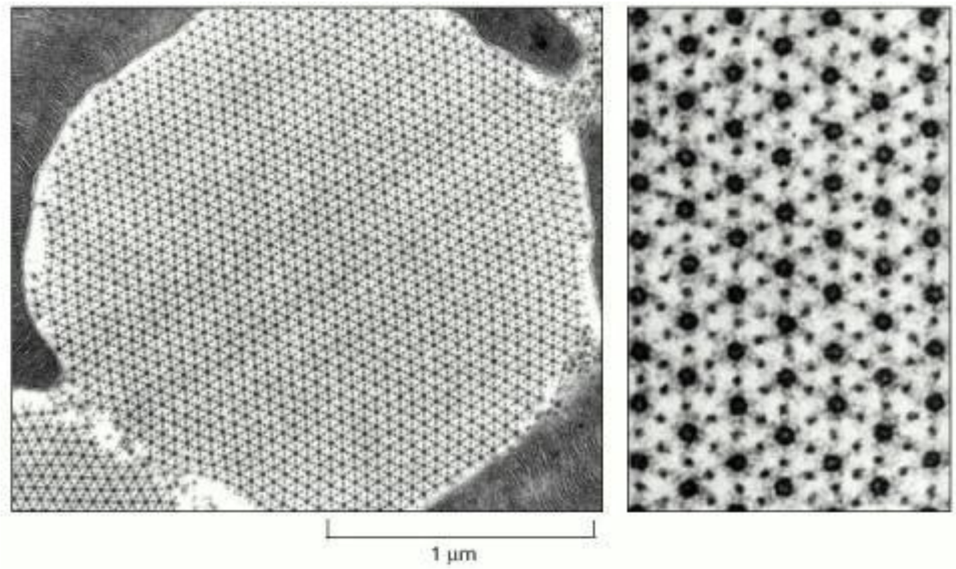
BIOLOGICAL FRAMEWORKS FOR ENGINEERS

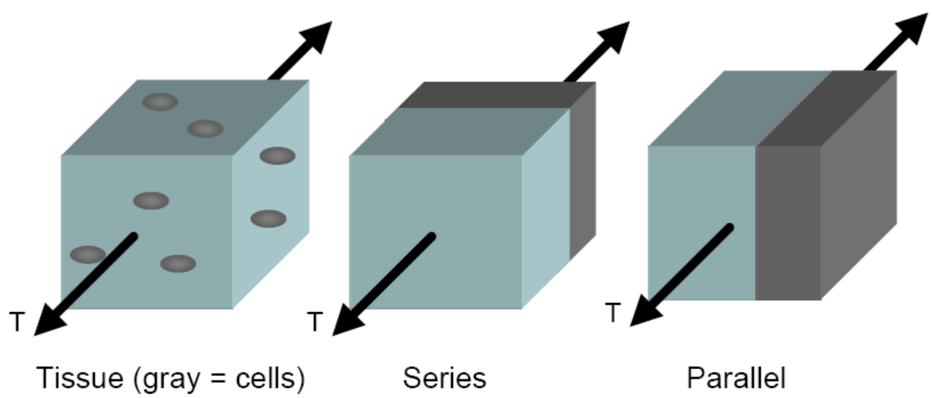
Homework #5 (due 11/16/11) [Cell & Tissue Mechanics]

This assignment aims to have you understand the mechanical properties of cells.

1. Shown in the figure below is a cross-sectional view through muscle, showing actin and myosin filaments. Assuming that muscle can generate a maximum force of 20 N/cm^2, determine the maximum force exerted by each myosin thick filament. Make and state appropriate assumptions.



2. A cartilage sample consists of cells (chondrocytes, with effective Young's modulus E_{cell}) and ECM (effective Young's modulus E_{ECM}) as shown in the figure below.



The volume fraction of the cells (cell volume/total tissue volume) is ϕ. The tissue is subject to a uniaxial tension, has unstretched length L, and cross sectional area A. We wish to determine the effective Young's modulus for the tissue sample E_{tissue}. In general, this is a complicated problem; however we can get bounds for E_{tissue} by considering two special cases. In the first case, we replace the real tissue configuration by a "series" configuration where a cell-containing volume is in series with an ECM-containing volume. In the second case, we replace the real configuration by a "parallel" arrangement. In both cases, we required the total tension T applied to the tissue to match that for the real tissue case, and the total elongation to also match that occurring in the real case.

(a) Considering the work done by the tension force T, show that the energy stored in the real tissue sample is $A*L*E_{tissue}*\varepsilon^2 / 2$.

(b) Consider the series configuration. You should first convince yourself that in this configuration, the stress σ is the same for both tissue components and is equal to the true stress in the tissue, but the strain ε is different for the two components. Thinking about the energy stored in each of the two tissue components, show that overall tissue modulus can be written as $1/E_{tissue} = \phi/E_{cell} + (1-\phi)/E_{ECM}$.

(c) Using a similar approach as for the series configuration, show that the overall tissue modulus is $E_{tissue} = \phi*E_{cell} + (1-\phi)*E_{ECM}$ for the parallel configuration.