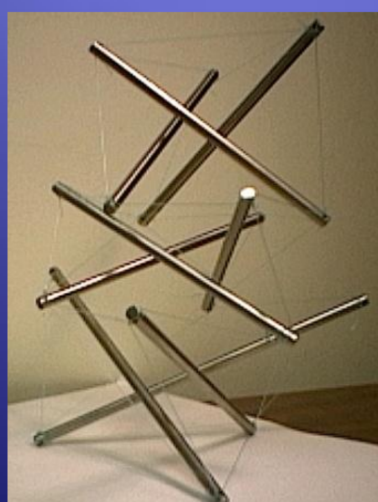
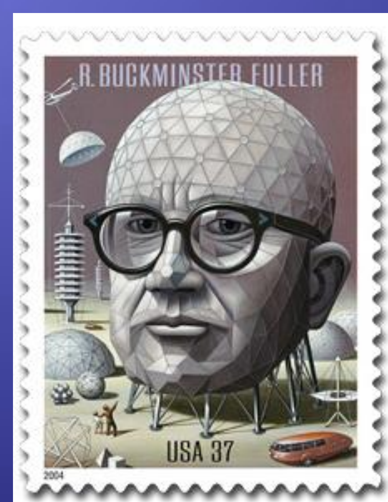
Session 14

TENSEGRITY

Tensegrity (*tens*ional inte*grity*)

- Stability of a structure based on a balanced synergy of continuous tension and discrete compression components
- Richard Buckminster Fuller coined tensegrity as "islands of compression inside an ocean of tension"





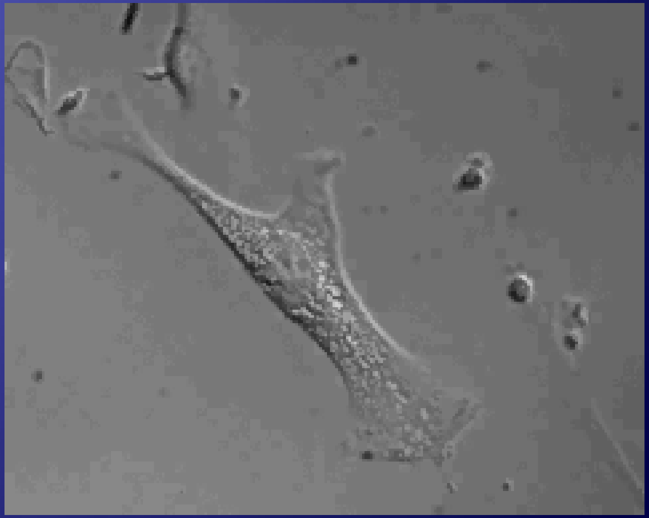
Tensegrity for cells

- Common cellular interpretation
  - Actin microfilaments – tension elements
  - Microtubules – compression elements

- Additional components
  - ECM – compression element
  - Intermediate filaments – tension elements
  - Cross-linked actin bundles – compression elements

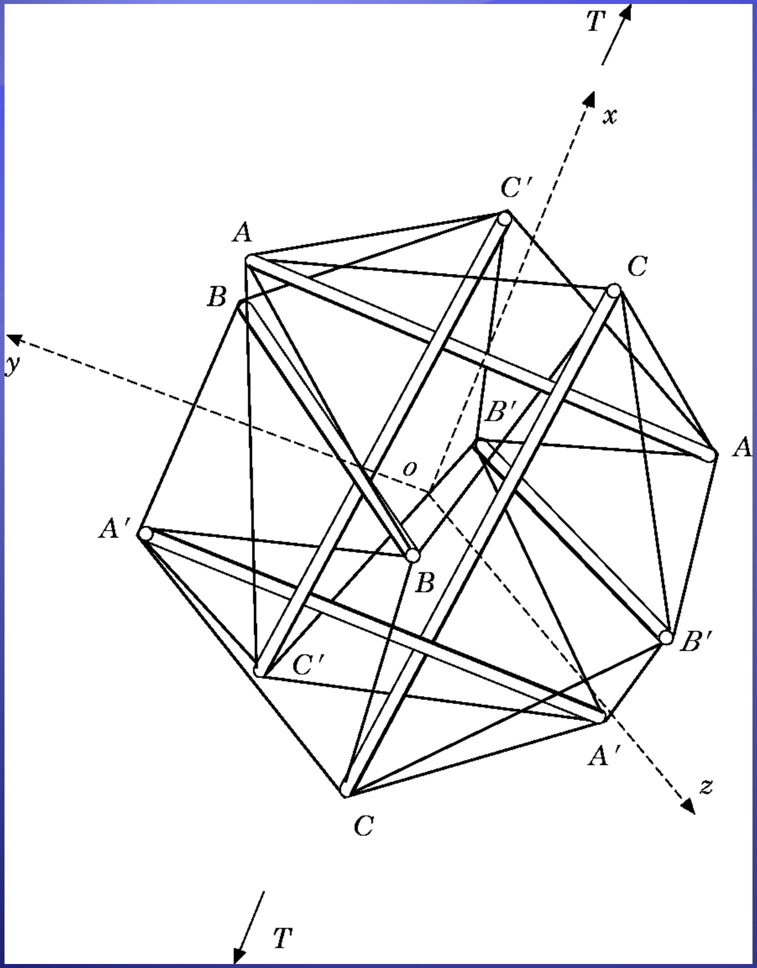
Prestress

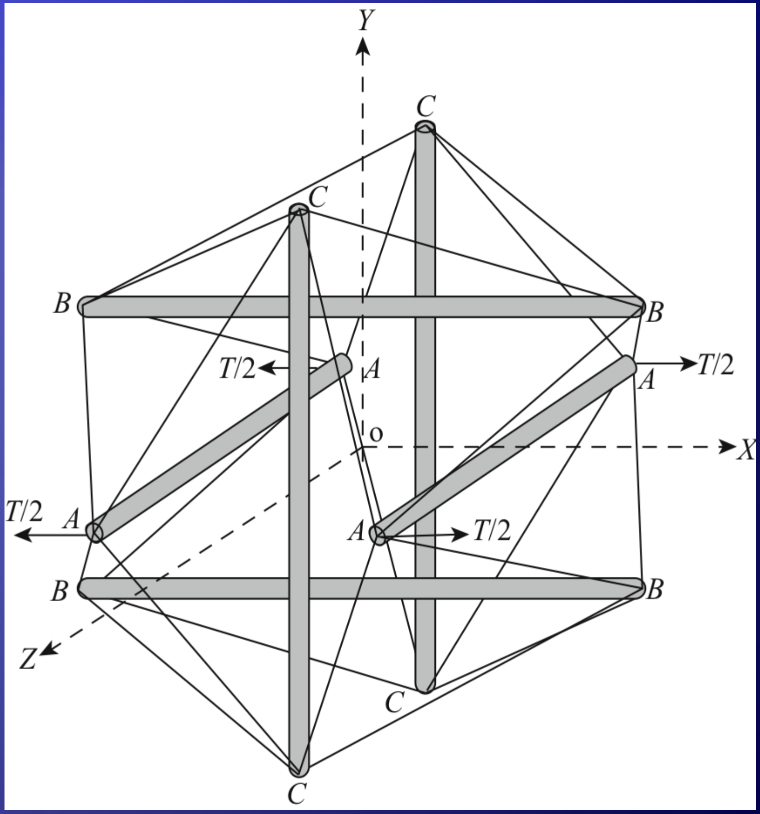
- Central to tensegrity is the amount of pre-loaded tension in the structure (prestress).
- Dictates the stiffness and mechanical response
- Cells have prestress that is balanced by ECM attachments
- Detach the cell from culture dish and it rounds up
- Akin to cortical tension in lipid drop models



Trypsin added to detach cell

The Model

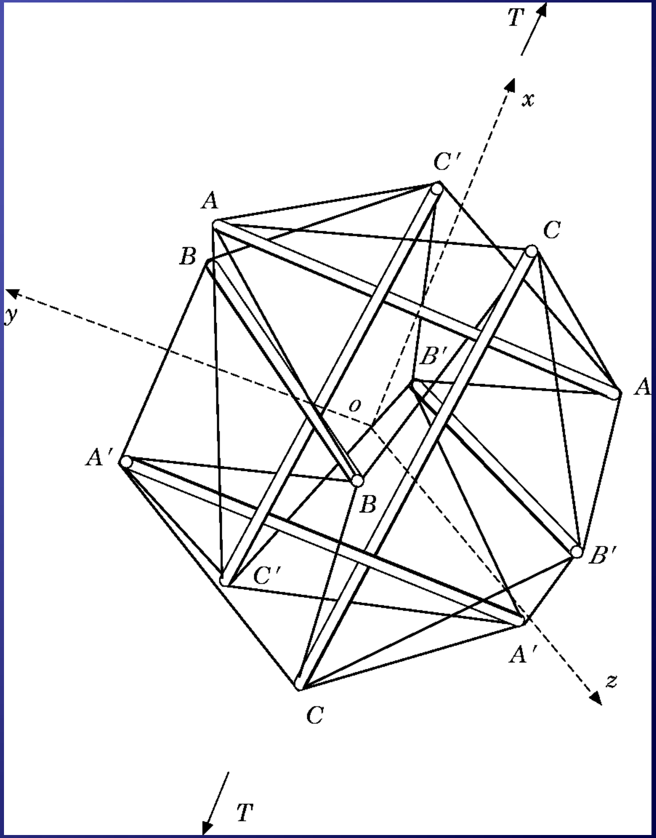




Here, B-B′ → C-C and C-C′ → B-B

Dimensions

- Element dimensions
  - $s_x \; s_y \; s_z$ strut-to-strut separation
  - $l_1 \; l_2 \; l_3$ tension lengths
  - $L_A \; L_B \; L_C$ strut lengths (=1)
- Recall vector notation:
  - Vector $\vec{l}_i = x\hat{i} + y\hat{j} + z\hat{k}$
  - Distance $l_i = \left|\vec{l}\right| = \sqrt{x^2 + y^2 + z^2}$
  - Direction $\hat{l}_i = \vec{l}_i / l_i$



Tension Element Length

- Based on model geometry

$$l_i = \sqrt{x^2 + y^2 + z^2}$$

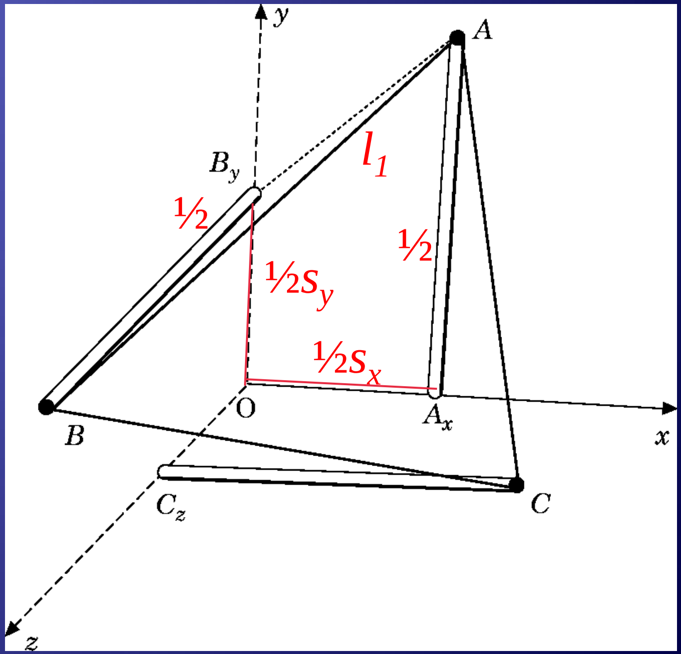
$$l_1 = \sqrt{\left(\tfrac{1}{2} s_x\right)^2 + \left(\tfrac{1}{2} - \tfrac{1}{2} s_y\right)^2 + \left(\tfrac{1}{2}\right)^2}$$

$$l_1 = \frac{1}{2}\sqrt{s_x{}^2 + s_y{}^2 - 2s_y + 2}$$

- Similarly

$$l_2 = \frac{1}{2}\sqrt{s_y{}^2 + s_z{}^2 - 2s_z + 2}$$

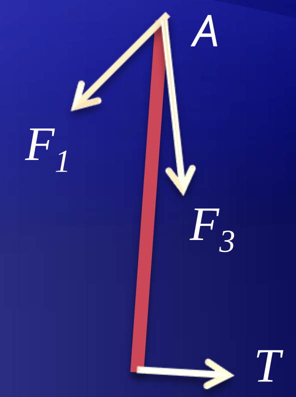
$$l_3 = \frac{1}{2}\sqrt{s_z{}^2 + s_x{}^2 - 2s_x + 2}$$



Equilibrium Equations

- Balance of forces on A-A

$$\vec{F}_i = F_i \hat{l}_i$$



$$\sum F_{x,AA} = 0$$

$$T + 4\vec{F}_3 \cdot \hat{i} + 4\vec{F}_1 \cdot \hat{i} = 0$$

$$T + 4F_3\left(\hat{l}_3 \cdot \hat{i}\right) + 4F_1\left(\hat{l}_1 \cdot \hat{i}\right) = 0$$

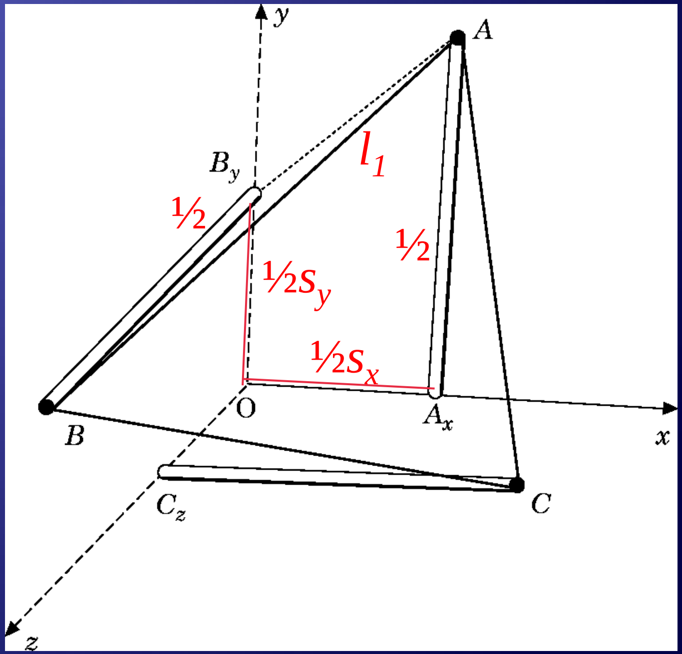
$$T + 4F_3\left(\frac{\frac{1}{2} - \frac{1}{2}s_x}{l_3}\right) - 4F_1\left(\frac{\frac{1}{2}s_x}{l_1}\right) = 0$$

By symmetry, 2 of 8 tension forces shown

- Similarly

$$\left(\sum F_{y,BB} = 0\right) \quad \Rightarrow \frac{F_1}{l_1}\left(1 - s_y\right) = \frac{F_2}{l_2}s_y$$

$$\left(\sum F_{z,CC} = 0\right) \quad \Rightarrow \frac{F_2}{l_2}\left(1 - s_z\right) = \frac{F_3}{l_3}s_z$$



Constitutive Relationships

- Struts are slender with zero transverse loading
- Cables are linear elastic springs

$$F = k\left(l_i - l_R\right)$$

where k is spring stiffness and l_R is unstressed cable length

- Define prestress (strain)

$$\xi = \frac{l_0 - l_R}{l_0}$$

where l_0 is uniform initial length of each cable for an initially isotropic structure

Approach

- At rest, structure is isotropic, therefore

$$L_A = L_B = L_C = 1$$

$$s_x = s_y = s_z = 1/2$$

$$l_0 = l_1 = l_2 = l_3 = \sqrt{0.375}$$

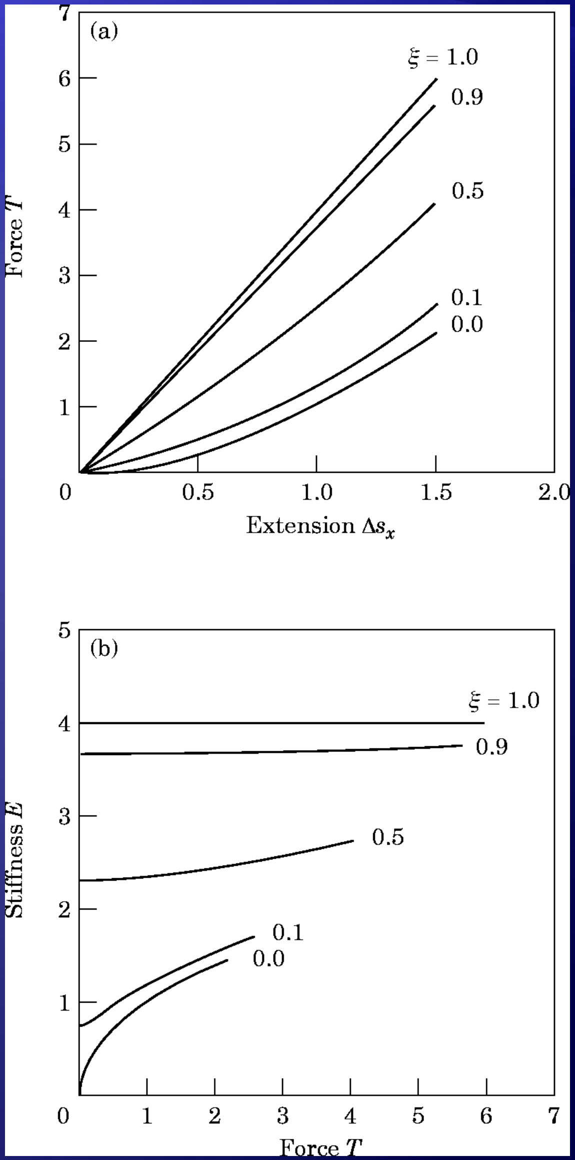
- For given applied force T and prestress (ξ , k, l_R)
  - Have 6 unknowns ($\Delta s_x \ \Delta s_y \ \Delta s_z \ \Delta l_1 \ \Delta l_2 \ \Delta l_3$)
  - Have 3 geometric length equations and 3 force balance equations
  - Can solve…

Stiffness

- Structural stiffness

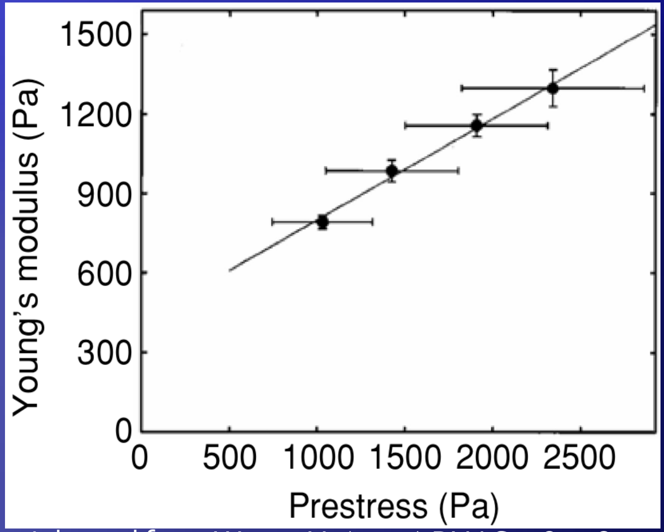
$$E \equiv \frac{T}{\Delta s_x}$$

- High prestress (ξ), stiffness is constant
  - Tensegrity structure is Hookean
- Low prestress, stiffness increases with applied force
  - Strain hardening seen by Petersen (1982) and Radmacher (1996)
- Stiffness increases with prestress

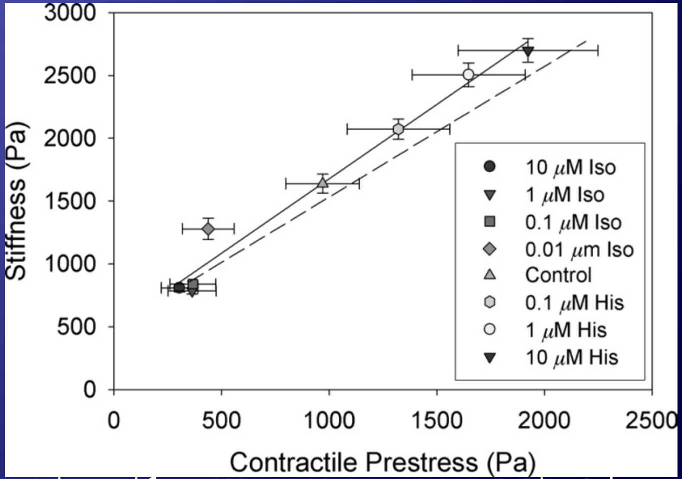


Controversy surrounds tensegrity

- Prestress:
  - Traction forces for cells on gels embedded with fluorescent microbeads
- Young's Modulus
  - Cell stiffness measured with magnetic twisting cytometry
- Tensegrity predict linear relationship between prestress and effective stiffness
- At top, slope is 0.4 but tensegrity predicts slope to be 1
- At bottom, dashed-line slope is 1.04.
  - Accounting for 14% loading in MTs not measurable by traction forces



Adapted from Wang, N. (2001) PNAS, 98:7765



Adapted from Wang, N. (2002) Am. J. Physiol. Cell Physiol, 282:C606

Criticisms of Tensegrity

- Class Discussion
  - Mechanotransduction shows that cells adapt to stress
  - Tension only, where is compression? Not only MT, could be ECM.
  - Cells are fluid filled, how does that relate to tensegrity?
  - Dampening consideration
  - What about AFM? Discrete points have high stiffness at struts, low stiffness at springs
  - FEM is more powerful to model cell mechanics
  - But disregards the mechanical nature of each protein (filaments)

Action at a distance

- Tensegrity: local perturbation results in global rearrangement
  - Local load should cause distal disturbance
- Continuum models show local response
  - Dissipate inversely with distance from point of load
- Examples
  - Mantiotis (1997) pulling on ECM coated bead causes nuclear distortion
  - Hu (2003) used MTC and MitoTracker and observed discrete displacements at distal pts
  - Both observations were actin-CSK dependent