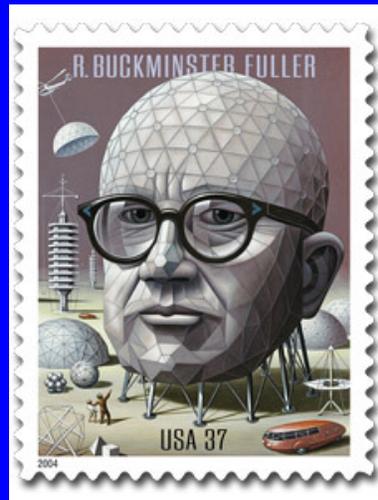
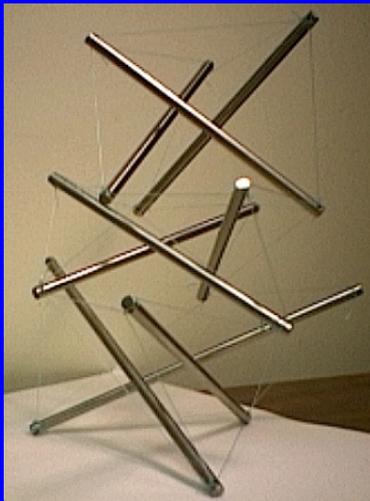


Session 14

TENSEGRITY

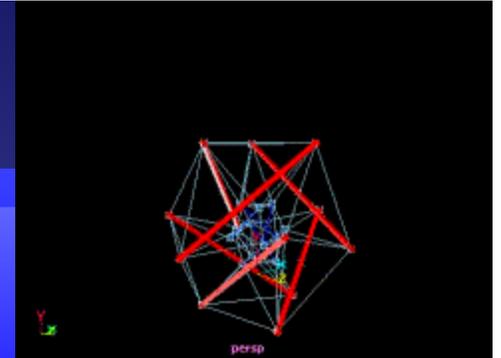
Tensegrity (*tensional integrity*)

- ◆ Stability of a structure based on a balanced synergy of continuous tension and discrete compression components
- ◆ Richard Buckminster Fuller coined tensegrity as “islands of compression inside an ocean of tension”



Tensegrity for cells

- ◆ Common cellular interpretation
 - ◆ Actin microfilaments – tension elements
 - ◆ Microtubules – compression elements
- ◆ Additional components
 - ◆ ECM – compression element
 - ◆ Intermediate filaments – tension elements
 - ◆ Cross-linked actin bundles – compression elements



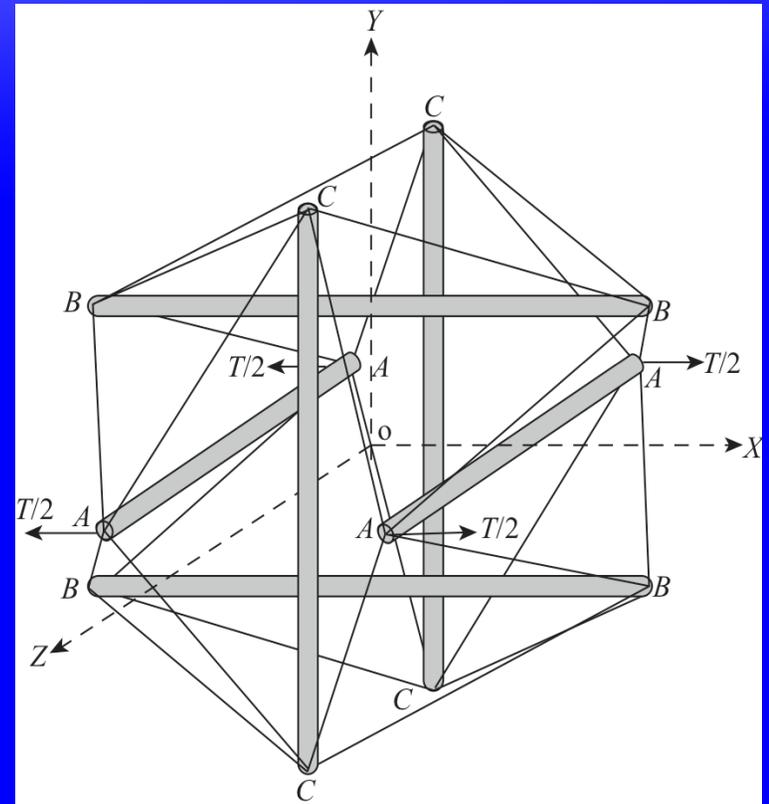
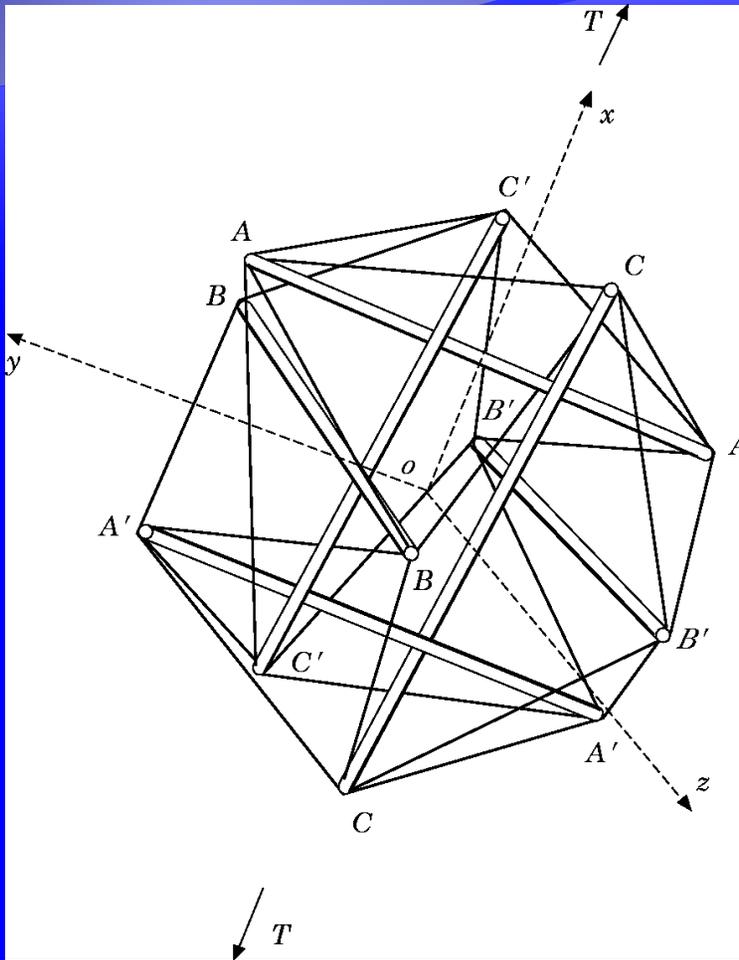
Prestress

- ◆ Central to tensegrity is the amount of pre-loaded tension in the structure (prestress).
- ◆ Dictates the stiffness and mechanical response
- ◆ Cells have prestress that is balanced by ECM attachments
- ◆ Detach the cell from culture dish and it rounds up
- ◆ Akin to cortical tension in lipid drop models



Trypsin added to detach cell

The Model



Here, $B-B' \rightarrow C-C$ and $C-C' \rightarrow B-B$

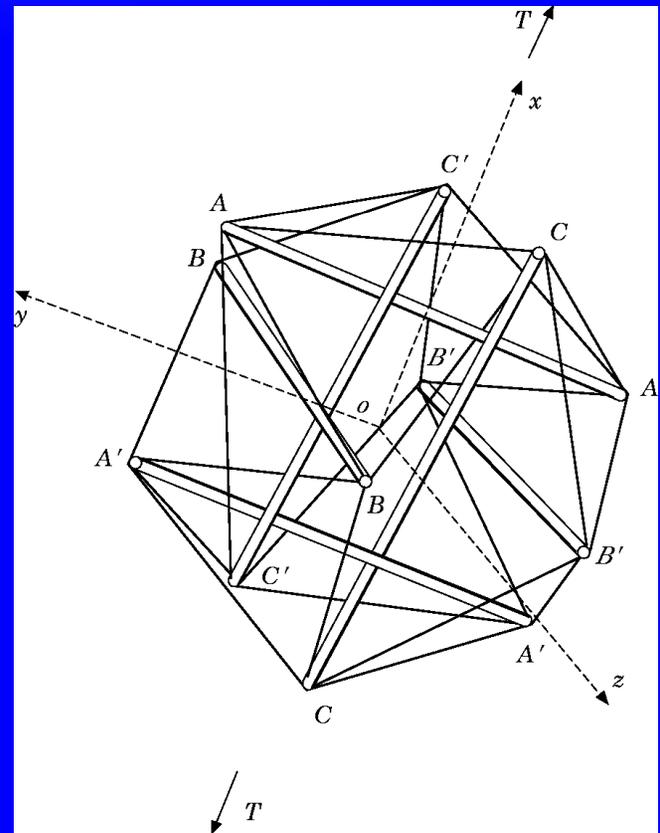
Dimensions

- ◆ Element dimensions

- ◆ s_x s_y s_z strut-to-strut separation
- ◆ l_1 l_2 l_3 tension lengths
- ◆ L_A L_B L_C strut lengths (=1)

- ◆ Recall vector notation:

- ◆ Vector $\vec{l}_i = x\hat{i} + y\hat{j} + z\hat{k}$
- ◆ Distance $l_i = |\vec{l}_i| = \sqrt{x^2 + y^2 + z^2}$
- ◆ Direction $\hat{l}_i = \vec{l}_i / l_i$



Equilibrium Equations

- ◆ Balance of forces on A-A

$$\sum F_{x,AA} = 0$$

$$T + 4\vec{F}_3 \cdot \hat{i} + 4\vec{F}_1 \cdot \hat{i} = 0$$

$$T + 4F_3 (\hat{l}_3 \cdot \hat{i}) + 4F_1 (\hat{l}_1 \cdot \hat{i}) = 0$$

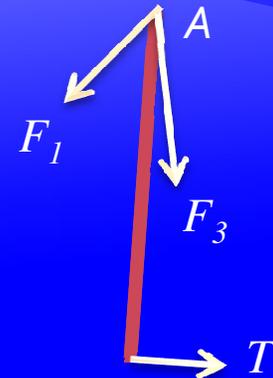
$$T + 4F_3 \left(\frac{\frac{1}{2} - \frac{1}{2}s_x}{l_3} \right) - 4F_1 \left(\frac{\frac{1}{2}s_x}{l_1} \right) = 0$$

- ◆ Similarly

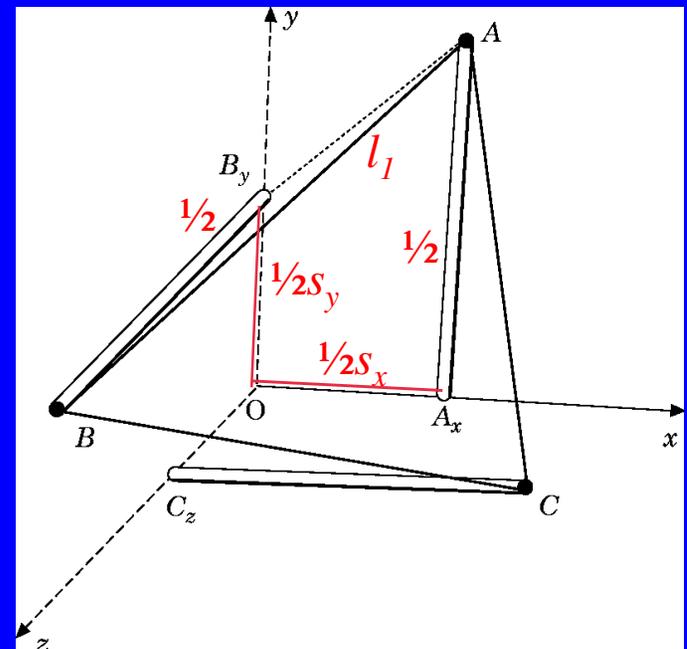
$$\left(\sum F_{y,BB} = 0 \right) \Rightarrow \frac{F_1}{l_1} (1 - s_y) = \frac{F_2}{l_2} s_y$$

$$\left(\sum F_{z,CC} = 0 \right) \Rightarrow \frac{F_2}{l_2} (1 - s_z) = \frac{F_3}{l_3} s_z$$

$$\vec{F}_i = F_i \hat{l}_i$$



By symmetry,
2 of 8 tension
forces shown



Constitutive Relationships

- ◆ Struts are slender with zero transverse loading
- ◆ Cables are linear elastic springs

$$F = k(l_i - l_R)$$

where k is spring stiffness and l_R is unstressed cable length

- ◆ Define prestress (strain)

$$\xi = \frac{l_0 - l_R}{l_0}$$

where l_0 is uniform initial length of each cable for an initially isotropic structure

Approach

- ◆ At rest, structure is isotropic, therefore

$$L_A = L_B = L_C = 1$$

$$s_x = s_y = s_z = 1/2$$

$$l_0 = l_1 = l_2 = l_3 = \sqrt{0.375}$$

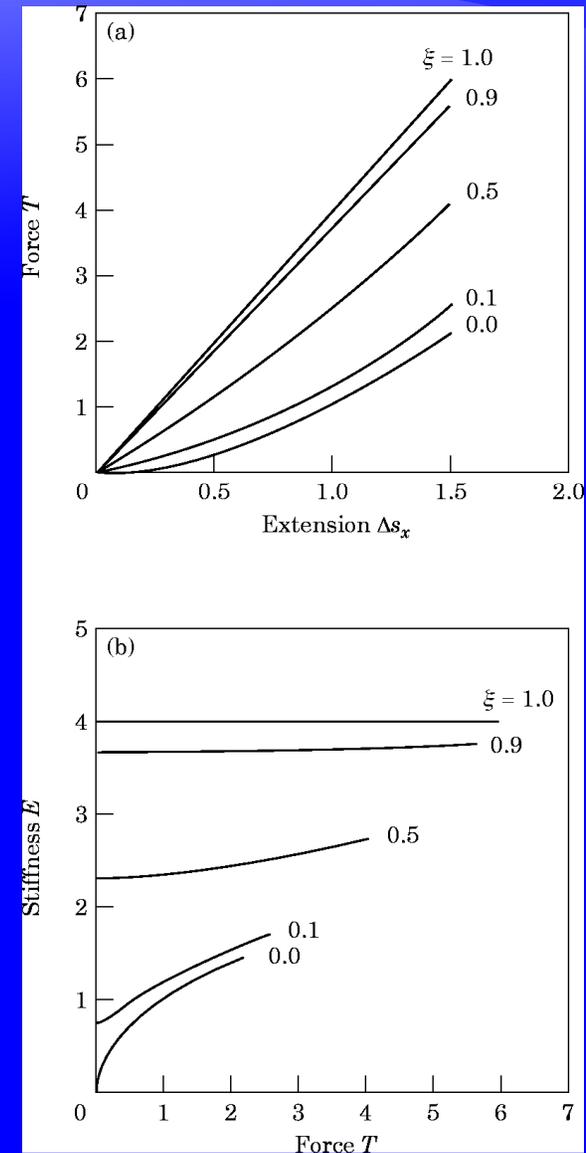
- ◆ For given applied force T and prestress (ξ, k, l_R)
 - ◆ Have 6 unknowns $(\Delta s_x, \Delta s_y, \Delta s_z, \Delta l_1, \Delta l_2, \Delta l_3)$
 - ◆ Have 3 geometric length equations and 3 force balance equations
 - ◆ Can solve...

Stiffness

- ◆ Structural stiffness

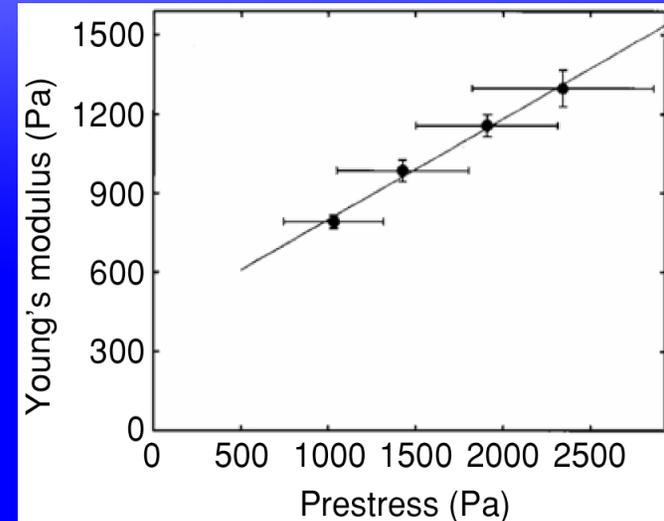
$$E \equiv \frac{T}{\Delta s_x}$$

- ◆ High prestress (ξ), stiffness is constant
 - ◆ Tensegrity structure is Hookean
- ◆ Low prestress, stiffness increases with applied force
 - ◆ Strain hardening seen by Petersen (1982) and Radmacher (1996)
- ◆ Stiffness increases with prestress

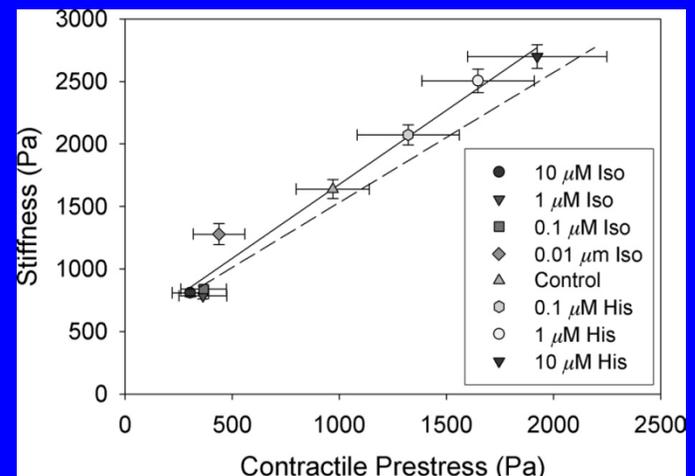


Controversy surrounds tensegrity

- ◆ Prestress:
 - ◆ Traction forces for cells on gels embedded with fluorescent microbeads
- ◆ Young's Modulus
 - ◆ Cell stiffness measured with magnetic twisting cytometry
- ◆ Tensegrity predict linear relationship between prestress and effective stiffness
- ◆ At top, slope is 0.4 but tensegrity predicts slope to be 1
- ◆ At bottom, dashed-line slope is 1.04.
 - ◆ Accounting for 14% loading in MTs not measurable by traction forces



Adapted from Wang, N. (2001) PNAS, 98:7765



Adapted from Wang, N. (2002) Am. J. Physiol. Cell Physiol, 282:C606

Criticisms of Tensegrity

- ◆ Class Discussion
 - ◆ Mechanotransduction shows that cells adapt to stress
 - ◆ Tension only, where is compression? Not only MT, could be ECM.
 - ◆ Cells are fluid filled, how does that relate to tensegrity?
 - ◆ Dampening consideration
 - ◆ What about AFM? Discrete points have high stiffness at struts, low stiffness at springs
 - ◆ FEM is more powerful to model cell mechanics
 - ◆ But disregards the mechanical nature of each protein (filaments)

Action at a distance

- ◆ Tensegrity: local perturbation results in global rearrangement
 - ◆ Local load should cause distal disturbance
- ◆ Continuum models show local response
 - ◆ Dissipate inversely with distance from point of load
- ◆ Examples
 - ◆ Mantiotis (1997) pulling on ECM coated bead causes nuclear distortion
 - ◆ Hu (2003) used MTC and MitoTracker and observed discrete displacements at distal pts
 - ◆ Both observations were actin-CSK dependent

