DEPARTMENTAL

Types of Engineering Structures

One type of engineering structure is one which is composed of a few simple elements but subjected to a complex loading condition as shown in Fig. 11.10. In this figure the loading condition involves a torque and bending moment and possibly an internal pressure. The stresses due to these loading conditions can be calculated and appropriately superposed before performing the transformations to determine the principal stresses.

Another type of engineering structure is one which is composed of many similarly loaded elements subjected to either a relatively simple or slightly more complex loading condition. Trusses (see Fig. 11.11) are an example of one of the major types of engineering structures, providing practical and economical solutions to many engineering situations. Trusses consist of straight members connected at joints (for example, see Figure 1). Note that truss members are connected at their extremities only: thus no truss members are continuous through a joint.

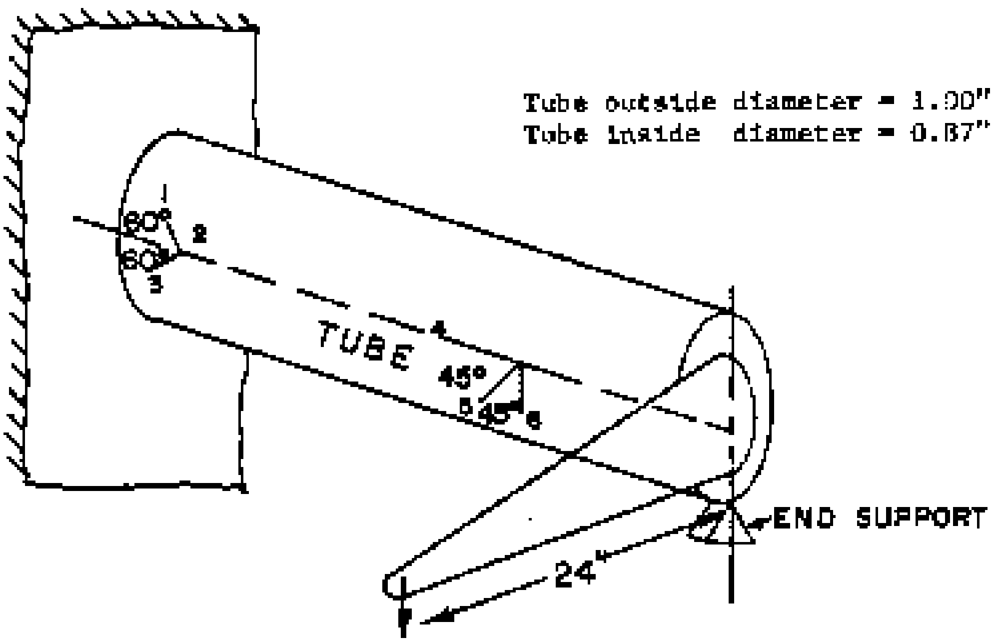


Figure 11.10 Relatively simple engineering component subjected to a complex loading condition

In general, truss members are slender and can support little lateral load. Therefore, major loads must be applied to the various joints and not the members themselves. Often the weights of truss members are assumed to be applied only at the joints (half the weight at each joint). In addition, even though the joints are actually rivets or welds, it is customary to assume that the truss members are pinned together (i.e., the force acting at the end of each truss member is a single force with no couple). Each truss member may then be treated as a two force member and the entire truss is treated as a group of pins and two-force members.

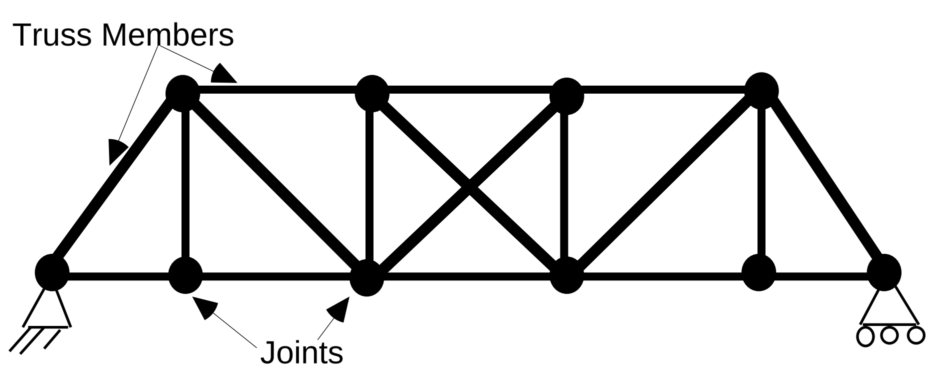


Figure 11.11 Example of a Simple Truss

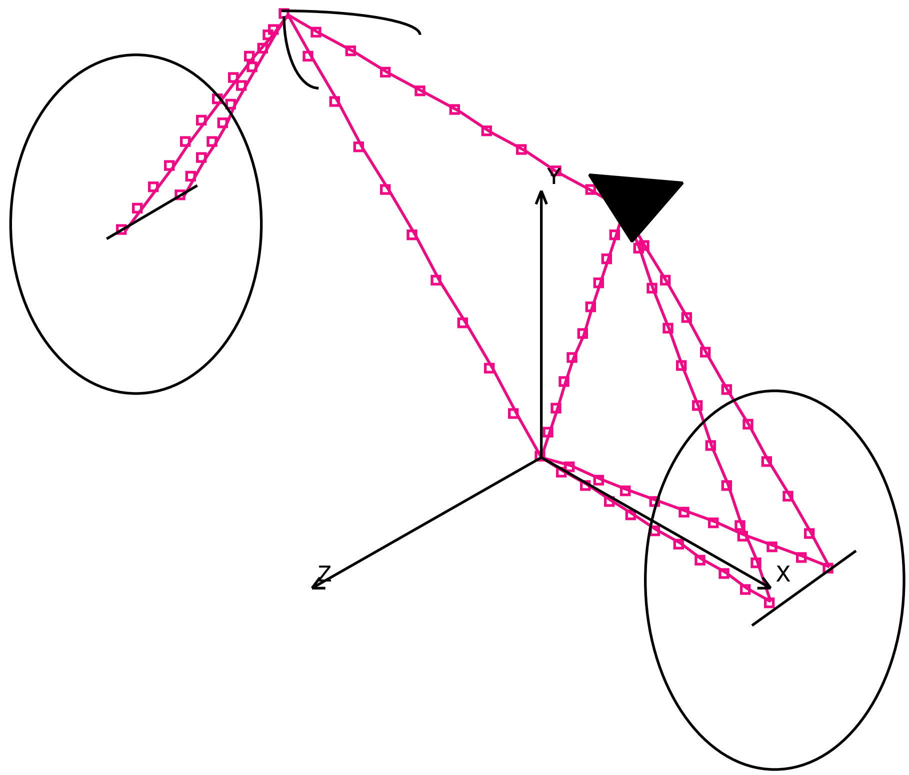


Figure 11.12 Illustration of a bicycle frame as a truss-like structure

A bicycle frame, on first inspection, appears to be an example of a truss (see Fig. 11.12) Each tube (truss member) is connected to the other at a joint, the principal loads are applied at joints (e.g., seat, steering head, and bottom bracket), and the reaction loads are carried at joints as well (e.g., front and rear axles). Although the joints are not pinned, a reasonable first approximation for analyzing forces, deflections, and stresses in the various tubes of the bicycle frame might be made using a simple truss analysis.

Forces in various truss members can be found using such analysis techniques as the method of joints or the method of sections. Deflections at any given joint may be found by using such analysis techniques as the unit load method of virtual work.

An example of the use of the method of joint to solve for the axial loads in each truss member is as follows. For the simple truss shown in Figure 11.13 the first step is to calculate the reactions at joints C and D. In this case, $\sum F = 0$ and $\sum M = 0$ such that

$$\sum M_C = 0 = PL - R_D L \qquad R_D = P \downarrow \tag{11.15}$$

and

$$\begin{aligned} \sum F = 0 \qquad & \sum F_x = 0 = -P + R_{xC} \qquad R_{xC} = P \rightarrow \\ & \sum F_y = 0 = -2P - P + R_{yC} \qquad R_{yC} = 3P \uparrow \end{aligned} \tag{11.16}$$

The resulting free body diagram is shown in Fig. 11.14

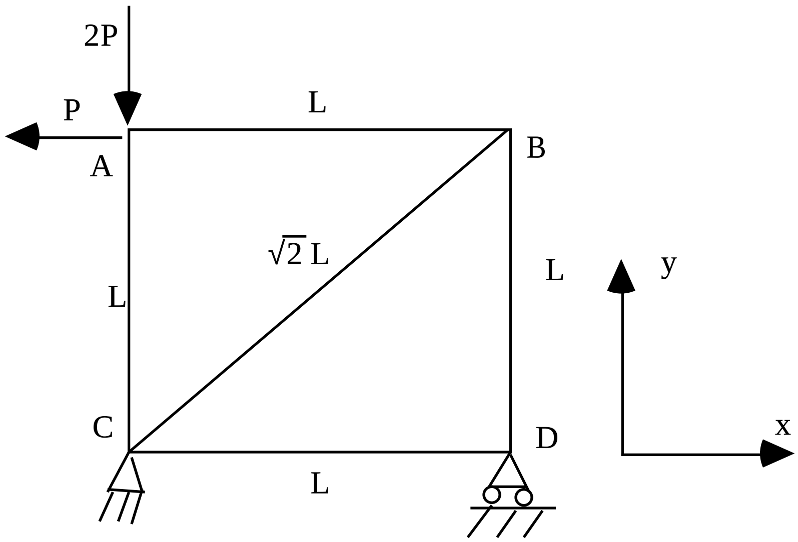
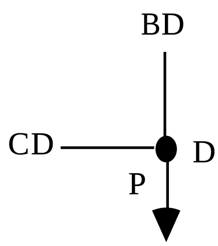


Figure 11.13 Example of a simple truss

Using the method of joints, $\sum F = 0$ at joint D such that



$$\sum F = 0 \qquad \begin{aligned} \sum F_x &= 0 = -F_{CD} \qquad F_{CD} = 0 \rightarrow \\ \sum F_y &= 0 = P + F_{BD} \qquad F_{BD} = P \uparrow \end{aligned} \tag{11.17}$$

and since F_{BD} pulls on the joint, then the joint must pull back on the member so member BD is in tension.

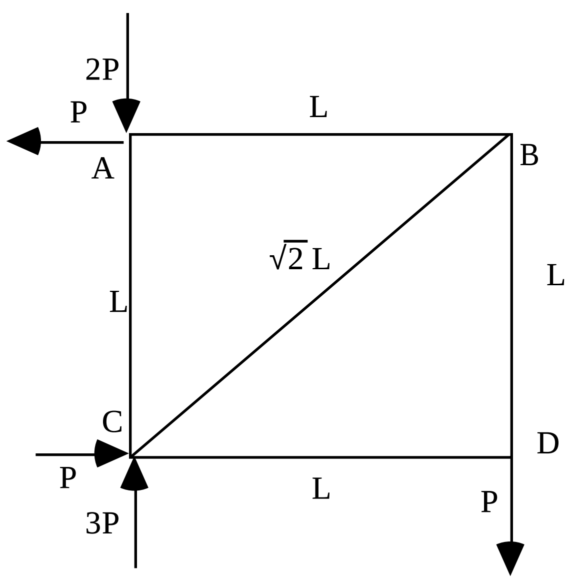
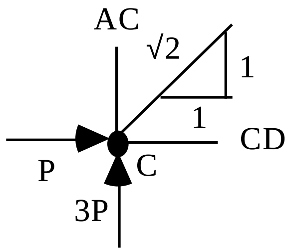


Figure 11.13 Free body diagram for simple truss

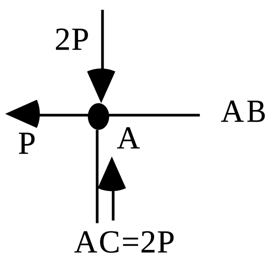
Using the method of joints, $\sum F$=0 at joint C such that



$$\sum F = 0 \qquad \sum F_x = 0 = P - \frac{1}{\sqrt{2}} F_{BC} \qquad F_{BC} = \sqrt{2}P \leftarrow$$
$$\sum F_y = 0 = 3P - \frac{1}{\sqrt{2}} F_{BC} - F_{AC} \qquad F_{AC} = P \downarrow \tag{11.18}$$

Since F_{AC} pushes on the joint, then the joint must push back on the member so member AC is in compression. Furthermore, since F_{CB} pushes on the joint, then the joint must push back on the member so member CB is in compression.

Finally, using the method of joints, $\sum F$=0 at joint A such that



$$\sum F = 0 \qquad \sum F_x = 0 = -P + F_{AB} \qquad F_{AB} = P \rightarrow$$
$$\sum F_y = 0 = -2P + 2P \qquad checks \tag{11.19}$$

Since F_{AB} pulls on the joint, then the joint must pull back on the member so member AB is in tension.

The summary of the member forces is shown in Table 11.1.

Although finding deflections in complex structures is more involved than finding deflections in simple components, it is not difficult. A useful technique is the unit load method in which the displacements can be found from simple deflection equations at joints which do not have forces acting on them. The unit load method works for linearly elastic materials and superposition applies.

Table 11.1 Summary of Truss Member Forces

Member	Force
AB	P (tension)
AC	2P (compression)
BC	√2P (compression)
BD	P (tension)
CD	0

For axially loaded members, the displacement is:

$$\Delta_N = \int \frac{N_U N_L}{EA} dx \tag{11.20}$$

where N_U is the axial force in the member due to a unit load applied at the point and direction of interest, N_L is the actual force in the member due to the actual applied load on the structure, E and A are the elastic modulus and cross sectional area of the individual member. The integral sign signifies that the calculated quantities for each member are summed via integration to give the final total deflection at the point and direction of interest.

For members subjected to bending moments, the displacement is:

$$\Delta_M = \int \frac{M_U M_L}{EI} dx \tag{11.21}$$

where M_U is the bending moment in the member due to a unit load applied at the point and direction of interest, M_L is the actual bending moment in the member due to the actual applied load on the structure, E and I are the elastic modulus and cross sectional moment of inertia of the individual member. The integral sign signifies that the calculated quantities for each member are summed via integration to give the final total deflection at the point and direction of interest.

For members subjected to torsion, the displacement is:

$$\Delta_T = \int \frac{T_U T_L}{GJ} dx \tag{11.22}$$

where T_U is the torque in the member due to a unit load applied at the point and direction of interest, T_L is the actual torque in the member due to the actual applied load on the structure, G and J are the shear modulus and polar moment of inertia of the individual member. The integral sign signifies that the calculated quantities for each member are summed via integration to give the final total deflection at the point and direction of interest.

For members subjected to transverse shear, the displacement is:

$$\Delta_v = \int \frac{V_U V_L}{GA} dx \tag{11.23}$$

where V_U is the transverse shear in the member due to a unit load applied at the point and direction of interest, V_L is the actual transverse shear in the member due to the actual applied load on the structure, G and A are the shear modulus and cross sectional area of the individual member. The integral sign signifies that the calculated quantities for each member are summed via integration to give the final total deflection at the point and direction of interest.

The total deflection due to each of these contributions can then be found by adding the individual contribution such that

$$\Delta_t = \int \frac{N_U N_L}{EA} dx + \int \frac{M_U M_L}{EI} dx + \int \frac{V_U V_L}{GA} dx + \int \frac{T_U T_L}{GJ} dx \qquad (11.24)$$

An example of the unit load method applied to the simple truss example is shown in Fig. 11.14 in which only the axial loading contributions are required since truss members are pinned and no bending moments, transverse shear, or torque can be carried in the members.

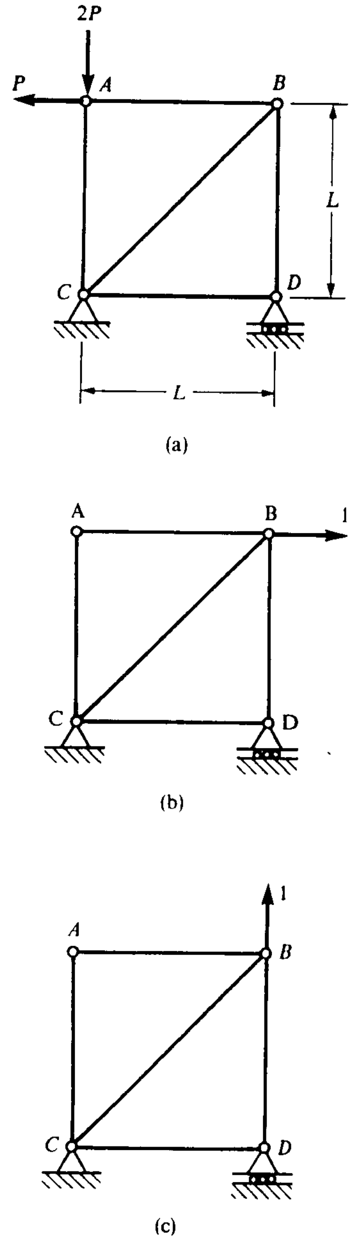


Fig. 10-4 Example 1. Displacements of a truss by the unit-load method

Example 1

The truss pictured in Fig. 10-4a is subjected to loads P and $2P$ at joint A. All members of the truss are assumed to be prismatic and to have the same axial rigidity EA. Calculate the horizontal and vertical displacements of joint B of the truss using the unit-load method.

Because the loads act only at the joints, the axial force in each member is constant throughout the length of the member. Therefore, we can use Eq. (10-5) to determine the desired deflections. It is helpful to record the calculations in a systematic manner, as shown in Table 10-2. The first two columns in the table identify the members of the truss and their lengths. The axial forces N_L, obtained by static-equilibrium analysis of the truss shown in Fig. 10-4a, are listed in column 3 of the table (tensile forces are positive).

Table 10-2 Calculations for Example 1

(1) Member	(2) Length	(3) N_L	(4) N_U	(5) $N_U N_L L$	(6) N_U	(7) $N_U N_L L$
AB	L	P	0	0	0	0
AC	L	$-2P$	0	0	0	0
BD	L	P	-1	$-PL$	1	PL
CD	L	0	0	0	0	0
CB	$\sqrt{2}L$	$-\sqrt{2}P$	$\sqrt{2}$	$-2.828PL$	0	0
				$-3.828PL$		PL

In order to find the horizontal displacement δ_h of joint B, we introduce a horizontal unit load on the structure at B (see Fig. 10-4b). The axial forces N_U produced by this unit load are given in column 4 of the table; again, tensile forces are positive. Next, the products $N_U N_L L$ are calculated for each member and summed (column 5). Dividing this result by EA gives the desired displacement (see Eq. 10-5):

$$\delta_h = -3.828\frac{PL}{EA}$$

The negative sign in this expression means that the displacement is in the direction opposite to the direction of the unit load; that is, toward the left.

The same general procedure is used to find the vertical displacement δ_v of joint B. The corresponding unit load (taken as positive when upward) is portrayed in Fig. 10-4c, and the axial forces N_U for this loading condition are listed in column 6 of the table. In the last column, the products $N_U N_L L$ are calculated and summed. Finally, dividing the sum by EA yields

$$\delta_v = \frac{PL}{EA}$$

Because this result is positive, we know that the vertical displacement of joint B produced by the loads P and $2P$ is upward.

Figure 11.14 Example of application of unit load method to find a deflection in a simple truss