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In Internet paid search advertising, marketers pay for search engines to serve text advertisements in response to keyword searches that are generic (e.g., “hotels”) or branded (e.g., “Hilton Hotels”). Although stand-alone metrics usually show that generic keywords have higher apparent costs to the advertiser than branded keywords, generic search may create a spillover effect on subsequent branded search. Building on the Nerlove–Arrow advertising framework, the authors propose a dynamic linear model to capture the potential spillover from generic to branded paid search. In the model, generic search advertisements serve to expose users to information about the brand’s ability to meet their needs, raising awareness that the brand is relevant to the search. In turn, this can induce additional future search activity for keywords that include the brand name. Using a Bayesian estimation approach, the authors apply the model to data from a paid search campaign for a major lodging chain. The results show that generic search activity positively affects future branded search activity through awareness of relevance. However, branded search does not affect generic search, demonstrating that the spillover is asymmetric. The findings have implications for understanding search behavior on the Internet and the management of paid search advertising.

Keywords: Internet advertising, paid search, spillover, awareness, Nerlove–Arrow model, Bayesian dynamic linear model

From Generic to Branded: A Model of Spillover in Paid Search Advertising

Consider the following scenario: A traveler is planning a vacation to Los Angeles and is unfamiliar with the hotel options available. The traveler begins his planning with an Internet search for the generic term “hotels Los Angeles.” He clicks on a sponsored text advertisement from Hilton, visits its Web site, but takes no further action. The next day the traveler continues his planning; recalling that Hilton operates in Los Angeles, he now searches for the branded term “Hilton Los Angeles.” The search engine again returns a sponsored text advertisement from Hilton. When the user

clicks through, he is taken to the Hilton Web site, where he places a reservation.

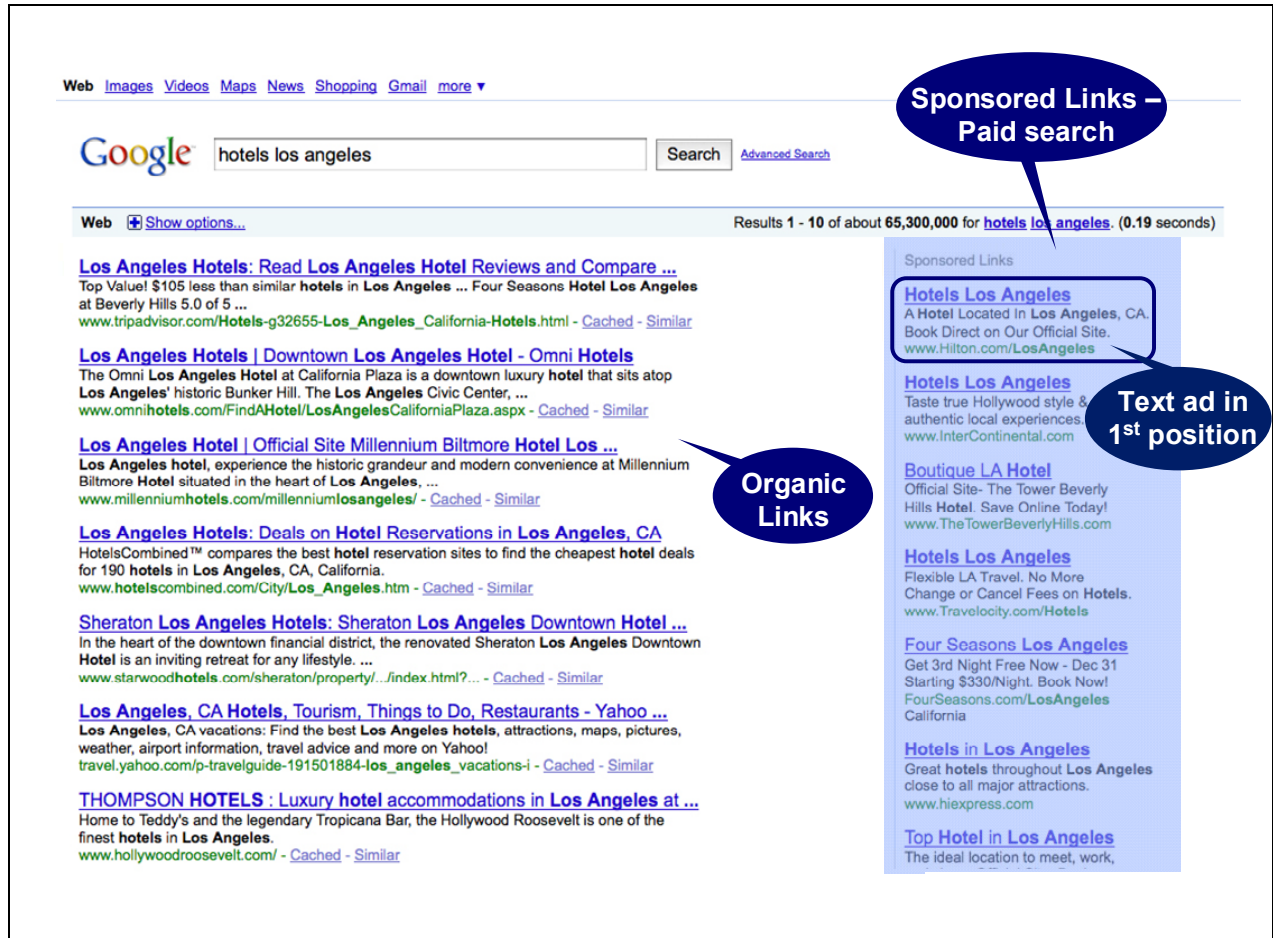
Figure 1 provides an illustration of the search results returned in response to the generic query “hotels Los Angeles.” In the hypothetical example, the initial generic search creates awareness that Hilton might be able to meet the traveler’s needs, which in turn helps lead to a subsequent search for the branded term “Hilton Los Angeles.”¹ In this scenario, a spillover effect has occurred from generic to branded search. The question we pose is this: To what extent does such spillover take place, and if so, what are the implications for the management of paid search ad campaigns and researchers’ understanding of consumer search behavior?

The rapidly growing role of paid search advertising in the marketing communications mix (e.g., PricewaterhouseCoopers 2008) points to the need to carefully examine the performance metrics for it. These metrics are derived from

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¹We thank an anonymous reviewer for suggesting the use of an illustrative scenario.

Figure 1
SEARCH RESULTS PAGE EXAMPLE FOR GOOGLE



the progression of activity through the consumer search funnel. The left panel of Figure 2 illustrates this for the case of a hotel chain. As the figure shows, searches lead to impressions, which could lead to clicks, which could lead to reservations.

To return to our example, hotels competing in Los Angeles might find generic search terms of interest and bid for placement of their text advertisements in the sponsored search listing. Search terms, including brand names such as "Hilton Los Angeles," attract more limited interest—primarily from the brand owner and channel intermediaries that broker or resell the named service or product. Indeed, legal restrictions on trademark use typically preclude competitors from "hijacking" a branded search term to promote their own products or services. Consequently, we might expect that branded keywords will have a lower cost per click than generic keywords. If branded terms are used more heavily closer to purchase times, conversion rates from click to reservation might also be higher for branded versus generic terms.

Tables 1 and 2 give descriptive statistics from our data set for the paid search campaign run for a major lodging chain on the Google and Yahoo search engines.² The metrics dif-

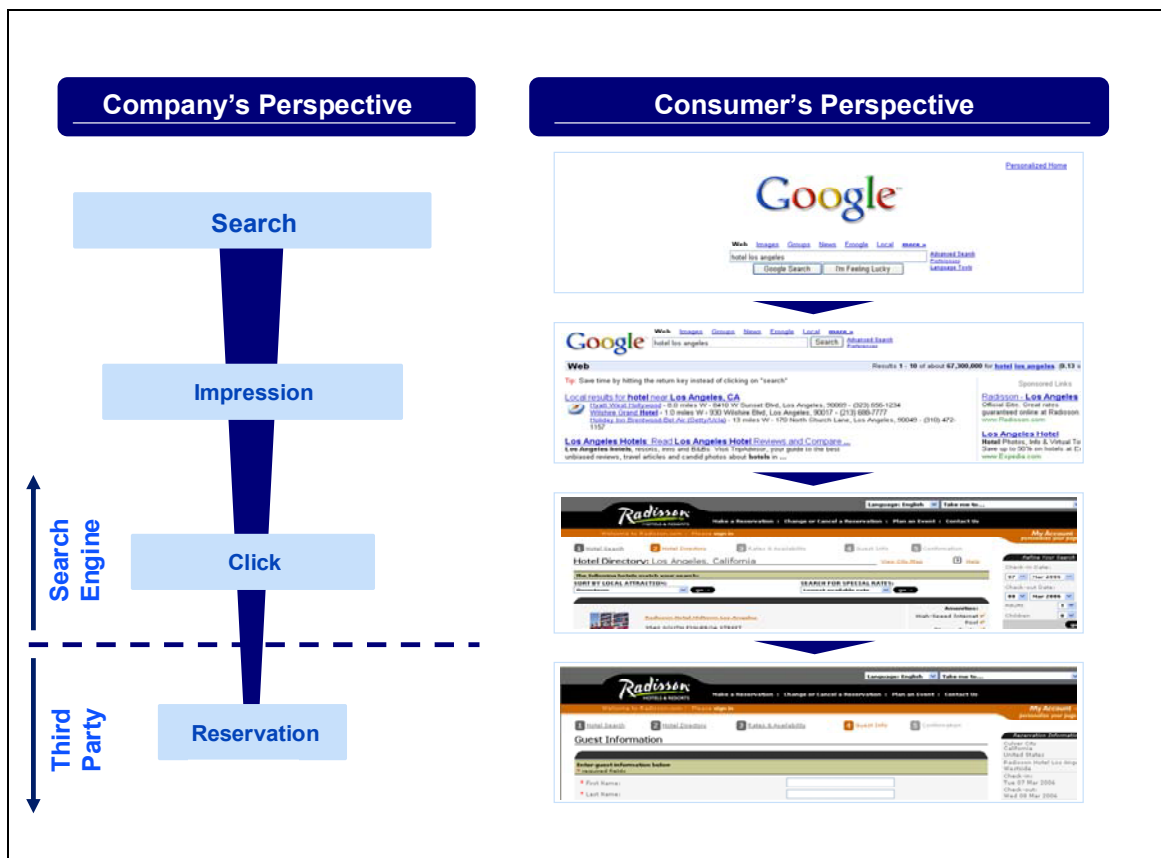
fer sharply between the generic and the branded keywords in the campaign. On Google, click-through rates (e.g., 13.68% versus .26%) and conversion rates (e.g., 6.03% versus 1.05%) are substantially higher for branded versus generic keywords, and the Yahoo data show a similar pattern. Furthermore, the cost per click for branded keywords is substantially lower than that for generic keywords (e.g., \$.18 versus \$.55 for Google). The resulting difference in the apparent cost per reservation for the two classes of keywords is striking (e.g., \$2.94 versus \$51.84 on Google). This difference initially led managers we worked with to question whether the use of generic keywords in the campaign should be heavily curtailed in favor of branded terms.

SPILOVER EFFECTS IN PAID SEARCH

One drawback of the metrics described in the preceding section is that they do not account for the potential dynamic interaction between generic and branded search activity. We propose that generic search can create awareness that the brand is relevant to the goals of the search and consequently spill over to influence subsequent branded search. This awareness can then lead to future branded keyword searches in which the user researches the brand's offering in more detail or perhaps goes on to complete a purchase.

²Data from other campaigns we analyzed show similar patterns, but confidentiality restrictions do not allow us to report details. Conversations with many practitioners indicate that this pattern is also a widespread phenomenon.

Figure 2
ILLUSTRATION OF THE SEARCH PROCESS



Our lodging company data show a pattern of modest spikes in generic search activity followed by increases in branded search. As an illustration, we take the three-week period leading up to the July 4 holiday weekend. Three weeks before the holiday, search activity on generic keywords ran at 114% of average, and branded search activity ran at 92%. Two weeks previous, generic activity dropped to 93% of average while branded activity rose to 110%. In the week just before the holiday, generic was 90% of average while branded was 98%. A similar pattern is present in the data for other preholiday periods.

We propose that a consumer planning a trip who searches using a generic keyword may not be aware that a specific brand (e.g., the anonymous lodging company we worked with) is relevant for his or her current search. Conversely, a consumer using a branded keyword is likely to be aware that the brand is relevant to the search. This difference in awareness of relevance should then translate into differences in the likelihood of purchase (in our case, a hotel reservation), given click-through. Note that we are not proposing that generic search makes a user aware of the brand for the first time. This may or may not be the case. Rather, we propose that it is the generic search that makes the user aware that the brand offers a potential solution relevant to the user's goals. This is how we distinguish between awareness and awareness of relevance.

Our objective is to develop a statistical model to determine whether the previously described spillover occurs in paid search advertising and, if so, to what extent. Our modeling approach builds on the so-called leaky bucket approach to advertising (e.g., Naik, Raman, and Srinivasan 2009; Nerlove and Arrow 1962), which posits a latent, decaying, unmeasured construct for awareness as part of the model. The premise of our approach is similar: Exposure to brand-related information ensuing from generic search increases awareness of relevance; this awareness also decays over time (i.e., leaks out of the bucket). In turn, greater awareness of relevance leads to an increase in subsequent branded search activity. Because we do not have measures for awareness of relevance, we need a model that can accommodate such a latent construct. Therefore, we turn to the Nerlove-Arrow approach.

To handle the dynamic nature of this process, we specify our proposed model in a multivariate time-series framework. Specifically, we use a dynamic linear model (DLM) estimated in a Bayesian framework following the procedures of West and Harrison (1997). We note that other multivariate time-series models, such as the vector autoregressive (VAR) model, are not set up to handle latent constructs. In this study, we test the performance of our DLM model versus a VAR model. The test assesses whether incorporating latent awareness of relevance adds significantly to model fit, and we find that it does.

Table 1
DESCRIPTIVE STATISTICS FOR GOOGLE DATA

	<i>Impressions</i>	<i>Clicks</i>	<i>Reservations</i>	<i>Cost</i>	<i>Impressions/ Day</i>	<i>Clicks/Day</i>	<i>Reservations/ Day</i>	<i>Cost/Day</i>	<i>Average Position</i>	<i>Click- Through Rate</i>	<i>Conversion Rate</i>	<i>Cost/Click</i>	<i>Cost/ Reservation</i>
Generic	37,059,020	98,162	1033	\$53,549.52	126,051	334	4	\$182.14	5.55	.26%	1.05%	\$.55	\$51.84
Branded	4,925,351	673,971	40,671	\$119,498.50	16,753	2292	138	\$406.46	1.55	13.68%	6.03%	\$.18	\$2.94
Total	41,984,371	772,133	41,704	\$173,048.02	142,804	2626	141	\$588.60					

Table 2
DESCRIPTIVE STATISTICS FOR YAHOO DATA

	<i>Impressions</i>	<i>Clicks</i>	<i>Reservations</i>	<i>Cost</i>	<i>Impressions/ Day</i>	<i>Clicks/Day</i>	<i>Reservations/ Day</i>	<i>Cost/Day</i>	<i>Average Position</i>	<i>Click- Through Rate</i>	<i>Conversion Rate</i>	<i>Cost/Click</i>	<i>Cost/ Reservation</i>
Generic	2,118,555	5608	108	\$2,372.42	7206	19	.4	\$8.07	4.92	.26%	1.93%	\$.42	\$21.97
Branded	3,378,749	361,828	25,889	\$118,024.09	11,492	1231	88	\$401.44	2.16	10.71%	7.16%	\$.33	\$4.56
Total	5,497,304	367,436	25,997	\$120,396.51	18,698	1250	88	\$409.51					

We structure the article as follows: After a brief literature review, we present our model specification, describe our data set in detail, and discuss the empirical results we obtained. Next, we quantify the extent of spillover and test for possible reverse causality. A concluding section summarizes our work, notes limitations of our approach, and discusses future research opportunities.

LITERATURE REVIEW

Much of the previous empirical research on online advertising in marketing has focused on display or banner advertising. Marketing and economics journals have only recently begun publishing work on paid search advertising. For example, theoretical studies have just recently focused on the paid search auction mechanism (e.g., Edelman and Ostrovsky 2007; Edelman, Ostrovsky, and Schwarz 2007), the role of paid search advertising in product differentiation (Chen and He 2009), and click fraud (Wilbur and Zhu 2009).

Empirical work has largely taken a keyword perspective and has treated paid search as a direct marketing channel. A focus is on estimating the response rate to paid search in terms of both click-through and conversion. A premise of this work is that consumers who do not purchase immediately after clicking are considered “lost”; that is, they do not generate any revenue (Ghose and Yang 2009a; Goldfarb and Tucker 2009; Rutz and Bucklin 2009; Yao and Mela 2009). These studies do not address the potential spillover from generic to branded search. In a related study, Ghose and Yang (2009b) investigate within-session spillover as the amount of cross-category purchase that occurs after an initial search brings a consumer to a Web site. For example, consider a consumer searching on a keyword related to the kitchen category. Ghose and Yang investigate the propensity with which this consumer buys a kitchen item as well as an item from another category (e.g., bedding) in the session begun by the kitchen-related keyword. They do not consider that the search could generate searches/visits in future periods that might lead to purchase.

MODELING APPROACH

We ground our modeling approach on the concept of awareness of relevance. We propose that such awareness parsimoniously captures the effects of exposure to brand-related information during search. Consumers who conduct a generic search might not be aware of the brand, or when they are, they might not be aware that the brand is relevant for the search. Generic search leads to brand-related exposures in the form of impressions (i.e., the text advertisements in the sponsored section of the search results page) and clicks (i.e., the searcher clicking on the advertisement and being taken to the advertiser’s Web site). These brand-related exposures may create and/or increase the consumers’ awareness that the brand is relevant for their search.³ We note that the impact of a text-ad impression versus a Web site exposure (following a click-through) might differ as well. An impression is a passive exposure to the brand’s text advertisement, whereas a click is an active option that leads to further information exposure at (and possi-

bly after) the landing page. In our model, we can investigate whether generic impressions versus generic clicks produce different spillover effects.

According to one industry study, 70% of searches begin with a generic, inclusive keyword, and as the search process continues, it tends to become increasingly specific (Search Engine Watch 2006). For example, consider a consumer search for a cruise vacation (Enquiro 2006). The observed user began his search using the keyword “cruise”—a generic keyword that returned a broad set of results. After reviewing the initial results, the user searched “Caribbean cruise.” In this initial phase, the user may develop awareness for new options and begin to narrow the scope of interest. He may also see that particular cruise lines, some of which he already knows, are offering Caribbean cruises. Conversion rates at this stage are usually low, consistent with these types of searches occurring early in the decision-making process.

The initial broad search is followed by narrower searches in which users research options in detail, review third-party testimonials, and choose a brand—often over multiple days or weeks. In this example, the observed user narrowed his search by reading third-party reviews on Panama cruises. After reading the reviews, he conducted a final search for the branded keyword “Princess Panama cruise.” This type of final, targeted search has a much higher conversion rate, possibly 30%–40%, according to the Enquiro study. Had “Panama cruises” and the availability of a Princess cruise not been introduced early in the awareness stage, the user would have been unlikely to have narrowed his search in that direction.

Search engines provide advertisers with data on impressions, clicks, positions, and costs on an aggregate level, typically reported on a daily basis. The data are aggregated on the basis of keywords. For each search keyword (e.g., hotels Los Angeles), campaign managers have daily information on cost (in U.S. dollars), average position served (given by daily average placement rank, e.g., 2.3), number of impressions and clicks, and number of sales or, in our case, reservations. We build our model using these paid search data.⁴

Researchers have modeled the effects of advertising with aggregate data in a wide variety of ways. One popular approach holds that advertising creates goodwill or “ad stock” for the firm or brand but that the goodwill decays over time. Originally described by Nerlove and Arrow (1962; hereinafter N-A), this so-called leaky-bucket model provides an elegant and parsimonious way to capture the effects of advertising over time. The model has served as the basis for recent empirical work in marketing. For example, Naik, Raman, and Srinivasan (2009) find that corporate advertising generates goodwill, which increases both brand sales due to direct spillover effects and brand advertising effectiveness due to indirect spillover effects. Bass and col-

³We use simply “awareness” in place of “awareness of relevance” for the remainder of the article.

⁴The major search engines (Google, Yahoo, and MSN) do provide advertisers with data on their own campaign performance (i.e., impressions, clicks, position, and cost). These data are aggregated on a daily keyword level. None of the major search engines provide competitive data or allow advertisers to “backtrack” which competitors were listed together with their own advertisements. We spoke with executives at both Google and Yahoo, and neither plan to make more detailed data available to either companies or academic institutions because of severe privacy concerns.

leagues (2007) link the N-A model with a demand model for telephone services. In their model, goodwill helps explain how different advertising themes affect demand for telephone services and interact with each other. Building on this research stream, we use an N-A-type model adapted to handle the spillover problem in paid search advertising. In particular, our goal is to capture the changes in awareness of relevance similar to how previous work has captured changes in goodwill.

Model Specification

In our model, changes to awareness (of relevance) depend on generic search activity (the advertising input) and a carryover effect (which captures decay). Following prior formulations (e.g., Naik, Mantrala, and Sawyer 1998; Nerlove and Arrow 1962), we specify the dynamic evolution of changes in awareness as follows:

$$(1) \quad \frac{dA_t}{dt} = \beta^{\text{gen}} \text{GEN}_t - \tilde{\alpha}^A A_t,$$

where A_t is awareness at time t , GEN_t is a vector of generic search activity variables at time t , and β^{gen} and $\tilde{\alpha}^A$ are parameters to be estimated. In discrete time, this model can be rewritten as follows:

$$(1a) \quad A_t = \beta^{\text{gen}} \text{GEN}_t + \alpha^A A_{t-1},$$

where $\alpha^A = (1 - \tilde{\alpha}^A)$ and α^A is the carryover rate of awareness (e.g., Naik, Mantrala, and Sawyer 1998).⁵ We can measure daily generic search activity GEN_t by the dollar amount spent or the number of daily impressions and clicks for generic keywords. This enables us to investigate whether the actual exposure information provides a better measure than dollar spending. We can also study possible saturation effects by modeling, for example, the log of impressions and the log of clicks.

The logic underlying our adaptation of the N-A model in Equation 1 is as follows: A consumer who begins his or her search with a generic keyword will be exposed to multiple brands that offer a service/product that might fit his or her needs. This activity creates or enhances an awareness that these brands offer the service for which the consumer is looking (and that might be relevant). If the search is a multi-stage process, a brand that has increased its awareness by bidding on a generic term might benefit in the later stages of the search. In contrast, if the searcher has never learned that a certain brand has an offering that fits his or her needs, chances are lower that he or she will search this brand later in the process versus brands that have increased awareness. In the aggregate, consumers search and ultimately decide to buy or not (thus dropping out of the market), and new consumers enter. The N-A-type model specification allows us to describe the aggregation of this individual behavior and to test whether the latent construct of awareness (of relevance) increases our ability to explain observed consumer behavior.

Equation 1 specifies the dynamics for how generic search activity affects awareness. Next, to capture the potential spillover, we need to specify the dynamics of how awareness affects branded search. The search activity of consumers can be measured along three dimensions: average

number of searches per keyword in a category (e.g., branded) per day (NS), click-through rate (CTR), and conversion rate (CR). The number of searches is recorded in the data as impressions. The clicks are the number of impressions served times the click-through rate. Last, the number of sales is the number of clicks times the conversion rate. In modeling search, we also need to take the firm's actions into account; these are reflected in the average position served for the branded keywords (POS) and the advertiser's bidding policy, summarized by the average cost per click (CPC), for branded keywords.

The system for describing branded search activity includes dynamic models for each of the five outcome variables: NS, CTR, CR, POS, and CPC. We use a simultaneous equation framework that allows for both endogenous relationships and exogenous effects through variables such as seasonal effects, lagged branded search activity, and latent awareness. The structure of our formulation closely follows the procedures that Naik, Mantrala, and Sawyer (1998), Naik, Raman, and Srinivasan (2009), and Bass and colleagues (2007) develop as well as the literature on structural VAR models (e.g., Hamilton 1994).

First, we model the evolution in NS for branded keywords as follows:

$$(2) \quad \text{NS}_t = \beta^{\text{NS}} I_t - \alpha^{\text{NS}} \text{NS}_{t-1}^{\text{imp}} + \gamma^{\text{NS}} A_t + \varepsilon_t^{\text{NS}},$$

where NS_t is the average number of searches at time t , I_t is a vector of indicator variables accounting for day of week and month (i.e., seasonality), and A_t is the latent awareness from Equation 1. The coefficient α^{NS} is the carryover rate, and the coefficient γ^{NS} reflects the spillover effect from generic search as captured through the impact of awareness, A_t . If there is no spillover from generic to branded, the γ^{NS} coefficient should not be significant. The model allows for a relatively stable NS over time (if α^{NS} is close to 1), a behavior we would expect from a well-known brand. Moreover, NS varies according to seasonal effects (e.g., most searches occur during the week and not on the weekend; Pauwels and Dans 2001). Most important for our purposes, NS might increase as a result of a positive change in awareness of relevance.

Second, we model the evolution in the CTR for branded terms as follows:

$$(3) \quad \text{CTR}_t = \beta^{\text{CTR}} I_t - \alpha^{\text{CTR}} \text{CTR}_{t-1} + \gamma^{\text{CTR}} A_t + \delta \text{POS}_t + \varepsilon_t^{\text{CTR}}.$$

Changes in CTR can come from a change in position because consumers' reaction to a search advertisement may differ depending on the position. Seasonality could also be a factor in CTR. For a well-established brand, CTR should be stable over time, implying α^{CTR} close to 1. Awareness could potentially affect CTR. For example, on the one hand, a consumer might be more prone to click on the advertiser's paid advertisement if awareness of relevance is greater. On the other hand, the narrower set of results typically associated with branded search might leave little room for variability in click-through rate. Thus, we expect that γ^{CTR} will be either positive or zero but not negatively signed.

Third, we model the evolution in CR as follows:

$$(4) \quad \text{CR}_t = \beta^{\text{CR}} I_t - \alpha^{\text{CR}} \text{CR}_{t-1} + \gamma^{\text{CR}} A_t + \varepsilon_t^{\text{CR}}.$$

We apply the same logic to the specification of Equation 4 that we applied for CTR. Seasonality and awareness might

⁵In discrete time, $\Delta A = C - \tilde{\alpha}^A A_t$, where $\Delta A = A_t - A_{t-1}$ and C represents all other terms. Thus, $A_t = C + (1 - \tilde{\alpha}^A) A_{t-1}$. It follows that $A_t = C + \alpha^A A_{t-1}$ and α^A is called the carryover rate.

affect CR. However, CR for a well-established brand should be stable over time, implying α^{CR} close to 1. Again, we make no specific prediction for γ^{CR} other than that it is nonnegative.

Our objective is to study the nature and extent of spillover effects in paid search, not to optimize bidding strategy. Nonetheless, to model this phenomenon, it is potentially important to account for the advertiser's bidding and the resulting positions of the text advertisements. In paid search, the underlying auction is not a pure second price auction as modeled in theory research (e.g., Edelman and Ostrovsky 2007; Edelman, Ostrovsky, and Schwarz 2007). From the advertiser's perspective, it is something of a "black box" that combines the bid amount with past performance of the advertisement as generally measured by previous CTR. The actual algorithms are proprietary and not shared with advertisers. As we noted previously, firms do not observe competitors' bids (see the subsequent discussion on the issue of missing competitive data). In light of these factors, we approximate the auction mechanism by modeling the change in average position as a function of carryover (to account for the firm's position strategy), current CPC, last period's CTR (as an approximation of the search engine's past performance measure), and indicators for seasonality. Note that Ghose and Yang (2009a, b) employ a similar approach to modeling the auction in their studies, though theirs is based only on weekly data. This is given by the following:

$$(5) \text{ POS}_t = \beta \text{POS}_{t-1} - \alpha \text{POS}_{t-1} + \phi \text{CPC}_t + \eta \text{CTR}_{t-1} + \varepsilon_t^{\text{POS}}$$

Last, we need to control for the company's advertising spending, which in this case is equal to CPC times clicks. We model changes in CPC as a function of the current average position and past CTR. Past CTR affects CPC through the auction mechanism (as modeled in Equation 5). A more successful advertisement in the past (i.e., higher past CTR) will require a lower CPC for a fixed position than a less successful advertisement. We also include CPC carryover to account for the firm's CPC strategy as well as indicators to account for seasonality:

$$(6) \text{ CPC}_t = \beta \text{CPC}_{t-1} - \alpha \text{CPC}_{t-1} + \iota \text{POS}_t + \kappa \text{CTR}_{t-1} + \varepsilon_t^{\text{CPC}}$$

Estimating a system that includes Equations 5 and 6 requires us to account for the contemporaneous relationship between current POS and current CPC. We discuss how this can be accomplished in the next section.

Our data do not include information on competitor advertising. Search engines do not share competitors' bids or provide the names of the firms bidding on keywords. Google actively discourages crawling Google results pages to obtain competitive information and has listed doing so as a violation of its terms of service.

Despite the data limitations, we believe that competitive information would be unlikely to change the substantive nature of our results. It is possible that changes in competitive bids for the same generic keywords (e.g., "hotels Los Angeles") could affect the position of the firm's generic text advertisements, leading to lower click-through rates. We are studying the link between generic search activity and—given the impression of the firm's text advertisement and, possibly, click-through—subsequent search activity for its branded keywords. Thus, the decline in generic search activity would simply be passed along in the form of a proportional

reduction in awareness and fewer branded searches. In this sense, the spillover phenomenon we study occurs regardless of competition, but competition can scale it up or down.

For several reasons, we believe that the extent of competitive interaction in this environment is actually fairly muted. First, currently at Google and Yahoo, text advertisements are placed in position rank order as a function of click-through rates, landing page quality scores, and bid amounts. This means that competitive bidding is not the sole determinant of text ad position. Second, the carryover effects we find are of short duration (most of the effect occurs within a few days). Even within the fast-paced world of paid search advertising, firms face many constraints on how quickly and how often they can reset their budgets, doing so on monthly or quarterly bases (as in the case of our collaborating firm). Third, to the extent that it occurs, competitive reaction is not head on. Firms maintain differentiated lists of keywords (i.e., they bid on different sets of search terms). In the case of our midprice lodging chain, it faced a host of different competitors in terms of price point, positioning, and geography (i.e., many of the search terms were location specific).

Finally, recent empirical literature suggests that competitive reaction is infrequent. Steenkamp and colleagues (2005) find that the most common reaction is no reaction. They also find that not all reaction is aggressive. In related literature, Pauwels (2004, 2007) finds that competitive harm is rather limited in magnitude—the net effect being about 10% of the initiating marketing action. Last, Srinivasan and colleagues (2004) report that when competitive action arises after a promotion, in the majority of cases, incremental revenue is generated by the end of the dust-settling period.

Bayesian DLM

We use a DLM implemented in a Bayesian framework to integrate the models in Equations 1–6. In marketing research, DLMs have been used to deal with scenarios in which a key component of the data is unobserved (e.g., Bass et al. 2007; Naik, Raman, and Srinivasan 2009), as is true in our model, and to handle time-varying parameters (e.g., Ataman, Mela, and Van Heerde 2008; Van Heerde, Mela, and Manchanda 2004). We estimate the model using Markov chain Monte Carlo methods (West and Harrison 1997). An appealing feature of the DLM is that it simultaneously captures the dynamic evolution of the branded search activities, the underlying mechanics (as defined previously), and latent awareness (allowing us to quantify the effect of generic search on branded search).

Equations 1–6 form a structural DLM given by the following⁶:

$$(7) \begin{pmatrix} 1 & & & & & \\ & 1 & -\delta & & & \\ & & & 1 & & \\ & & & & 1 & -\phi \\ & & & & -\tau & 1 \\ & & & & & & 1 \end{pmatrix} \begin{pmatrix} \text{NS} \\ \text{CTR} \\ \text{CR} \\ \text{POS} \\ \text{CPC} \\ \text{A} \end{pmatrix}_t$$

⁶In time-series literature, the term "structural" refers to the notion that we estimate a system given by $Ay_t = By_{t-1} + X\beta + \varepsilon$. This system is generally unidentified, and identification restrictions must be imposed (e.g., Hamilton 1994).

$$= \begin{pmatrix} \alpha^{NS} & & & & & & & & & \gamma^{NS} \\ & \alpha^{CTR} & & & & & & & & \gamma^{CTR} \\ & & \alpha^{CR} & & & & & & & \gamma^{CR} \\ & & & \eta & & & & & & \\ & & & \kappa & & & & & & \\ & & & & \alpha^{POS} & & & & & \\ & & & & & \alpha^{CPC} & & & & \\ & & & & & & \alpha^A & & & \end{pmatrix} \begin{pmatrix} NS \\ CTR \\ CR \\ POS \\ CPC \\ A \end{pmatrix}_{t-1} \\
 + \begin{pmatrix} d^{NS} \\ d^{CTR} \\ d^{CR} \\ d^{POS} \\ d^{CPC} \\ d^A \end{pmatrix} + \begin{pmatrix} \varepsilon^{NS} \\ \varepsilon^{CTR} \\ \varepsilon^{CR} \\ \varepsilon^{POS} \\ \varepsilon^{CPC} \\ \varepsilon^A \end{pmatrix},$$

where the drift vector, d_t^{\cdot} , for NS, CTR, CR, POS, and CPC is given by

$$(8a) \quad d_t^{\cdot} = \sum_{\text{days}} I_t^{\text{DAY}} \lambda_{\text{DAY}}^{\cdot} + \sum_{\text{months}} I_t^{\text{MONTH}} \lambda_{\text{MONTH}}^{\cdot}$$

and for A is given by

$$(8b) \quad d_t^A = \text{GEN}_t^{\text{IMP}} \lambda_{\text{GEN}}^{\text{IMP}} + \text{GEN}_t^{\text{CL}} \lambda_{\text{GEN}}^{\text{CL}}$$

In Equations 8a and 8b, I_t^{DAY} is an indicator for weekday (e.g., Monday), I_t^{MONTH} is an indicator for month (e.g., July), $\text{GEN}_t^{\text{IMP}}$ are the generic impressions at time t , and GEN_t^{CL} are the generic clicks at time t . The correlated error terms, ε_t^{\cdot} , capture the effect of other factors not included in the model and $\underline{\varepsilon} \sim N(\underline{0}, V_\varepsilon)$, where V_ε is a full covariance matrix to be estimated.

To estimate the model, we need to transform the structural DLM given in Equation 7 into a reduced-form DLM (for details, see Appendix Web Appendix A at <http://www.marketingpower.com/jmrfeb11>):

$$(9) \quad \begin{pmatrix} NS \\ CTR \\ CR \\ POS \\ CPC \\ A \end{pmatrix}_t \\
 = \begin{pmatrix} \alpha^{NS} & & & & & & & & & \gamma^{NS} \\ & \hat{\alpha}^{CTR} & & \beta^{24} & \beta^{25} & & & & & \gamma^{CTR} \\ & & \alpha^{CR} & & & & & & & \gamma^{CR} \\ & & & \beta^{42} & \hat{\alpha}^{POS} & \beta^{45} & & & & \\ & & & \beta^{52} & \beta^{54} & \hat{\alpha}^{CPC} & & & & \\ & & & & & & \alpha^A & & & \end{pmatrix} \begin{pmatrix} NS \\ CTR \\ CR \\ POS \\ CPC \\ A \end{pmatrix}_{t-1} \\
 + \begin{pmatrix} \hat{d}^{NS} \\ \hat{d}^{CTR} \\ \hat{d}^{CR} \\ \hat{d}^{POS} \\ \hat{d}^{CPC} \\ \hat{d}^A \end{pmatrix} + \begin{pmatrix} \hat{\varepsilon}^{NS} \\ \hat{\varepsilon}^{CTR} \\ \hat{\varepsilon}^{CR} \\ \hat{\varepsilon}^{POS} \\ \hat{\varepsilon}^{CPC} \\ \hat{\varepsilon}^A \end{pmatrix}.$$

To complete the setup of the DLM, we specify the observation equation linking the state variables to the observed quantities—branded impressions (IMP), branded clicks (CL), branded reservations (RES), POS, and total branded cost (COST)⁷:

$$(10) \quad \begin{pmatrix} \text{IMP} \\ \text{CL} \\ \text{RES} \\ \text{POS} \\ \text{COST} \end{pmatrix}_t \\
 = \begin{pmatrix} \text{NW} & & & & & & & & & \\ & \text{IMP} & & & & & & & & \\ & & \text{CL} & & & & & & & \\ & & & 1 & & & & & & \\ & & & & \text{CL} & & & & & \\ & & & & & & & & & \end{pmatrix} \begin{pmatrix} NS \\ CTR \\ CR \\ POS \\ CPC \\ A \end{pmatrix}_t + \begin{pmatrix} V^{\text{IMP}} \\ V^{\text{CL}} \\ V^{\text{RES}} \\ V^{\text{POS}} \\ V^{\text{BID}} \end{pmatrix}_t,$$

where NW is the number of keywords in the campaign, IMP is the number of impressions, CL is the number of clicks, and the v_t^{\cdot} are error terms and $\underline{v} \sim N(\underline{0}, V_v)$.⁸

We estimate the DLM (Equations 9 and 10) using sequential Gibbs sampling. For a complete description of the estimation procedure following the DLM framework, see Web Appendix B (West and Harrison 1997). For details on the recovery of the structural parameters from Equation 7, see Web Appendix A (<http://www.marketingpower.com/jmrfeb11>).

We base our empirical identification on the variation in the data that stems from the number of daily searches consumers conduct on the company’s generic keywords. (The company maintained a fixed list of keywords over the observation period.) This produces the daily variation in generic clicks (see Figure 3). Exploratory data analysis reveals strong correlation between lagged generic activity and branded activity, as in the July 4 holiday example discussed previously. In a regression, we find significant effects of lagged generic activity on branded activity, for both Google and Yahoo. We also fail to find the reverse effect (i.e., lagged branded activity did not significantly affect generic activity). As another check, we also run a series of Granger causality tests. Generic impressions did not Granger-cause any branded search activity (impressions, clicks, or resulting reservations), but generic clicks did Granger-cause branded impressions and branded clicks (but not branded reservations). Branded activity did not Granger-cause any generic activity (impressions, clicks, or reservations).

In addition to these tests, we undertake two additional analyses to provide further confidence in the empirical identification of our results. First, we investigate whether a reverse spillover effect exists from generic to branded and test this using our proposed model. As described subsequently, we find that reverse spillover is not present. Second, our data from two search engines, Yahoo and Google,

⁷Impressions are number of words times average number of searches per word, clicks are impressions times CTR, reservations are clicks times CR, and total cost is clicks times CPC.

⁸Although it is linear in parameter specification, the DLM does not force a linear structure on the data. At each point, the DLM would approximate a nonlinear function with a linear piece. This provides for a flexible evolution pattern of the search activities and the latent awareness.

allow us to test for a potential identification problem by leveraging information from the nonfocal search engine. Specifically, we investigate whether generic activity on the nonfocal search engine spills over into branded activity on the focal search engine. We find that spillover is not present in these tests, reducing the likelihood that spillover effects are due to an omitted variable that affects both generic and subsequent branded activity.

Alternative Models

One alternative model is a traditional time-series approach, such as an autoregressive distributed lag (ARDL) model with correlated errors. In such a model, there would be no latent construct for awareness present. Branded search activity (i.e., impressions, clicks, reservations, average position, and costs) is dependent on its own lagged values, seasonal effects, and lagged generic search activity (i.e., generic impressions and generic clicks). Note that in using this approach, we would not be able to deconstruct search activities as proposed previously; that is, we would not be able to model NS, CTR, and CR. An advantage of our DLM is that it allows us to estimate the underlying structural parameters. Observed clicks are matched to $NS \times CTR$, which allows us to decompose the effects of generic search on NS and CTR. A traditional approach needs to model impressions, clicks, and reservations directly together with position and cost to account for the firm's paid search strategy. Next, we test whether our DLM with awareness fits better than an alternative ARDL specification.

A second alternative is the VAR approach. Here, we do not need to formulate how different search activities interact with each other, as described in Equation 7. Rather, the standard VAR model allows the data to reveal relationships among the different search activities by treating each as endogenous. (For a more detailed discussion of alternative models, see Web Appendix C at <http://www.marketingpower.com/jmrfeb11>.)

Test for Reverse Spillover

It is possible that generic search is affected by spillover from branded search. Because the awareness concept is related to a brand, not a generic entity, it is unclear how branded search activity would lead to greater generic search activity (through awareness). Nevertheless, we test for the possible effects of branded search on generic search using our DLM and an ARDL model analogous to the branded ARDL.⁹ Both models use generic search activity as the dependent variable to test whether past generic search and past branded search affect it.

EMPIRICAL APPLICATION

Paid Search Lodging Data

Our data include daily information on a paid search campaign for a major lodging chain. For each search keyword (e.g., "hotels Los Angeles"), we know its generic or branded designation, daily information on cost (in U.S. dollars), average position served (given by daily average placement rank, e.g., 2.3), and number of impressions, clicks, and

reservations. The data set includes campaign information from both Google and Yahoo. The Google data run from March 1, 2004, to December 20, 2004, and the Yahoo data run from May 6, 2004, to August 31, 2004. Both campaigns included several hundred generic and branded keywords. (The exact number is proprietary but stable over the sample.) Tables 1 and 2 give summary statistics.

Impressions, clicks, and reservations pertaining to both generic and branded search fluctuate by day of week. In Figure 3, we present a representative time-series snapshot from the Google data for four weeks. (The Yahoo data show the same pattern.) The point of greatest activity is usually on Monday. Activity declines modestly up to Thursday. Beginning on Friday, the weekend brings a steep drop. This pattern is consistent, with most online traffic coming from the workplace (Pauwels and Dans 2001); therefore, we include indicator variables for day of week in the model. In addition to the day-of-week fluctuation, there is also substantial variation in the level of generic search activity over time (e.g., from preholiday trip planning).

Model Comparison: Within Sample

We estimate the DLM, ARDL, and VAR models in a Bayesian framework and compare them using log Bayes factors. We test for autocorrelation using a Durbin-Watson test and regress the residuals on their own lags. Using both methods, we find that there is no autocorrelation in the residuals.¹⁰ (This holds for both the Google and the Yahoo data.) In Table 3, Panel A, we report the results of the model comparisons for the Google data.¹¹ For both the ARDL and the VAR models, the formulation with one lag term for both branded and generic activity provides the best fit.

Table 3, Panel A, shows the best-fitting ARDL model (one lag generic and one lag branded; log marginal density:

¹⁰Serial correlation also can be modeled in the DLM framework. Consider the example model, given by $y_t = x_t\beta + \theta_t + \varepsilon_t$ and $\theta_t = \delta\theta_{t-1} + v_t$, where y and x are observed, θ_t is latent, and $v_t \sim N(0, \sigma_v^2)$. If serial correlation is present, it follows that $\varepsilon_t = \rho\varepsilon_{t-1} + \xi_t$, where $\xi_t \sim N(0, \sigma_\xi^2)$. To handle it, we can augment the state space as follows:

$$y_t = x_t\beta + \theta_t + \varepsilon_t$$

$$\begin{pmatrix} \theta_t \\ \varepsilon_t \end{pmatrix} = \begin{pmatrix} \delta & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} \theta_{t-1} \\ \varepsilon_{t-1} \end{pmatrix} + \begin{pmatrix} v_t \\ \xi_t \end{pmatrix}.$$

We can estimate the augmented model in the DLM framework and, if necessary, incorporate higher-order serial correlation as well (Naik and Raman 2003). As Naik and Raman (2003, p. 384) illustrate in their Equations 25, 26, and 27, both the lagged dependent variable and the serial correlation can be incorporated in an appropriate DLM formulation. As a special case of their equations, we consider the following model: $y_t = \lambda y_{t-1} + x_t\beta + \theta_t + \varepsilon_t$. Then we can express it in DLM as follows:

$$y_t = Z\alpha_t + \zeta_t,$$

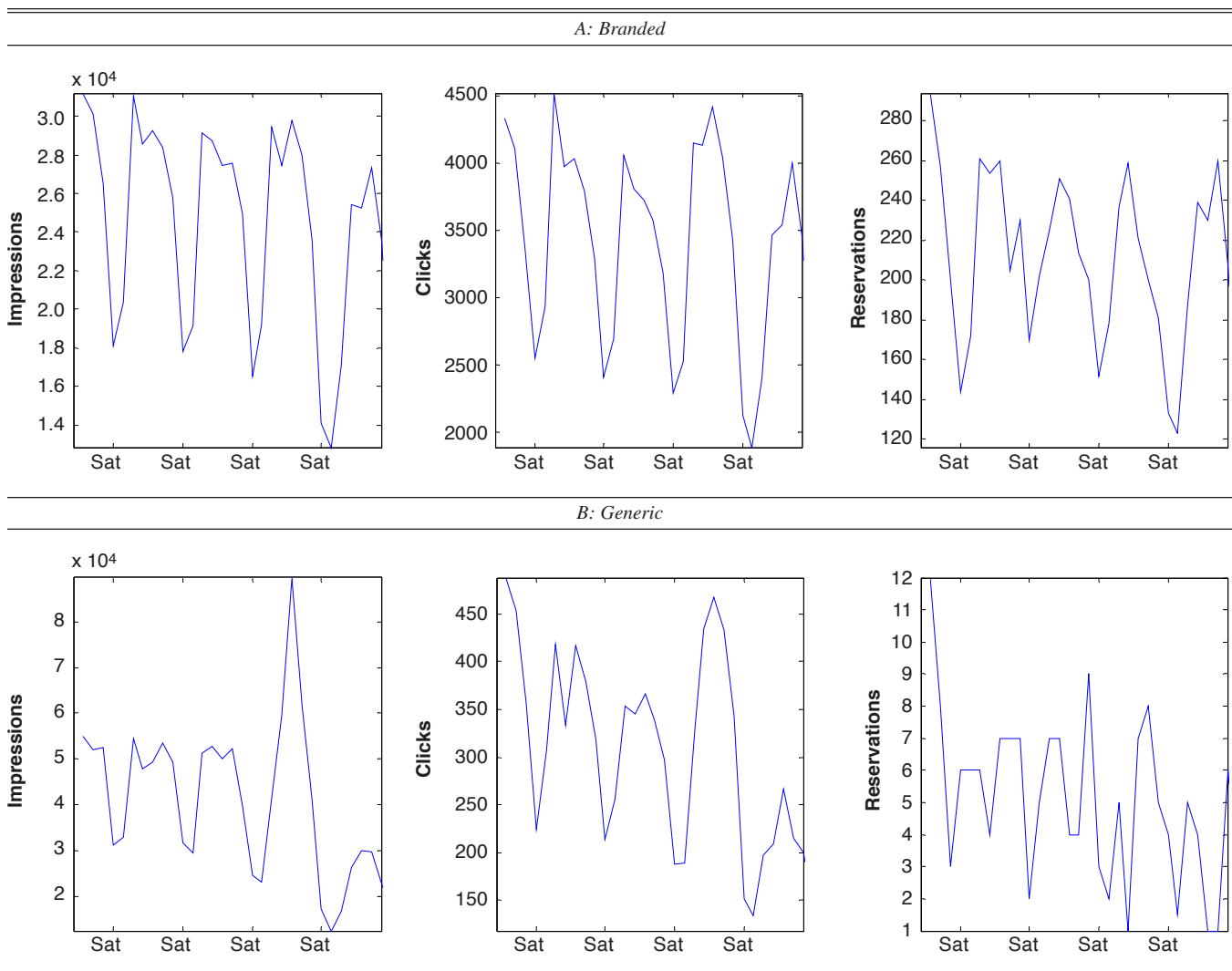
$$\alpha_t = \begin{bmatrix} y_t \\ \theta_{t+1} \\ \varepsilon_{t+1} \end{bmatrix} = \begin{bmatrix} \lambda & 1 & 1 \\ 0 & \delta & 0 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \theta_t \\ \varepsilon_t \end{bmatrix} + \begin{bmatrix} x'\beta \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ v_{t+1} \\ \xi_{t+1} \end{bmatrix},$$

where $Z = [1 \ 0 \ 0]$.

¹¹The results for the Yahoo data are similar and available on request.

⁹The specification simply exchanges branded and generic in Equation 11. Generic search activities become the dependent variables, and we use branded impressions and clicks as predictors.

Figure 3
DAILY ACTIVITY FOR BRANDED AND GENERIC KEYWORDS FOR GOOGLE DATA (FOUR-WEEK PERIOD)



-11,958), VAR model (one lag generic and one lag branded, log marginal density: -11,312), and DLM model (log marginal density: -11,245). We find that the best ARDL and VAR models are rejected in favor of the integrated DLM.

We test for diminishing returns in the awareness part of our model by using the log of impressions and the log of clicks as measures of generic activity. We find that the models with impressions and clicks provide better fits than the log models (see Table 3, Panel B). We also test whether dollars spent on generic search provides a better fit than the observed count of generic impressions and clicks. Table 3, Panel B, shows that the count data provide a better fit.

Model Comparison: Out of Sample

We assessed the out-of-sample forecast performance for the DLM, ARDL, and VAR models on the Google data. We estimated all models with two sample cutoff points: The first sample has 100 data points, and the second has 200. For both cases, we generate an out-of-sample forecast for ten periods (Bass et al. 2007). In Table 4, we report the mean absolute percentage error in the forecast period for each

branded search activity for each model. In all cases, the DLM outperforms the ARDL and VAR models. The selected lag structure for the ARDL and VAR models is also confirmed by the holdout test. (The best-fitting ARDL and VAR models use one lag branded and one lag generic.)

In summary, our DLM model outperforms the alternative time-series models within sample and out of sample. This suggests that incorporating the latent construct of awareness (DLM) offers a superior approach to one in which generic search directly enters the model through a specified lag structure or in a VAR. It is noteworthy that we find that measuring generic search activity on the basis of absolute impressions and clicks is preferred to doing so on the basis of dollars spent.

Parameter Estimates

We first discuss the parameter estimates for the Google data set (Table 5, Panels A and B). Google had a significantly higher level of daily activity than Yahoo (at least for this search campaign) and also provided a somewhat longer time series. Next, we briefly examine the Yahoo results to corroborate the findings from the Google data (Table 6, Pan-

Table 3
MODEL COMPARISON

A: In-Sample Fit Measures			
		Model Fit	
		Log Marginal Density	Log Bayes Factor
DLM model		-11,245	—
ARDL model	1 Lag BR 1 Lag GEN	-11,958	713
VAR model	1 Lag BR 1 Lag GEN	-11,312	67
B: Measurement of Generic Activity in the DLM Model			
		Model Fit	
Generic Activity Measured By		Log Marginal Density	Log Bayes Factor
DLM model	Impressions and clicks	-11,245	—
	ln(impressions) and ln(clicks)	-11,307	62
	Cost	-11,339	94

Notes: For Panel A, generic activity is measured by number of impressions and number of clicks, the log Bayes factor is expressed in relation to the best model (i.e., the DLM), and the ARDL and VAR models are the best-fitting models. Fits for other lag formulations are available on request. For Panel B, the log Bayes factor is expressed in relation to the best model (i.e., the model that uses impressions and clicks as measures of generic activity).

els A and B).¹² Table 5, Panel A, and Table 6, Panel A, show the estimates from the reduced-form model given in Equations 9 and 10, and Table 5, Panel B, and Table 6, Panel B, give the estimates for the structural DLM specified in Equation 7 (for details, see Web Appendix A at <http://www.marketingpower.com/jmrfeb11>).

Indicator variables for day of week and month. All models include indicator variables to control for differences in search activity by day of week and month. The significant (and, therefore, retained) covariates are the same for Google and Yahoo. As we expected, the indicator variables capture the lower weekend search activity. We find a negative effect for Saturday (or the start of the weekend, WE) and a positive effect for Monday (or the start of the week, WK). Only the indicator variables for NS are significant (see Table 5, Panel B, and Table 6, Panel B). Thus, day-of-the-week effects in branded search activity occur only in the number of searches (i.e., the search volume), not in the differences in click-through or conversion (see Table 5, Panel A, and Table 6, Panel A). In other words, search volume is affected by day of the week, as previous research has found (Pauwels and Dans 2001). However, the propensity to click through or convert is not affected by day of the week. We find similar patterns for position and CPC: There are no significant differences across days of the week (see Table 5, Panel A, and Table 6, Panel A). We did not find any significant patterns of monthly seasonality in either data set.

Average number of branded searches. We turn to the results for NS reported in Table 5, Panel B. We find that lagged NS has a coefficient of .8367 (α^{NS} ; see Table 5, Panel B). Our data come from a well-established firm in the U.S. lodging industry. The high carryover rate indicates that there

Table 4
MODEL COMPARISON: FORECAST PERFORMANCE

A: $t = 100$				
		MAPE		
		Impressions	Clicks	Reservations
DLM model		.1012	.0129	.0014
ARDL model	1 Lag BR 1 Lag GEN	.1113	.0153	.0028
VAR model	1 Lag BR 1 Lag GEN	.1087	.0138	.0021
B: $t = 200$				
		MAPE		
		Impressions	Clicks	Reservations
DLM model		.1257	.0108	.0009
ARDL model	1 Lag BR 1 Lag GEN	.1521	.0236	.0016
VAR model	1 Lag BR 1 Lag GEN	.1284	.0128	.0012

Notes: MAPE = mean absolute percent error. We assessed forecast performance with two scenarios: using the first 100 data points ($t = 100$) and the first 200 data points ($t = 200$). In each scenario, we forecast the next ten periods and compare models. We report fit results for the best-fitting lag specification for the ARDL and VAR models.

is a significant and stable base level of searches for the firm over time. We find that the latent construct for awareness positively affects current NS ($\gamma^{NS} = .0514$; see Table 5, Panel B). This indicates that greater awareness leads to more searches (i.e., impressions) for keywords that include the brand name. (We discuss the effect of generic search activity on awareness after findings for branded search.)

CTR. The carryover from the previous period is $\alpha^{CTR} = .8836$ (see Table 5, Panel B). Again, we find a high carryover rate for CTR. This is because our firm is well known and offers a stable service. We expect low variation in click-through across time because the service offering is not changing. As we expected, current position has an effect on CTR. We find that the effect of current position on CTR is negative ($\delta = -.0166$, see Table 5, Panel B). A higher position (e.g., 5) decreases CTR compared with a lower position (e.g., 2). Unlike NS, CTR is not affected by awareness (γ^{CTR} is not significant; see Table 5, Panel B). In other words, awareness (potentially generated by exposure to branded materials after a generic search) leads to an increase in searches. However, it does not lead to an increase in CTR.

CR. We also find a high carryover rate for CR ($\alpha^{CR} = .8149$; see Table 5, Panel B), though slightly lower than for NS and CTR. Again, this result is driven by our lodging company being well known and offering a clear value proposition. Thus, our finding of a relatively stable CR over time is not surprising. There is also no significant effect of awareness on CR (γ^{CR} is not significant; see Table 5, Panel B).

POS. We find that $\alpha^{POS} = .7975$, indicating a stable positioning strategy over time. Managers verified our finding: Position targets remained mainly stable over the period of our data. Next, η allows us to understand better how Google has “tweaked” the second price auction to include past ad performance (generally assumed in practice to be measured by past CTR). We find that $\eta = -12.8446$. In other words, an increase in past CTR leads to a lower current position. (A lower position means a better position; e.g., 2 is a better position than 5, all else being equal.) This is what Google’s algorithm is doing—rewarding better-targeted advertisements (i.e., those with higher CTR) with lower position at the same cost or a constant position at a lower cost (see the

¹²At the time we collected our data, Yahoo had not introduced its new auction system Panama. Panama mimics Google’s system (i.e., incorporating past ad performance) and was using a pure second price auction system. Therefore, no interaction between past ad performance and the auction exists. Translated to our framework, this means that η and κ are zero.

Table 5
ESTIMATES FOR THE GOOGLE DATA SET

A: Reduced-Form DLM Parameter Estimates for the Google Data Set			B: Structural DLM Parameter Estimates for the Google Data Set		
Parameter	Estimate		Parameter	Estimate	
	Mean	95% Coverage Interval		Mean	95% Coverage Interval
<i>Carryover</i>			<i>Carryover</i>		
α^{NS}	.8367	(.7841, .8766)	α^{NS}	.8367	(.7841, .8766)
$\hat{\alpha}^{CTR}$.9198	(.7826, .9972)	α^{CTR}	.8836	(.7550, .9661)
α^{CR}	.8149	(.6929, .9828)	α^{CR}	.8149	(.6929, .9828)
$\hat{\alpha}^{POS}$.9741	(.8977, .9994)	α^{POS}	.7975	(.7364, .8210)
$\hat{\alpha}^{CPC}$.9238	(.8403, .9983)	α^{CPC}	.7564	(.5706, .8210)
α^A	.3354	(.3302, .3486)	α^A	.3354	(.3302, .3486)
<i>Interaction</i>			<i>Interaction</i>		
β^{24}	-.0162	(-.0294, -.0102)	δ	-.0166	(-.0304, -.0104)
β^{25}	.1494	(.0549, .1977)	ϕ	-1.4592	(-18.8231, -4.6312)
β^{42}	-2.1852	(-3.1003, -2.0026)	ι	-.0198	(-.0369, -.0100)
β^{45}	1.7340	(1.4048, 2.3912)	η	-12.8456	(-21.2735, -6.7291)
β^{52}	-1.0193	(-1.1579, -.8925)	κ	-.1490	(-.1973, -.0546)
β^{54}	.1218	(.1076, .1552)	γ^{NS}	.0514	(.0416, .0653)
γ^{NS}	.0514	(.0416, .0653)	γ^{CTR}	4.20E-06	(-2.15E-05, 3.16E-05)
γ^{CTR}	4.20E-06	(-2.15E-05, 3.16E-05)	γ^{CR}	1.91E-05	(-4.63E-07, 4.51E-05)
γ^{CR}	1.91E-05	(-4.63E-07, 4.51E-05)	<i>Drift</i>		
<i>Drift</i>			λ_{WK}^{NS}	59.6532	(48.9035, 69.7489)
λ_{WK}^{NS}	59.6532	(48.9035, 69.7489)	λ_{WE}^{NS}	-47.1453	(-56.9338, -34.6855)
λ_{WE}^{NS}	-47.1453	(-56.9338, -34.6855)	λ_{WK}^{CTR}	-.0029	(-.0180, .0128)
$\hat{\lambda}_{WK}^{CTR}$	-.0034	(-.0184, .0124)	λ_{WE}^{CTR}	.0114	(-.0047, .0266)
$\hat{\lambda}_{WE}^{CTR}$.0110	(-.0154, .0261)	λ_{WK}^{CR}	-.0028	(-.0186, .0129)
λ_{WK}^{CR}	-.0028	(-.0186, .0129)	λ_{WE}^{CR}	.0001	(-.0159, .0163)
λ_{WE}^{CR}	.0001	(-.0159, .0163)	λ_{WK}^{POS}	.0308	(-.0265, .0920)
$\hat{\lambda}_{WK}^{POS}$.0308	(-.0265, .0920)	$\hat{\lambda}_{WE}^{POS}$.0246	(-.0324, .0806)
$\hat{\lambda}_{WE}^{POS}$.0246	(-.0324, .0806)	λ_{WK}^{CPC}	.0063	(-.0124, .0250)
$\hat{\lambda}_{WK}^{CPC}$.0063	(-.0124, .0250)	λ_{WE}^{CPC}	.0061	(-.0123, .0254)
$\hat{\lambda}_{WE}^{CPC}$.0061	(-.0123, .0254)	λ_{WK}^{IMP}	.0012	(-.0236, .1102)
λ_{WK}^{IMP}	.0012	(-.0236, .1102)	λ_{GEN}^{CL}	1.0873	(.9812, 1.1828)
λ_{GEN}^{CL}	1.0873	(.9812, 1.1828)			

Notes: WK = start week (i.e., Monday), and WE = start weekend (i.e., Saturday). Indicator variables for months were not significant, and we removed them from the model. Parameters in boldface are significant.

next section for results for CPC). The effect of current CPC on current POS is $\phi = -10.4592$, so a higher current CPC leads to a lower position. In turn, the effect of current POS on current CPC is $\iota = -.0198$, meaning that an increase in POS reduces CPC. As with many time-series models, these parameters are not meaningful in themselves; in other words, comparing η and ι will not lead to insights other than whether the parameters are significant. Next, we use an impulse response approach to shed light on their meaning.

Note that we did not set out to optimize the firm's bids. We included POS and CPC in our model to account for the firm's campaign management strategy and to allow simultaneous interactions among POS, CPC, and CTR. Given the structure of paid search in general, and Google's system in particular, a model that investigates spillover effects between generic and branded needs to address these interactions.

CPC. We find that $\alpha^{CPC} = .7564$ and $\kappa = -.1490$. The high carryover rate for CPC, α^{CPC} , again indicates that the firm's bidding strategy was relatively stable over the period of the data. The parameter κ gives us another glimpse into Google's black box: A higher past CTR leads to a lower CPC going forward. Google "punishes a bad advertisement" (i.e., low past CTR) by charging more for the same position. The effect of current POS on current CPC is $\iota = -.0198$, meaning that a higher position is cheaper.

Awareness. In the leaky-bucket model formulation, changes in awareness are a function of the carryover rate

and changes in the brand-related exposure generic search activity. Table 5, Panel B, reports the parameter estimates and coverage intervals for this component of the model. We find that approximately 30% of current awareness is "carried over" to the next period (carryover rate $\alpha^A = .3354$; see Table 5, Panel B). In our framework, this means that consumers have, on average, a relatively short search process.

We believe that our results using daily search data provide a lower bound for spillover. To be sure, more spillover could be occurring intraday or even within a given user session. Our finding of significant spillover across days highlights the importance of this phenomenon. Although some advertisers are now able to examine intraday data (and analyzing this should be a topic for further research), we note that it is likely to bring other modeling challenges along with it (e.g., sparse data in the overnight hours).¹³

At the heart of our study is the question whether generic search activity spills over into branded search activity through awareness. Our modeling results indicate that this is

¹³Although we acknowledge the limitations of daily data, we note that other recent studies on paid search have worked with weekly or monthly data (e.g., Ghose and Yang 2009a, b; Yao and Mela 2009). We conducted an analysis in which we aggregated our data to the weekly level and estimated exploratory models. We found no significant effect for generic-to-branded spillover in any case. Thus, access to daily data has at least enabled documentation of the spillover effect, its asymmetry, and an initial estimate of its magnitude and decay over time.

Table 6
ESTIMATES FOR THE YAHOO DATA SET

A: Reduced-Form DLM Parameter Estimates for the Yahoo Data Set			B: Structural DLM Parameter Estimates for the Yahoo Data Set		
Parameter	Estimate		Parameter	Estimate	
	Mean	95% Coverage Interval		Mean	95% Coverage Interval
<i>Carryover</i>			<i>Carryover</i>		
α^{NS}	.9396	(.9001, .9732)	α^{NS}	.9396	(.9001, .9732)
$\hat{\alpha}^{CTR}$.9680	(.8786, .9994)	α^{CTR}	.9680	(.8786, .9994)
α^{CR}	.8911	(.8015, .9945)	α^{CR}	.8911	(.8015, .9945)
$\hat{\alpha}^{POS}$.8912	(.8091, .9832)	α^{POS}	.6845	(.5754, .7736)
$\hat{\alpha}^{CPC}$.8773	(.7687, .9937)	α^{CPC}	.6696	(.5022, .7674)
α^A	.4134	(.3326, .4773)	α^A	.4134	(.3326, .4773)
<i>Interaction</i>			<i>Interaction</i>		
β^{24}	-.0110	(-.0142, -.0091)	δ	-.0124	(-.0160, -.0105)
β^{25}	.0394	(.0131, .0570)	φ	-3.7261	(-5.9309, -1.1162)
β^{42}		N.A.	ι	-.0488	(-.1296, -.0335)
β^{45}	1.0334	(.0992, 1.9018)	η		N.A.
β^{52}		N.A.	κ		N.A.
β^{54}	.0122	(.0017, .0341)	γ^{NS}	.0747	(.0524, .1061)
γ^{NS}	.0747	(.0524, .1061)	γ^{CTR}	.0003	(-.0001, .0008)
γ^{CTR}	.0003	(-.0001, .0008)	γ^{CR}	.4134	(-.0003, .0005)
γ^{CR}	.4134	(-.0003, .0005)	<i>Drift</i>		
<i>Drift</i>			λ_{WK}^{NS}	28.1173	(21.5866, 34.3440)
λ_{WK}^{NS}	28.1173	(21.5866, 34.3440)	λ_{WE}^{NS}	-15.1429	(-21.4095, -9.3381)
λ_{WE}^{NS}	-15.1429	(-21.4095, -9.3381)	$\hat{\lambda}_{WK}^{CTR}$.0100	(-.0277, .0500)
$\hat{\lambda}_{WK}^{CTR}$.0090	(-.0291, .0489)	$\hat{\lambda}_{WE}^{CTR}$.0243	(-.0142, .0655)
$\hat{\lambda}_{WE}^{CTR}$.0241	(-.0150, .060)	λ_{WK}^{CR}	-.0030	(-.0407, .0355)
λ_{WK}^{CR}	-.0030	(-.0407, .0355)	λ_{WE}^{CR}	.0164	(-.0212, .0551)
λ_{WE}^{CR}	.0164	(-.0212, .0551)	$\hat{\lambda}_{WK}^{POS}$.0773	(-.1464, .3268)
$\hat{\lambda}_{WK}^{POS}$.0773	(-.1464, .3268)	$\hat{\lambda}_{WE}^{POS}$.0222	(-.2002, .2524)
$\hat{\lambda}_{WE}^{POS}$.0222	(-.2002, .2524)	$\hat{\lambda}_{WK}^{CPC}$.0039	(-.0372, .0458)
$\hat{\lambda}_{WK}^{CPC}$.0039	(-.0372, .0458)	$\hat{\lambda}_{WE}^{CPC}$.0006	(-.0390, .0430)
$\hat{\lambda}_{WE}^{CPC}$.0006	(-.0390, .0430)	λ_{GEN}^{IMP}	6.89E-07	(-3.48E-06, 3.28E-05)
λ_{GEN}^{IMP}	6.89E-07	(-3.48E-06, 3.28E-05)	λ_{GEN}^{CL}	.1677	(.0054, .7139)
λ_{GEN}^{CL}	.1677	(.0054, .7139)			

Notes: WK = start week (i.e., Monday), and WE = start weekend (i.e., Saturday). Indicator variables for months were not significant, and we removed them from the model. N.A. = not applicable because Yahoo's auction mechanism at this time did not take past performance into account; that is, past CTR does not affect current position or CPC. Parameters in boldface are significant.

indeed the case. First, as we discussed previously, awareness has a positive impact on current branded searches, NS ($\gamma^{NS} = .0514$). Second, we find that generic clicks have a strong positive effect on awareness ($\lambda_{GEN}^{CL} = 1.0873$; see Table 5, Panel B). In contrast, generic impressions do not have a significant effect on awareness ($\lambda_{GEN}^{IMP} = .0012$; see Table 5, Panel B) because the 95% coverage interval $(-.0236, .1102)$ includes zero. This means that simply being exposed to the company's text advertisement after a generic search does not spill over to increase branded search activity. However, if the user clicks on the advertisement and visits the company Web site, this leads—through awareness—to an increase in branded searches going forward. We can hypothesize that inspecting a brand after a generic click-through might lead the consumer to become aware of its relevance for current search goals. The user might search for the brand again, next time using a query that includes the brand name.

Results for the Yahoo Data

We also estimated the DLM model on the Yahoo data set to provide a validation test across search engines. Yahoo used a different method to rank sponsored links on its site at the time we collected our data set. Yahoo had not introduced its Panama system, which mimics the Google system, but was using a pure second price auction system with no performance feedback through past CTR. Note that the feed-

back parameters, η and κ , that link past CTR to current POS and CPC are set to zero in the Yahoo model.¹⁴ The two search engines also differed in site design and appearance and might attract different types of online users. (We have no direct evidence of user differences other than management's belief and industry white papers.) Panels A and B in Table 6 report the parameter estimates for the Yahoo data.

A comparison of Table 5 (Google) and Table 6 (Yahoo) shows that all the key findings are corroborated. We find that the effects for Saturdays and Monday are similar among the indicator variables. We also find no significant seasonality in monthly effects. As with the Google data, lagged branded activity has carryover coefficients in the .90–.95 range (see Table 6, Panel B). In both cases, CR has a somewhat lower carryover rate than NS and CTR. Awareness also significantly affects NS, but not CTR and CR. (Because awareness is dimensionless, the coefficients are not directly comparable between the two data sets.)

The estimated awareness carryover rate for the Yahoo data is of similar magnitude (Yahoo $\alpha^A = .4134$ versus Google $\alpha^A = .3354$) to the one estimated for Google. The parameter estimates for generic impressions and clicks in

¹⁴Estimating the model on the Yahoo data without these constraints produced similar results, and neither parameter was significantly different from zero.

the Yahoo data also parallel the results in the Google data. At both search engines, generic impressions had no significant effect on awareness, but generic clicks did. Thus, we conclude that the estimates from the Yahoo data corroborate the nature of the spillover effects found in the Google data.

Testing for Reverse Spillover

Our results show that generic search positively affects branded search through awareness. However, this finding could be due to a general correlation between generic and branded search: On days with high search activity in the category, both generic and branded search might be similarly affected. To address this alternative explanation, we also investigate whether branded search influences generic search. We estimate a DLM and an ARDL model with generic search activity measures as the dependent variables (in place of branded search measures). Analogous to the previously described models, we include daily and monthly indicator variables along with lagged generic and branded activity as covariates.

In both the DLM and the ARDL models, we do not find that branded search activities affect generic search activity; that is, all the relevant parameters are insignificant for both the Google and the Yahoo data. Table 7 presents these parameters and their coverage intervals. Collectively, our results show that spillover is asymmetric (i.e., generic affects branded but not vice versa), consistent with the conceptual and modeling frameworks we propose.

Omitted Variable Bias

Another potential alternative explanation for our results is omitted variable bias. Suppose that there is no spillover from generic search activity to branded search. We still might find such an effect if generic search activity in our model proxies for an omitted variable, which in turn affects branded search. Although we cannot completely eliminate this possibility, we can take advantage of the availability of activity data for two search engines. (Thus far, all our results have been based on models estimated on the data within a given search engine, i.e., using either Google or Yahoo data alone.)

Industry white papers have reported that consumers are loyal to their search engine and that limited cross-engine search takes place (i.e., users become accustomed to employing one engine or another). If such is the case, examining whether generic search activity on Yahoo creates a spillover effect on branded activity on Google (and vice versa) enables us to assess the potential for omitted variable bias. This is because such an omitted variable (e.g., a major offline advertising campaign) would be likely to have similar effects on activity at both search engines. We reestimated our models by switching the generic activity variables between search engines. For both Google and Yahoo, we find that generic activity on the nonfocal search engine does not spill over to branded search activity on the focal search engine. Table 8 shows that the relevant parameters that capture the impact of generic activity on awareness are not significant at the 95% coverage level in either case.

The Extent of Generic-to-Branded Spillover

The foregoing results demonstrate that spillover is significant and asymmetric. However, it is difficult to tell from the DLM parameters what the extent of this effect is likely to be. To investigate the magnitude of the estimated spillover effect, we use an impulse response approach. Because generic impressions do not have a significant effect in the model, we focus our analysis on generic clicks. Specifically, we ask how much spillover to branded search would be generated by an additional ten generic clicks on the Google or Yahoo search engine. To do this, we shock the system in the first period with ten generic clicks. (Such a shock could come from, for example, advertising through an additional set of generic keywords.) Using the existing data and our model estimates, we then calculate how the one-time increase in generic clicks affects reservations stemming from branded search activity.

We find that the shock in Period 1 produces an increase in branded search activity. In Figure 4, we plot the incremental reservations stemming from activity at each search engine for the following 30 days. For Google, the effect peaks on Day 3 and decays afterward. For Yahoo, the effect peaks on Day 5. After 12 days, nearly 95% of the spillover

Table 7

STRUCTURAL SPILLOVER PARAMETER ESTIMATES FOR THE GENERIC DLM

Parameter	A: Google	
	Estimate	
	Mean	Coverage Interval
<i>Drift</i>		
λ_{BR}^{IMP}	.0021	(-.0030, .0093)
λ_{BR}^{CL}	.0047	(-.0004, .0085)
Parameter	B: Yahoo	
	Estimate	
	Mean	Coverage Interval
<i>Drift</i>		
λ_{BR}^{IMP}	.0013	(-.0006, .0022)
λ_{BR}^{CL}	-.0115	(-.0180, .0049)

Notes: For expositional ease, we only report the relevant parameters (i.e., the parameters that describe spillover from branded to generic). Parameters in boldface are significant.

Table 8

STRUCTURAL SPILLOVER PARAMETER ESTIMATES FOR THE GENERIC DLM USING NONFOCAL GENERIC ACTIVITY

Parameter	A: Google	
	Estimate	
	Mean	Coverage Interval
<i>Drift</i>		
$\lambda_{GE_Yahoo}^{IMP}$	-.0075	(-.0092, .0023)
$\lambda_{GE_Yahoo}^{CL}$	-.0900	(-.2752, .2789)
Parameter	B: Yahoo	
	Estimate	
	Mean	Coverage Interval
<i>Drift</i>		
$\lambda_{GE_Google}^{IMP}$	-.0678	(-.3666, .2573)
$\lambda_{GE_Google}^{CL}$.0144	(-.0601, .0615)

Notes: For expositional ease, we only report the relevant parameters (i.e., the parameters that describe spillover from branded to generic). Parameters in boldface are significant.

has been realized at Google. The total effect of the ten additional generic clicks is to produce .68 (.51) incremental reservations in the Google (Yahoo) case. Of these, .56 (.32) are spillover reservations from branded search, and the balance are reservations that flowed directly from generic search conversions.

The pattern of results in Figure 4 is consistent with the notion that the search for lodging is relatively short—primarily occurring over the course of a couple of days to two weeks. We would expect spillover timing to differ across products (e.g., for new cars, it might be longer and more evenly spread out). Analysts should be able to use this approach to estimate the role of spillover in a paid search advertising campaign and then put it into a decision support system for optimizing paid search ad spending allocations. We leave this endeavor as an important topic for further research.

Consumer Search and Awareness of Relevance

A key tenet of our approach is that the latent construct in the DLM represents what we refer to as awareness of relevance; that is, generic search informs consumers that the brand is relevant to their search goals. Thus, we might expect to observe that the effects of generic search are more pronounced when the underlying level of brand awareness is high. Although we lack information on awareness for our lodging chain, we do know that it is viewed as a stronger brand in rural areas than major cities. To explore this, we split our generic search variables into search activity for keywords with locations in major cities and activity for all other U.S. locations. We estimate the DLM model using both sets of new variables. We find that the effect of generic clicks is significant in each case but much stronger for generic searches for the rural locations.¹⁵ Though exploratory, this result is consistent with our interpretation of the latent construct as awareness of relevance.

CONCLUSION

In this study, we examine two categories of paid search advertising: the text advertisements linked to generic keyword searches and those linked to branded keyword searches. Our objective is to investigate the dynamic rela-

tionship between these two categories of search activity. Specifically, we study whether generic search activity spills over to branded search and, if so, whether the effect is asymmetric (i.e., reverse spillover does not occur).

To investigate these questions, we develop a modeling framework based on the established notion that exposure to brand-related information creates awareness. Applying this idea to paid search, our approach holds that the exposure to brand-related information due to paid search (e.g., impressions from text advertisements, company Web pages after click-through) can create an awareness of relevance that the brand provides a solution to the searcher's problem. As users continue their search process, this awareness can then spill over to future branded search activity.

Following Naik, Mantrala, and Sawyer (1998), Naik, Raman, and Srinivasan (2009), and Bass and colleagues (2007), we model the dynamic evolution of awareness of relevance using a leaky-bucket model. In place of traditional (indirect) measures of brand-related exposures (e.g., gross rating point, dollar expenditures), we use the direct measures of impressions and clicks that occur as a result of search activity. We combine the awareness model with a dynamic branded search activity model and estimate the two components together in a Bayesian framework. Our approach accounts for the firm's endogenous campaign management strategy by including POS and CPC in the model. Our model also accounts for the Google "enhanced" auction, which uses past CTR to either lower position (at the same CPC) or lower CPC (at the same position).

We apply the modeling approach to daily data from paid search campaigns on Google and Yahoo run by a major lodging chain. We model five measures of branded search activity as dependent variables simultaneously: impressions, clicks, reservations, position, and costs. We find that generic search activity, specifically generic clicks, has a significant, positive effect on awareness of relevance. In turn, we find that this awareness significantly influences the number of branded searches, though not the CTR or CR. Thus, increases in the number of clicks and conversions come from the increased number of branded searches taking place in response to an increase in awareness of relevance.

We compare our DLM with two time-series models that do not use a latent construct for awareness. Instead, these alternative models—a standard VAR and an ARDL—represent the effect of generic search on branded search directly within a lag structure. We find that our DLM outperforms both alternative models within sample as well as out of sample.

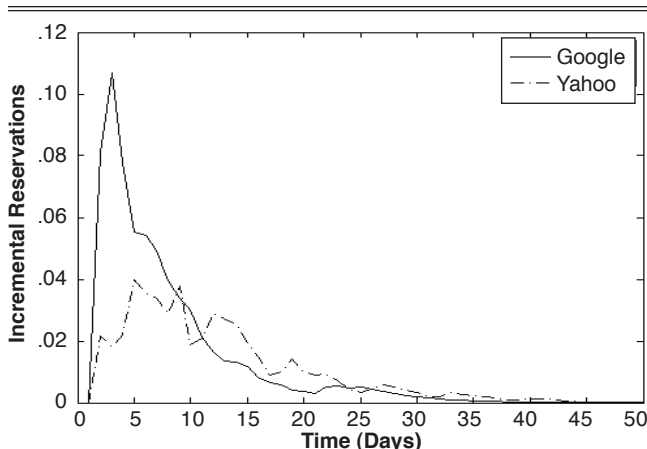
We also investigate the asymmetry in spillover (i.e., whether branded search also spills over to generic search). We find that branded search has no significant impact on generic search, showing the expected asymmetry. In addition, we test for whether generic activity potentially proxies for omitted variables. Our data allow us to test whether generic search on Yahoo (Google) affects branded search on Google (Yahoo). In both cases, we find that generic search on the nonfocal search engine does not influence branded search on the focal search engine.

Last, we measure the extent of the spillover effects over time by employing an impulse response approach. We find that increases in generic search generate sizable spillover to branded search. The peak impact occurs roughly two to five days out, and the bulk of the impact is realized within two weeks. The magnitude of these effects is large enough to

¹⁵Detailed results are available on request.

Figure 4

IMPULSE RESPONSE FOR NUMBER OF RESERVATIONS



have implications for spending allocations and search ad campaign management. We leave exploration of this as an important topic for further research.

We develop and test our model using the type of information that managers use on a day-to-day basis. Thus, our key goal is to make the model useful in practice without having to obtain additional data. However, a drawback of this approach is that it does not track the search activity of individual users. Thus, we have refrained from attempting to develop a comprehensive theory of Internet search behavior as applied to paid search advertising. Instead, we propose the more general conceptualization of awareness of relevance for the search, distinguishing generic from branded terms by the notion that only a brand that has awareness of relevance will be searched using a branded term. The exploratory results we obtained by stratifying the keywords by geographic location also lend support to this interpretation of consumer search behavior. Further research should develop and test a theory-driven model of individual-level paid search.

Although we have data from both Google and Yahoo, we lack information that would enable us to explore, in more detail, the differences between the search engines and their user bases. Given the innovation and competition currently taking place among search engines, we believe that this also represents an important topic for further research.

Links to the advertiser's Web site may also appear in the organic search results that are returned along with the paid search advertisements. Unfortunately, we have no information on organic search in our data set. This means that we cannot assess whether it is necessary for managers to buy branded keyword advertisements to reap the benefits of the spillover from generic search. To the extent that organic listings do meet this need, our approach will overstate the contribution of generic paid search to branded paid search. In addition, an increase in awareness could lead to an increase in direct visits to the Web site (i.e., when visitors type in the URL of the Web site directly). This could also affect the performance of generic search as we measure it.

In this article, we evaluate performance at the level of keyword categories (i.e., aggregations across keywords sharing certain characteristics). Managers also have the capability to make changes at the individual keyword level if they so choose. Developing models to help evaluate the performance of individual keywords would also be worthwhile. For example, it may turn out that the significance and extent of spillover from generic to branded may vary with the characteristics of the generic search term.

Last, we use aggregate data and a latent construct in our modeling approach. We have labeled this latent construct awareness of relevance and believe that, in so doing, it is consistent with both the overall approach and our empirical findings. An important next step would be to obtain actual measures for awareness (e.g., by using survey methods) to validate the construct in future modeling applications.

REFERENCES

- Ataman, Berk, Carl F. Mela, and Harald J. van Heerde (2008), "Building Brands," *Marketing Science*, 27 (6), 1036–1054.
- Bass, Frank, Norris Bruce, Sumit Majumdar, and B.P.S. Murthi (2007), "A Dynamic Bayesian Model of the Advertising-Sales Relationship," *Marketing Science*, 26 (2), 179–95.
- Chen, Yongmin and Chuan He (2009), "Paid Placement: Advertising and Search on the Internet," working paper, Department of Economics, University of Colorado–Boulder.
- Edelman, Benjamin and Michael Ostrovsky (2007), "Strategic Bidder Behavior in Sponsored Search Auctions," *Decision Support Systems*, 43 (1), 192–98.
- , ———, and Michael Schwarz (2007), "Internet Advertising and the Generalized Second Price Auction," *American Economic Review*, 97 (1), 373–99.
- Enquiro (2006), "Inside the Mind of the Searcher," (accessed October 18, 2010), [available at <http://pages.enquiro.com/whitepaper-inside-the-mind-of-the-searcher.html>].
- Ghose, Anindya and Sha Yang (2009a), "An Empirical Analysis of Search Engine Advertising: Sponsored Search," *Management Science*, 55 (10), 1605–622.
- and ——— (2009b), "Modeling Cross-Category Purchases in Sponsored Search Advertising," working paper, Leonard N. Stern School of Business, New York University.
- Goldfarb, Avi and Catherine Tucker (2009), "Search Engine Advertising: Pricing Ads to Context," working paper, Rotman School of Management, University of Toronto.
- Hamilton, James D. (1994), *Time Series Analysis*. Princeton, NJ: Princeton University Press.
- Naik, Prasad A., Murali K. Mantrala, and Alan G. Sawyer (1998), "Planning Media Schedules in the Presence of Dynamic Advertising Quality," *Marketing Science*, 17 (3), 214–35.
- and Kalyan Raman (2003), "Understanding the Impact of Synergy in Multimedia Communications," *Journal of Marketing Research*, 40 (November), 375–88.
- , ———, and Shuba Srinivasan (2009), "Managing Corporate and Product Brands," working paper, Graduate School of Management, University of California, Davis.
- Nerlove, Marc and Kenneth J. Arrow (1962), "Optimal Advertising Policy Under Dynamic Conditions," *Economica*, 29 (114), 129–42.
- Pauwels, Koen (2004), "How Dynamic Consumer Response, Competitor Response, Company Support, and Company Inertia Shape Long-Term Marketing Effectiveness," *Marketing Science*, 23 (4), 596–610.
- (2007), "How Retailer and Competitor Decisions Drive the Long-Term Effectiveness of Manufacturer Promotions for Fast Moving Consumer Goods," *Journal of Retailing*, 83 (3), 297–308.
- and Enrique Dans (2001), "Internet Marketing the News: Leveraging Brand Equity from Market Place to Market Space," *Journal of Brand Management*, 8 (4–5), 303–314.
- PricewaterhouseCoopers (2008), "IAB Internet Advertising Revenue Report," (accessed October 15, 2010), [available at http://www.iab.net/media/file/IAB_PwC_2008_full_year.pdf].
- Rutz, Oliver J. and Randolph E. Bucklin (2009), "A Shrinkage-Based Approach to Measuring Keyword Conversion Rates in Paid Search," working paper, Yale School of Management, Yale University.
- Search Engine Watch (2006), "Delving Deep Inside the Searcher's Mind," special report presented at Search Engine Strategies Conference, San Jose, CA (August 2–5, 2004).
- Srinivasan, Suba, Koen Pauwels, Dominique Hanssens, and Marnik Dekimpe (2004), "Do Promotions Benefit Retailers, Manufacturers, or Both?" *Management Science*, 50 (5), 617–29.
- Steenkamp, Jan-Benedict, Vincent Nijs, Dominique Hanssens, and Marnik Dekimpe (2005), "Competitive Reactions and Advertising and Promotion Shocks," *Marketing Science*, 24 (1), 35–54.
- Van Heerde, Harald J., Carl F. Mela, and Puneet Manchanda (2004), "The Dynamic Effect of Innovation on Market Structure," *Journal of Marketing Research*, 41 (May), 166–83.
- West, Mike and Jeff Harrison (1997), *Bayesian Forecasting and Dynamic Models*. New York: Springer.
- Willbur, Kenneth C. and Yi Zhu (2009), "Click Fraud," *Marketing Science*, 28 (2), 293–308.
- Yao, Song and Carl F. Mela (2009), "A Dynamic Model of Sponsored Search Advertising," working paper, Fuqua School of Business, Duke University.