore, $\mathbf{A}^{-1} \gg 0$ if $\rho(\tilde{\mathbf{B}}) < 1$, where $\rho(\tilde{\mathbf{B}})$ is the spectral radius of $\tilde{\mathbf{B}}$. If $\rho(\tilde{\mathbf{B}}) < 1$, the power vector $\vec{P}^{opt} = \mathbf{A}^{-1}\vec{\zeta} \gg 0$ since $\vec{\zeta} \gg 0$.

For reasons that will be apparent when we derive our second lemma, we strengthen the condition $\rho(\tilde{\mathbf{B}}) < 1$ with $\parallel \tilde{\mathbf{B}} \parallel_\infty < 1$. Note that, by Theorem 1, $\parallel \tilde{\mathbf{B}} \parallel_\infty < 1 \ \Rightarrow \ \rho(\tilde{\mathbf{B}}) < 1$. We therefore have the following lemma:

Lemma 1: The matrix $\mathbf{A} = (\mathbf{I} - \tilde{\mathbf{B}})$ is inverse-positive $\Rightarrow$ $\vec{P}^{opt}$ is strictly positive if:

$$\parallel \tilde{\mathbf{B}} \parallel_\infty < 1 \quad (56)$$

where $\tilde{\mathbf{B}}$ is as defined in (53).

While Lemma 1 is a sufficient condition for ensuring positivity of the transmit power vector, it depends critically on the accuracy of the $\tilde{\mathbf{B}}$ matrix. In practice, since it is impossible to know the link gains and the fading factors precisely, a slight perturbation of $\tilde{\mathbf{B}}$ might cause the spectral radius of the perturbed matrix to be greater than 1, in which case a finite positive power vector solution may not exist. This problem will be especially acute for heavily packed slots, which we characterize as those for which the difference $1 - \rho(\tilde{\mathbf{B}})$ is small. Smaller this difference, the higher is the sensitivity of the optimal power vector to perturbations of the matrix $\tilde{\mathbf{B}}$. As mentioned in Section III-A, it might therefore be prudent to remove one[10] or more transmissions from heavily packed slots so that the resultant slot schedule remains feasible in the event of expected perturbations.

The preceding paragraph also indicates the need for a fast iterative algorithm which could be run at the scheduler for quick and accurate determination of the spectral radius of $\tilde{\mathbf{B}}$. Such an algorithm was proposed by Hall and Porsching [20] for positive matrices. Their algorithm is computationally cheaper than the traditional power method and converges much faster. Moreover, the authors claim that the rate of convergence of their algorithm does not seem to depend on the dominance ratio of the matrix.

Next, we look at the sufficient condition for ensuring that $\vec{P}^{opt} \leq P^{max}$. For this condition to be satisfied, we must have:

$$max_i\left(P_i^{opt}\right) \overset{\triangle}{=} \parallel \vec{P}^{opt} \parallel_\infty \leq \ P^{max} \quad (57)$$

[10]For example, the transmission whose removal causes the greatest reduction in $\parallel \tilde{\mathbf{B}} \parallel_\infty$ may be deleted.

For any matrix $\mathbf{X} \in \mathbf{R}^{n\times n}$ and a conforming vector $\vec{y}$, we know that (see Section 5.6.5, pp. 295 of [19] for a proof):

$$\parallel \mathbf{X}\cdot\vec{y} \parallel_\infty \leq \parallel \mathbf{X} \parallel_\infty \cdot \parallel \vec{y} \parallel_\infty \tag{58}$$

Applying (58) to (55), we have:

$$\parallel \vec{P}^{opt} \parallel_\infty \leq \parallel (\mathbf{I}-\tilde{\mathbf{B}})^{-1} \parallel_\infty \cdot \parallel \vec{\zeta} \parallel_\infty \tag{59}$$

A sufficient condition for ensuring that $\vec{P}^{opt} \leq P^{max}$ is therefore given by:

$$\parallel (\mathbf{I}-\tilde{\mathbf{B}})^{-1} \parallel_\infty \cdot \parallel \vec{\zeta} \parallel_\infty \leq P^{max} \tag{60}$$

For any matrix $\mathbf{X} \in \mathcal{R}^{n\times n}$ such that $\parallel \mathbf{X} \parallel_\infty < 1$, it can be shown that[11] (see pp. 301, [19])):

$$\frac{1}{1+\parallel \mathbf{X} \parallel_\infty} \leq \parallel (\mathbf{I}-\mathbf{X})^{-1} \parallel_\infty \leq \frac{1}{1-\parallel \mathbf{X} \parallel_\infty} \tag{61}$$

Since $\parallel \tilde{\mathbf{B}} \parallel_\infty < 1$ by Lemma 1, we can apply the upper bound in (61) to (59) and (60). We therefore have:

$$\begin{aligned}\parallel \vec{P}^{opt} \parallel_\infty &= \parallel (\mathbf{I}-\tilde{\mathbf{B}})^{-1}\cdot\vec{\zeta} \parallel_\infty \\ &\leq \parallel (\mathbf{I}-\tilde{\mathbf{B}})^{-1} \parallel_\infty \cdot \parallel \vec{\zeta} \parallel_\infty \\ &\leq \frac{1}{1-\parallel \tilde{\mathbf{B}} \parallel_\infty}\cdot \parallel \vec{\zeta} \parallel_\infty \end{aligned}\tag{62}$$

An alternate upper bound to $\parallel \vec{P}^{opt} \parallel_\infty$, which can be shown to be at least as tight as (62), can be derived by considering the properties of the M-matrix $\mathbf{A} = \mathbf{I} - \tilde{\mathbf{B}}$. We denote the ith row sums of $\mathbf{A}$ and $\tilde{\mathbf{B}}$ by $\Sigma_i(\mathbf{A})$ and $\Sigma_i(\tilde{\mathbf{B}})$ respectively. Clearly,

$$\Sigma_i(\mathbf{A}) = \sum_j \mathbf{A}_{ij} = 1 - \sum_j \tilde{\mathbf{B}}_{ij} = 1 - \Sigma_i(\tilde{\mathbf{B}}) \tag{63}$$

$$\Rightarrow \min_i\{\Sigma_i(\mathbf{A})\} = 1 - \max_i\{\Sigma_i(\tilde{\mathbf{B}})\} = 1 - \parallel \tilde{\mathbf{B}} \parallel_\infty \tag{64}$$

the last equality in (64) following from (39). From Lemma 1 and the positivity property of $\tilde{\mathbf{B}}$, we know that $0 < \Sigma_i(\tilde{\mathbf{B}}) < 1$ for all i. Using (63), we therefore have: $0 < \Sigma_i(\mathbf{A}) < 1$.

We now consider the matrix equation $\mathbf{A}\cdot\vec{P}^{opt} = \vec{\zeta}$ and write the ith component of $\vec{\zeta}$ as follows:

$$\begin{aligned}&\sum_j \mathbf{A}_{ij}P_j^{opt} = \zeta_i \\ \Rightarrow &\mathbf{A}_{ii}P_i^{opt} + \sum_{j\neq i}\mathbf{A}_{ij}P_j^{opt} = \zeta_i \\ \Rightarrow &\sum_j \mathbf{A}_{ij}P_i^{opt} - \sum_{j\neq i}\mathbf{A}_{ij}P_i^{opt} + \sum_{j\neq i}\mathbf{A}_{ij}P_j^{opt} = \zeta_i \\ \Rightarrow &P_i^{opt}\sum_j \mathbf{A}_{ij} = \sum_{j\neq i}\mathbf{A}_{ij}(P_i^{opt} - P_j^{opt}) + \zeta_i \\ \Rightarrow &P_i^{opt}\Sigma_i(\mathbf{A}) = \sum_{j\neq i}\mathbf{A}_{ij}(P_i^{opt} - P_j^{opt}) + \zeta_i \\ \Rightarrow &P_i^{opt} = \sum_{j\neq i}\frac{\mathbf{A}_{ij}}{\Sigma_i(\mathbf{A})}(P_i^{opt} - P_j^{opt}) + \frac{\zeta_i}{\Sigma_i(\mathbf{A})}\end{aligned}\tag{65}$$

[11] In general, the result holds for $n \times n$ complex matrices. Also, it holds for any induced matrix norm, not just the ∞-norm.

Assume $i = \text{argmax}(\vec{P}^{opt})$. Then the first term on the r.h.s of (65) is nonpositive since $P_i^{opt} - P_j^{opt} \geq 0$ and $\mathbf{A}_{ij} \leq 0$ for all $j \neq i$ and $\Sigma_i(\mathbf{A}) > 0, \forall i$. Consequently, P_i^{opt} is upper bounded by $\frac{\zeta_i}{\Sigma_i(\mathbf{A})}$.

If, on the other hand, $i = \text{argmin}(\vec{P}^{opt})$, the first term on the r.h.s of (65) is nonnegative since $P_i^{opt} - P_j^{opt} \leq 0$ and $\mathbf{A}_{ij} \leq 0$ for all $j \neq i$ and $\Sigma_i(\mathbf{A}) > 0, \forall i$. Consequently, P_i^{opt} is lower bounded by $\frac{\zeta_i}{\Sigma_i(\mathbf{A})}$. Since we do not know in general whether the ith term of $\vec{P}^{opt}$ is maximum, we can compute $\frac{\zeta_i}{\Sigma_i(\mathbf{A})}$ for all i, the maximum of which is an upper bound for any P_i^{opt}. Thus:

$$\parallel \vec{P}^{opt} \parallel_\infty \leq \max_i\left(\frac{\zeta_i}{\Sigma_i(\mathbf{A})}\right) \tag{66}$$

By a similar argument, it is easy to see that $\vec{P}^{opt}$ is lower bounded[12] by $\min_i\left(\frac{\zeta_i}{\Sigma_i(\mathbf{A})}\right)$ and therefore:

$$\min_i\left(\frac{\zeta_i}{\Sigma_i(\mathbf{A})}\right) \leq \vec{P}^{opt} \leq \max_i\left(\frac{\zeta_i}{\Sigma_i(\mathbf{A})}\right) \tag{67}$$

Before showing that the upper bound in (66) is at least as tight as (62), we state the following lemma:

Lemma 2: Provided Lemma 1 is satisfied, the solution $\vec{P}^{opt}$ in (55) is upper bounded by P^{max} if:

$$\max_i\left(\frac{\zeta_i}{\Sigma_i(\mathbf{A})}\right) \leq P^{max} \tag{68}$$

To prove that the upper bound in (66) is at least as tight as (62), we first define two diagonal matrices $\mathbf{D}_\zeta = \text{diag}(\zeta_i)$ and $\mathbf{D}_\Sigma = \text{diag}(\frac{1}{\Sigma_i(\mathbf{A})})$. Clearly, $\parallel \mathbf{D}_\zeta \parallel_\infty = \parallel \vec{\zeta} \parallel_\infty$ and $\parallel \mathbf{D}_\Sigma \parallel_\infty$ is equal to the maximum of $(\frac{1}{\Sigma_i(\mathbf{A})})$. Since $\max_i(\frac{\zeta_i}{\Sigma_i(\mathbf{A})})$ is equal to the maximum row sum of the matrix product $\mathbf{D}_\zeta\mathbf{D}_\Sigma$, or equivalently, the ∞-norm of $\mathbf{D}_\zeta\mathbf{D}_\Sigma$, we have the following sequence of inequalities:

$$\begin{aligned}\max_i\left(\frac{\zeta_i}{\Sigma_i(\mathbf{A})}\right) &= \parallel \mathbf{D}_\zeta\mathbf{D}_\Sigma \parallel_\infty \\ &\leq \parallel \mathbf{D}_\zeta \parallel_\infty \parallel \mathbf{D}_\Sigma \parallel_\infty \\ &= \parallel \vec{\zeta} \parallel_\infty \max_i\left(\frac{1}{\Sigma_i(\mathbf{A})}\right) \\ &= \frac{\parallel \vec{\zeta} \parallel_\infty}{\min_i(\Sigma_i(\mathbf{A}))} = \frac{\parallel \vec{\zeta} \parallel_\infty}{1-\parallel \tilde{\mathbf{B}} \parallel_\infty}\end{aligned}\tag{69}$$

where the inequality follows from the submultiplicative property of matrix ∞-norms and the final equality follows from (64). This proves our claim that the upper bound in (66) is at least as tight as (62). We note however that the two upper bounds are exactly the same if all noise components are equal, *i.e.*, if $\vec{\eta} = \eta\cdot\vec{e} \Rightarrow \mathbf{D}_\zeta = (\eta/S^2)\mathbf{I}$, since,

$$\parallel \mathbf{D}_\zeta\mathbf{D}_\Sigma \parallel_\infty = (\eta/S^2)\cdot \parallel \mathbf{D}_\Sigma \parallel_\infty = \parallel \mathbf{D}_\zeta \parallel_\infty \cdot \parallel \mathbf{D}_\Sigma \parallel_\infty$$

With the above lemmas in place, we approximate the maximal feasible schedule for slot s, $\mathcal{RA}(s)$, by a maximum cardinality

[12] The lower bound might be useful for choosing the starting vector in iterative computation of transmit powers.

subset of $\mathcal{MR}(s)$ satisfying the conditions of Lemmas 1 and 2. Recall that $\mathcal{MR}(s)$ is a maximum matching of the request list at the start of iteration s. We emphasize that constraining $\mathcal{RA}(s) \subseteq \mathcal{MR}(s)$ may not be an optimal policy. However, this restriction is necessary since it is certainly possible for a pair of conflicting transmissions to satisfy the conditions of both lemmas.

If the matrices $\mathbf{A}$ and $\tilde{\mathbf{B}}$ corresponding to $\mathcal{MR}(s)$[13] satisfy the lemmas, we define a request list for the next iteration as in (23) and proceed to the next iteration. If not, we seek to identify the *highest order principal sub-matrix* of $\tilde{\mathbf{B}}$ (and correspondingly that of $\mathbf{A}$) which satisfies the conditions of Lemmas 1 and 2. This is done sequentially as shown in Figure 5. The first **while** loop in Figure 5 ensures that Lemma 1 is satisfied and the second **while** loop ensures that Lemma 2 is satisfied. If $|\mathcal{MR}(s)| = 1$ at any stage during the loops, we proceed directly to step 3. Following Theorem 5, the inverse-positivity of $\mathbf{A}$, which is assured by the first loop, is preserved during execution of the second loop. Within a loop, the row which most violates the lemma condition is deleted during each iteration. Note that deleting the kth row in $\tilde{\mathbf{B}}$ (or $\mathbf{A}$) corresponds to deleting the kth transmission from its corresponding request list. This deletion policy is intuitively very simple and parallels the weakest receiver removal policy suggested for distributed cellular power control [15]. We are currently evaluating other schemes, $e.g.$, strongest interferer removal as opposed to weakest receiver removal, to determine the best approximation to the optimal highest order principal sub-matrices of $\tilde{\mathbf{B}}$ and $\mathbf{A}$ which satisfy Lemmas 1 and 2.

/* `The first` **`while`** `loop ensures that Lemma 1 is satisfied.` */

1. **while** $\left(\| \tilde{\mathbf{B}} \|_\infty \geq 1\right)$
   - Identify the row (say 'k') which corresponds to the maximum row sum of $\tilde{\mathbf{B}}$; $k = \text{argmax}(\sum_j \tilde{\mathbf{B}}_{ij})$
   - Delete the kth row (transmission request) from $\mathcal{MR}(s)$;
   - Delete the kth row and column from $\tilde{\mathbf{B}}$;
   - Delete the kth row and column from $\mathbf{A}$;

   end while

/* `The next` **`while`** `loop ensures that Lemma 2 is satisfied.` */

2. **while** $\left(\max_i(\frac{\zeta_i}{\Sigma_i(\mathbf{A})}) > P^{max}\right)$
   - Let $k = \text{argmax}(\frac{\zeta_i}{\Sigma_i(\mathbf{A})})$
   - Delete the kth row from $\mathcal{MR}(s)$;
   - Delete the kth row and column from $\mathbf{A}$;

   end while
3. Assign $\mathcal{RA}(s) \leftarrow \mathcal{MR}(s)$;
4. Define a request list for the next iteration as in (23) and proceed to the next iteration.

Fig. 5. Sequential row/column deletion algorithm to ensure Lemmas 1 and 2 are satisfied.

[13]We assume that $|\mathcal{MR}(s)| > 1$. It can be easily verified that both lemma conditions are automatically satisfied if $|\mathcal{MR}(s)| = 1$. Also see item 15, Section II.

V. Conclusion

In this paper, we considered the problem of power controlled minimum frame length scheduling for TDMA wireless networks. Two different versions of the problem were studied, a per-slot version and a per-frame version. We developed mixed integer linear programming models for both versions which can be used for solving the problems optimally. In addition, we proposed a heuristic algorithm for the per-slot version. We are currently studying the convergence rates for iterative power vector solution methods when slots are heavily packed. This issue, along with detailed simulation results, will be addressed in a subsequent paper.

References

1) Tamer Elbatt and Anthony Ephremides, "*Joint Scheduling and Power Control for Wireless Ad-Hoc Networks*", Proc. IEEE INFOCOM, 2002.
2) Tamer Elbatt and Anthony Ephremides, "*Joint Scheduling and Power Control for Wireless Ad-Hoc Networks*", IEEE Trans. on Wireless Communications, Vol. 3, No. 1, Jan. 2004.
3) Jimmi Gronkvist and Anders Hansson, "*Comparison between Graph-based and Interference-based STDMA Scheduling*", Proc. of MOBIHOC, 2001.
4) Patrik Bjorklund, Peter Varbrand and Di Yuan, "*Resource Optimization of Spatial TDMA in Ad-Hoc Radio Networks: A Column Generation Approach*", Proc. IEEE INFOCOM, 2003.
5) Arash Behzad and Izhak Rubin, "*Power Controlled Multiple Access Control for Wireless Access Nets*", Proc. of IEEE Vehicular Technology Conference, 2003.
6) J. Monks, V. Bharghavan, and W. Hwu, "*A Power Controlled Multiple Access protocol for Wireless Packet Networks*", Proc. IEEE INFOCOM, Apr. 2001.
7) Sudhir Grandhi, Rajiv Vijayan, David J. Goodman and Jens Zander, "*Centralized Power Control in Cellular Radio Systems*", IEEE Trans. on Vehicular Technology, Vol. 42, No. 4, November 1993.
8) Sudhir Grandhi, Rajiv Vijayan and David J. Goodman, "*Distributed Power Control in Cellular Radio Systems*", IEEE Trans. on Communications, Vol. 42, No. 2/3/4, February/March/April 1994.
9) Sudhir Grandhi and Jens Zander, "*Constrained Power Control in Cellular Radio Systems*", IEEE Trans. on Communications, Vol. 42, No. 2/3/4, February/March/April 1994.
10) S. A. Grandhi, J. Zander, and R. Yates, "*Constrained Power Control*", Wireless Personal Communications, Vol. 1, 1995.
11) R. Yates, "*A Framework for Uplink Power Control in Cellular Radio Systems*", IEEE Journal on Selected Areas in Communications, Vol. 13, 1995.
12) G.J. Foschini and Z. Miljanic, "*A simple Distributed Autonomous Power Control algorithm and its Convergence*", IEEE Trans. on Vehicular Technology, Vol. 42, Issue 4, Nov. 1993.

13) R. Jantti and Seong-Lyun Kim, "*Second-Order Power Control with Asymptotically Fast Convergence*", IEEE Journal on Selected Areas in Communications, Vol. 18, No. 3, March 2000.
14) Z. Uykan and H.N. Koivo, "*Sigmoid-Basis Nonlinear Power-Control Algorithm for Mobile Radio Systems*", IEEE Trans. on Vehicular Technology, Vol. 53, Issue 1 , Jan. 2004.
15) J. Zander, "*Distributed cochannel interference control in cellular radio systems*," IEEE Trans. on Vehicular Technology, Vol. 41, Aug. 1992.
16) C.-Y. Huang and R. D. Yates, "*Rate of Convergence for Minimum Power Assignment in Cellular Radio Systems*", ACM/Baltzer Wireless Networks Journal, Vol. 1, 1998.
17) R. Jantti and Seong-Lyun Kim, "*On the Convergence of Linear Power Control for Cellular Radio Systems*", IEEE Vehicular Technology Conference, Spring 2001, May 2001.
18) Abraham Berman and Robert J. Plemmons, "*Nonnegative matrices in the Mathematical Sciences*", SIAM, 1994.
19) Roger A. Horn and Charles R. Johnson, "*Matrix Analysis*", Cambridge University Press, 1999.
20) C. A. Hall and T. A. Porsching, "*Computing the Maximal Eigenvalue and Eigenvector of a Positive Matrix*", SIAM J. Numerical Analysis, Vol. 5, pp. 269-274, 1968.
21) Theodore S. Rappaport, "*Wireless Communications: Principles and Practice*", Prentice Hall PTR, 1996.
22) *<http://www.lindo.com>*