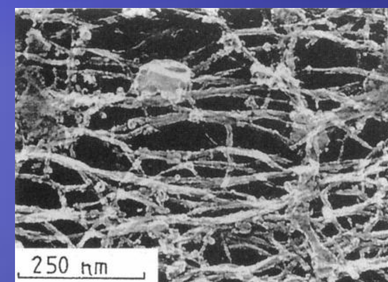
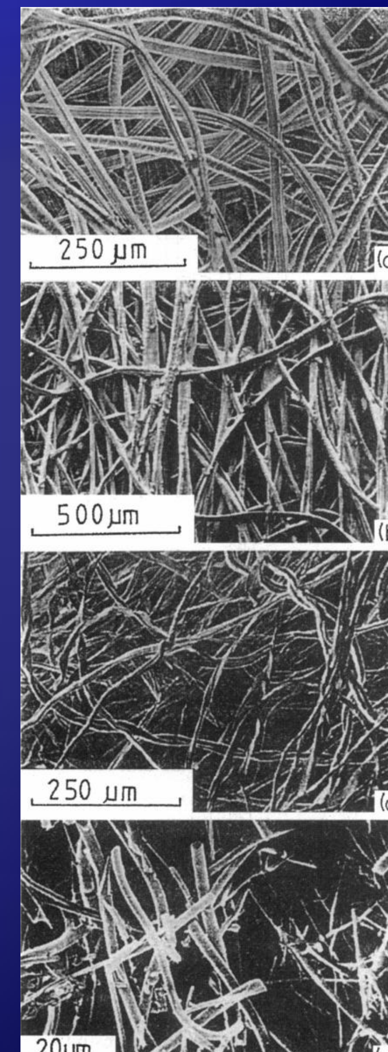
Session 15

FOAMS

Microscopic Structural Comparisons

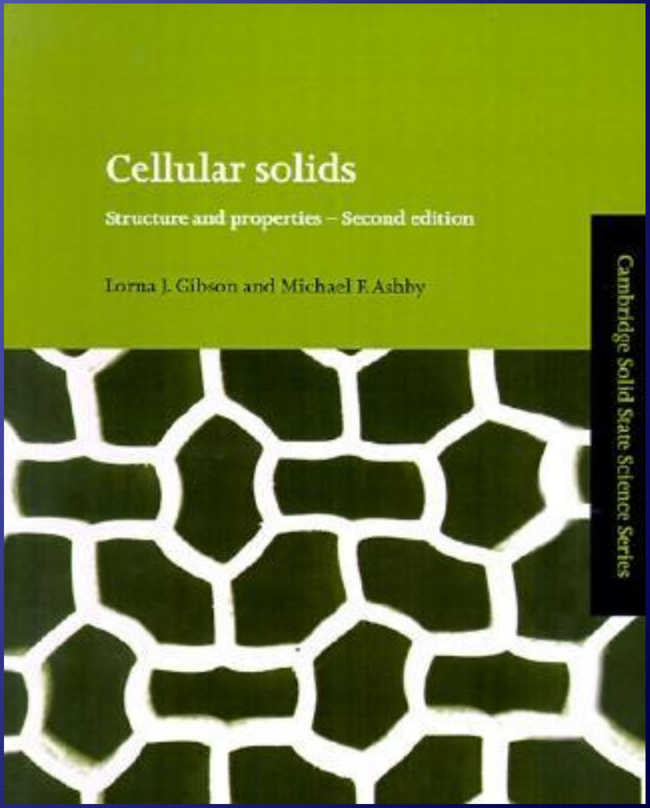
- Cellular CSK resembles natural and synthetic materials
  - Felt
  - Paper
  - Cotton
  - NASA Shuttle Tile
- Components
  - Actin
  - IF





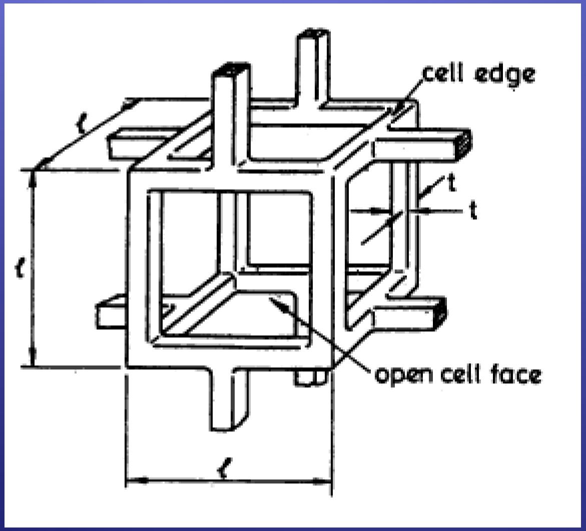
Foam Mechanics

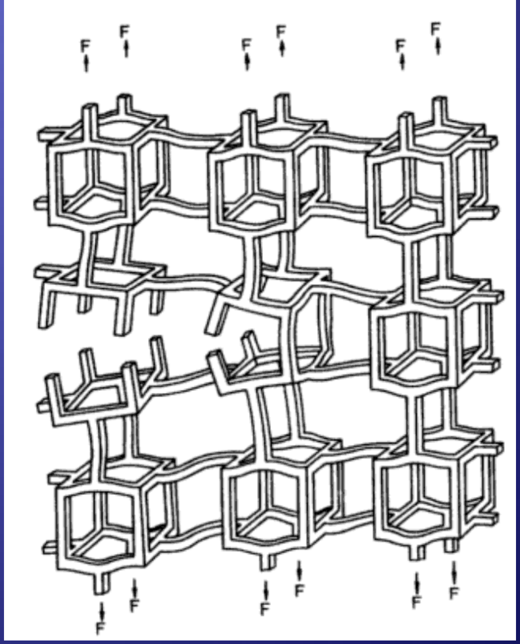
- Gibson & Ashbury (1997) "Cellular Solids: Structure and Properties" Cambridge University Press
- Types
  - Plastic Foams
  - Metallic Foams
  - Ceramic Foams



Structure Classes

- Honeycomb
- Open Cell
- Closed Cell



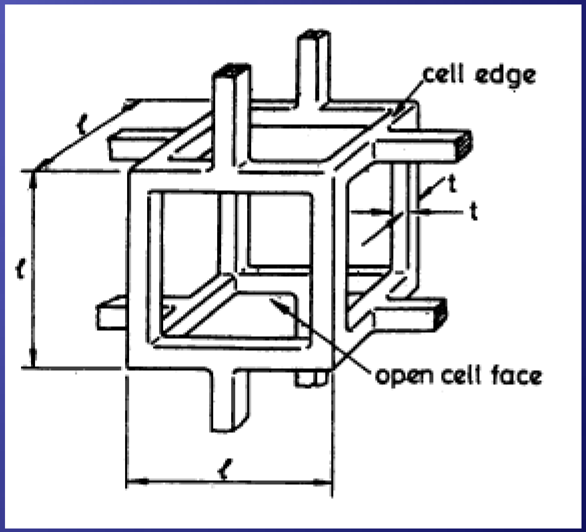


Relative Density

- Ratio (ρ^*/ρ_s)
  - ρ^* is overall foam density
  - ρ_s is constituent's bulk density, e.g. F-actin

$$\left(\frac{\rho^*}{\rho_s}\right) \propto \frac{A_t}{A_l}$$

$$\left(\frac{\rho^*}{\rho_s}\right) = C_0 \frac{t^2}{l^2}$$



  - C_0 is constant (≈1) that depends on foam cell shape

Moment of Inertia

- (a/k/a Second Moment of Area)
- Geometric resistance of a beam to bending

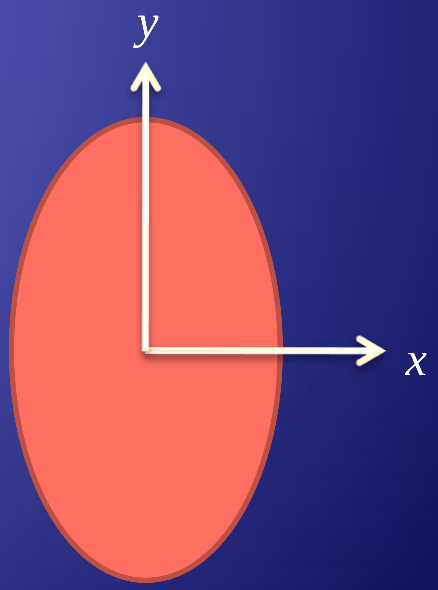
$$I_x = \iint_A y^2 dA$$

$$I_y = \iint_A x^2 dA$$

$$I_{square} = \frac{1}{12} t^4$$

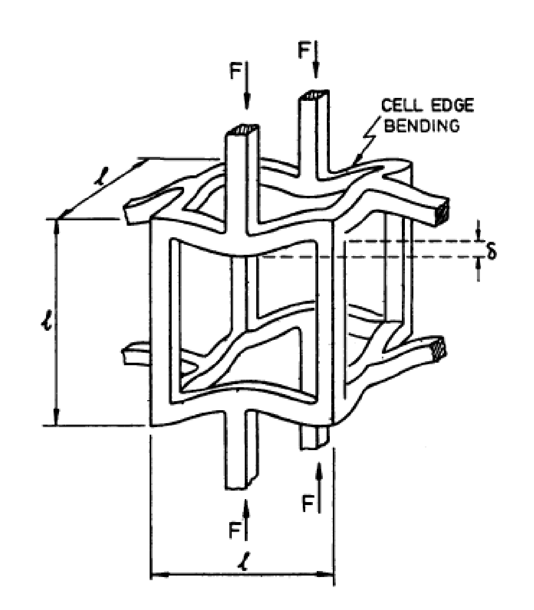
$$I_{circle} = \frac{\pi}{4} r^4$$

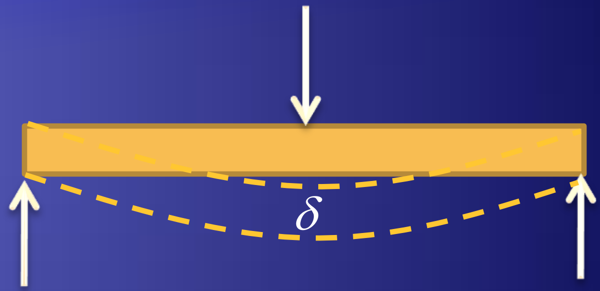
$$I_{foam} \propto t^4$$



Young's Modulus

- Linear Beam Theory
  - Pinned-pinned, three-point bending





$$\delta = \frac{F}{48 E_s I} l^3$$

  - E_s is modulus of elasticity for beam (F-actin ≈ GPa)

Young's Modulus

- Stress remote from the cell (σ) is related to force

$$F \propto \sigma l^2$$

- Strain (ε) related to beam displacement

$$\varepsilon \propto \delta / l$$

- By Hooke's law, the effective modulus is

$$E^* = \frac{\sigma}{\varepsilon}$$

$$\propto \frac{F}{l^2} \frac{E_s I}{F l^2}$$

$$\propto \frac{E_s I}{l^4}$$

Young's Modulus

- Using relative density and moment of inertia

$$\frac{E^*}{E_s} = \frac{I}{l^4} = \left(\left(\frac{t}{l} \right)^2 \right)^2$$

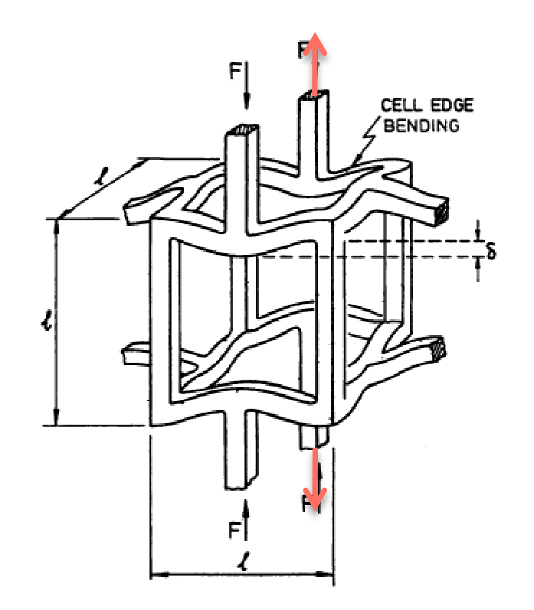
$$\frac{E^*}{E_s} = C_1 \left(\frac{\rho^*}{\rho_s} \right)^2$$

  - C_1 =1 for open cell foams

- Valid for small strains: Large compressive load, side struts will buckle

Shear Modulus

- Similar derivation



- Displacement

$$\delta = \frac{F}{48E_s I} l^3$$

- Stress

$$\tau = F/l^2$$

- Strain

$$\gamma = \delta/l$$

Shear Modulus

- Thus, as before

$$G^* = \frac{\tau}{\gamma}$$

$$\propto \frac{E_s I}{l^4}$$

$$\propto E_s \frac{t^4}{l^4}$$

$$\frac{G^*}{E_s} = C_2 \left(\frac{\rho^*}{\rho_s} \right)^2$$

- C_2 =3/8 for open cell foams

Poisson's Ratio

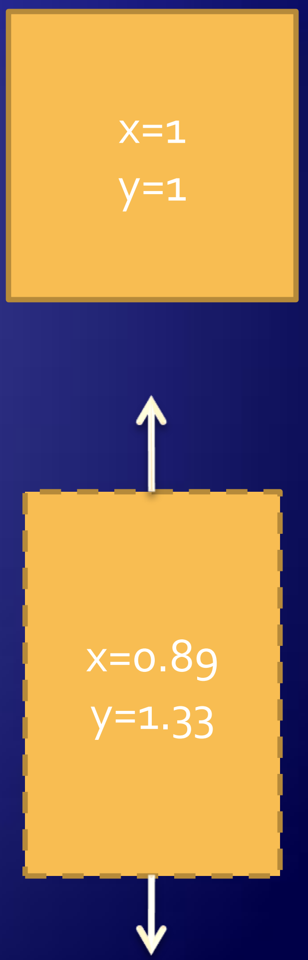
- For linear elastic, isotropic material

$$G = \frac{E}{2(1-\nu)}$$

- Using this for foams

$$\nu^* = \frac{E^*}{2G^*} - 1$$
$$= \frac{C_1}{2C_2} - 1 = C_3$$

- C_3 =1/3 for open cell foams



Buckling

- Elastic collapse occurs when

$$F_{crit} \propto \frac{\pi^2 E_s I}{l^2}$$

- Stress at elastic collapse

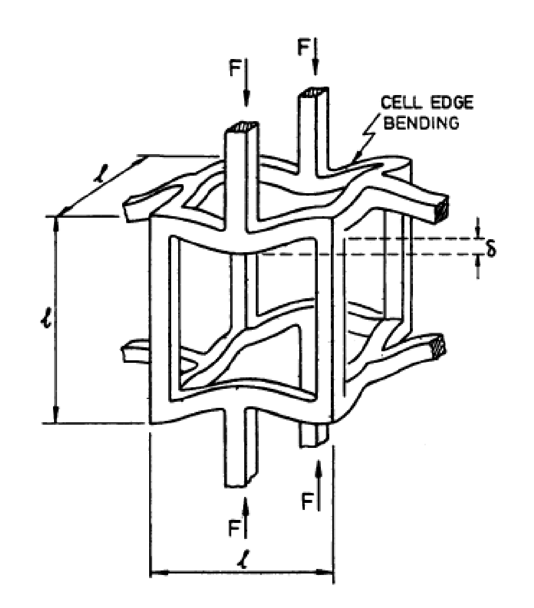
$$\sigma_{el}^* \propto \frac{F_{crit}}{l^2}$$

$$\propto \frac{E_s I}{l^4}$$

- For foams

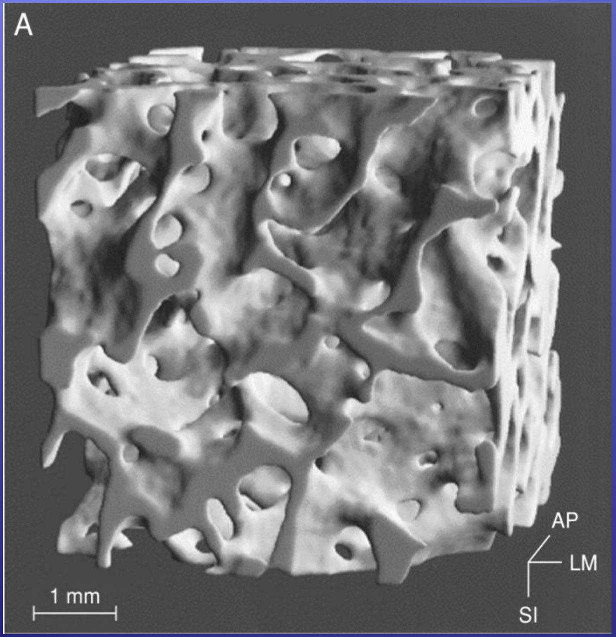
$$\frac{\sigma_{el}^*}{E_s} = C_4 \left(\frac{\rho^*}{\rho_s}\right)^2$$

  - C_4 =0.05 for open cell foams



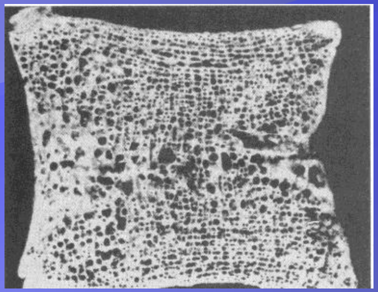
Bone

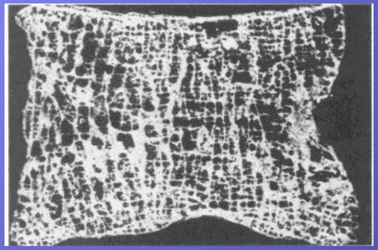
- Structure
  - Cortical Bone – laminate outside
  - Trabecular Bone – foam inside



Trabecular

- Elastic
- Collapse
- Densify





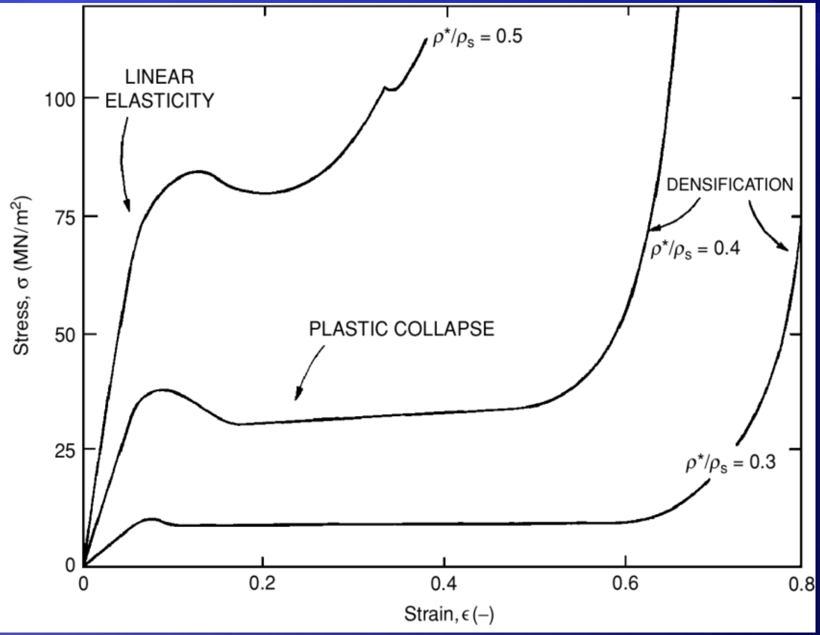


Table 9.4. Summary of mean compressive properties of human trabecular bone from different anatomic locations. Values in parentheses are standard deviations. Femur specimens were pooled from both the proximal and distal femur. The specimens from the tibia, distal femur, and spine were tested in the longitudinal (inferior-superior) direction. The proximal femur specimens were oriented along the neck of the femur. Adapted from Keaveny [11]. Copyright 2001 from *Bone Mechanics Handbook* by Cowin. Reproduced by permission of Routledge/Taylor & Francis Group, LLC.

Anatomic site	Relative density	Modulus (MPa)	Ultimate stress (MPa)	Ultimate strain (%)
Proximal tibia	0.16 (0.056)	445 (257)	5.33 (2.93)	2.02 (0.43)
Femur	0.28 (0.089)	389 (270)	7.36 (4.00)	Not reported
Lumbar spine	0.094 (0.022)	291 (113)	2.23 (0.95)	1.45 (0.33)

Actin Foams

- ρ_s = 700-800 mg/ml
- ρ^* = 10-20 mg/ml

$$\left(\frac{\rho^*}{\rho_s}\right) = 1 - 3\%$$

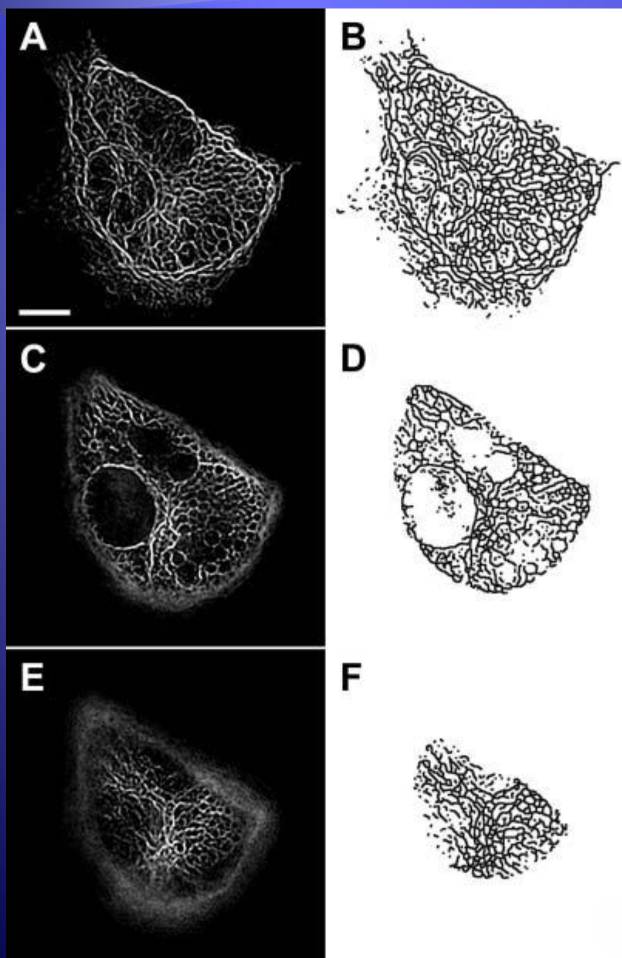
- E_s = 1.4 GPa

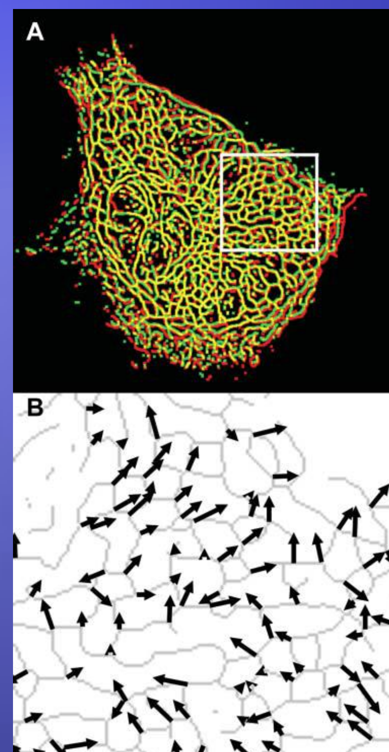
$$E^* = 20 - 40 \text{ MPa}$$
$$G^* = 20 - 40 \text{ MPa}$$

- That's a bit stiff!

Intermediate filament structure

- Shear flow induces IF strain in network





GFP-Vimentin
Before shear
3 min shear (L-R)

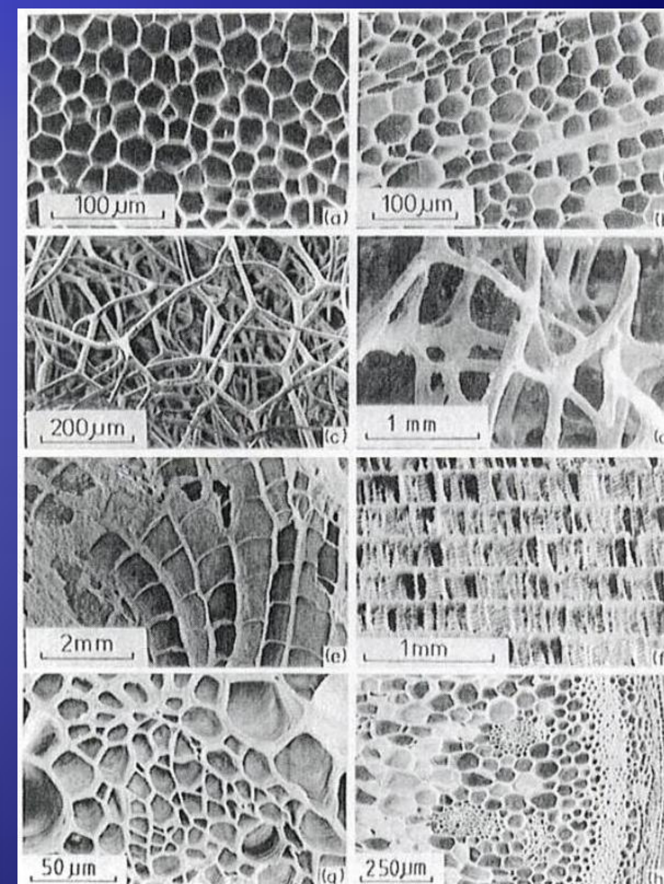


Figure 2.6 Natural cellular materials: (a) cork, (b) balsa, (c) sponge, (d) cancellous bone, (e) coral, (f) cuttlefish bone, (g) iris leaf, (h) stalk of a plant.

Helmke, et al., (2003) Biophys J, 84(4):2691

Additional Improvements?

- Make the model match the data
- Get better filament measurements (IF, maybe actin)
- Scalable issues?
- Node mechanics
- Buckling effects
- Over discretization by including actin binding proteins or other protein interactions