BIOLOGICAL FRAMEWORKS FOR ENGINEERS

Homework #6 (due 11/21/11)

In this assignment, we will be working with Hill's parameterized model for muscle.

When muscle is stimulated, it produces a short-lived rise in tension known as a twitch. The strength of the stimulation must be strong enough to depolarize the cell membrane of the muscle (sacrolemma) in order to trigger the release of Ca^{2+} from the sarcoplasmic reticulum. Increasing the stimulation strength will recruit more muscle fibers to help in contraction and thereby increase the force generation. At a high level of stimulation, all muscle fibers become activated and a maximum twitch force is reached. Further increase in the strength of the stimulation beyond this level does not increase the tension in the twitch.

When two identical stimulations are separated by a suitable period of time, they produce identical force responses. However, when the second stimulation starts before the first is over, a larger peak of tension results. It can be conceptualized that as the tension from the first begins to decay, it is superimposed with the second twitch response and results in a larger combined tension. The effect becomes more pronounced as the timing between stimulations is brought closer together. When stimulations are given at 50-60 Hz in mammalian muscle, the force produced reaches a steady maximum, which is known as tetanic fusion or tetanus

It can be observed that muscle exerts its greatest tetanic force when the speed of shortening is zero (isometric condition). This observation led A.V. Hill to postulate that stimulation always brings about an instantaneous development of force in the contractile machinery. When muscle shortens, some of the force is dissipated in overcoming the inherent viscous resistance. However, if the muscle is not allowed to shorten, then the maximum force is conserved. He proposed representing muscle as a pure force generator in parallel with a nonlinear dashpot.

As we know now, muscle cannot be viewed in such simple terms as a force-generator and a dashpot. It is the biochemical reactions that control the rate of energy release and hence the mechanical properties. However, Hill's model is a useful approximation for the pure mechanics of skeletal muscle working against an applied load.

Take the linear three-element model below and answer the following questions:

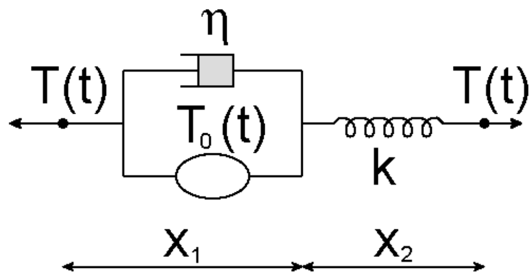

Figure 1. Three-element model of muscle. Contractile force is produced by the force-generator T_0. The dashpot resists length changes with a force F = η*dx_1/dt and the spring resists length changes with a force T = k*x_2. Muscle length is held under isometric conditions so $x_1 + x_2$ = constant but develops a tension T.

1) Derive the first order differential equation that describes the tension T from the system

2) Assume that there is no contractile force at the start and so we have no tension, i.e. initial condition is T(0) = 0. Immediately after the stimulus arrives (t > 0), the force-generator instantaneously increases to a maximum value ($T_0(t) = T_0$ for t>0). What is the function for developed tension over time?

3) Suppose the force-generator stays on for a period of time (t = 0.4 s) and then shuts off. What is the tension when the generator shuts off? Let η= 0.06 dynes*s/cm, k = 0.3 dyne/cm and T_0 = 4 dyne.

4) What is the function for the decay in tension for t > 0.4 s?

5) Suppose the force-generator is stimulated a second time at t = 0.6 s in order to create another twitch to combine with the first. What is the tension prior to the second stimulation? Use the same values for η, k, and T_0 as before.

6) What is the combined tension from the decaying first twitch and the new second twitch? Use superposition of the equations for force production and decay (questions 2 and 4), but be careful in matching up your time intervals between the two twitch functions.

7) Suppose the force-generator is shutoff a second time after 0.4 s of production. What is the tension at t = 1 s?