Stress and Strains in a Three-Point Loaded Beam

By

___________________________________ (Name)

Date of this Lab Exercise: ___________________________

Date of Lab Report Submission: ___________________________

Executive Summary:

The executive summary provides the essential information of the report. You should provide a brief description of what main experiments were conducted for the report. The objectives, procedures, results and conclusions of the laboratory exercise are to be briefly summarized. Total length should be 300-500 words and on its own separate, single page. Final quantitative results (e.g., % error in strain, modulus of elasticity for each material) should be provided to add quantitative detail and credibility to the conclusions.

This section is the only part of a report that very busy, higher-level managers will read so you need to carefully capture the essence of the work, results, and key recommendations. Avoid technical jargon.

A. Introduction

Two candidate beams have been selected for use in an aircraft landing gear. Scaled-down prototypes of the beams with strain-gages were subjected to applied forces in a three-point bending configuration. The beams had similar lengths and moment of inertias, but different cross-sectional designs. Uniaxial, biaxial, and rosette strain gauges were placed at various locations and orientations along the beams. A deflection gauge was used to measure the displacement at the point of loading. Constitutive relations were used to calculate the predicted stresses and strains and compared to the strain measurements. The goal is to determine which beam design will have the most mechanically stability. The working hypothesis is that the same loading condition for each beam will produce different stress states for each beam design. Specific learning objectives include a) familiarizing the user with strain gages and associated instrumentation, b) measuring deflections and compare these to predicted deflections, and c) verifying aspects of stress-strain relations and simple beam theory.

B. Procedure

Three point bending tests were conducted on two beams made from Aluminum 6061-T6. One of the beams was a hollow tube ("Round tube") and the other had a prismatic cross-section with a horizontal span and two vertical flanges ("C-Channel"). The beams and testing apparatus are shown in Figure 1. The dimensions are given in Appendix A. Both beams had a series of strain gauges situated at different points along the beam to measure the internal strain at that point. The Round tube had four pairs of biaxial strain gauges on it. The C-Channel had 2 rosette strain gauges and two uniaxial strain gauges. The beams were subjected to a downward applied load using a turnbuckle device located a fixed postion along the span of the beam. The applied load was indirectly measured using a load cell located at one end of the simply supported beam. Four loads were applied that caused reaction forces at the load cell that read between 100-400 N. The computer then reported the strain at each strain gauge using the Strain Smart data acquisition system and the values were recorded.

Insert illustration or digital image of the apparatus.

Figure 1. Three-point testing apparatus.

C. Results:

From the data collected during the experiments on the two beam designs, the predicted deflections and strains were calculated. The measured values of deflection and strain were then compared to their corresponding predicted values using percent error.

C.1 Deflections

The measured deflections at each applied load are shown in Figure 2. To compare these deflections to the predicted values, the applied load is first calculated from the free body diagram, the equations of static equilibrium, and the load cell readings (Appendix B). The centroid of each beam design and its moment of inertia was then determined (Appendix C) so that the predicted deflection of each beam could be calculated (Appendix D). Comparisons between the measured and predicted deflections for each beam design are tabulated in Tables 1 and 2.

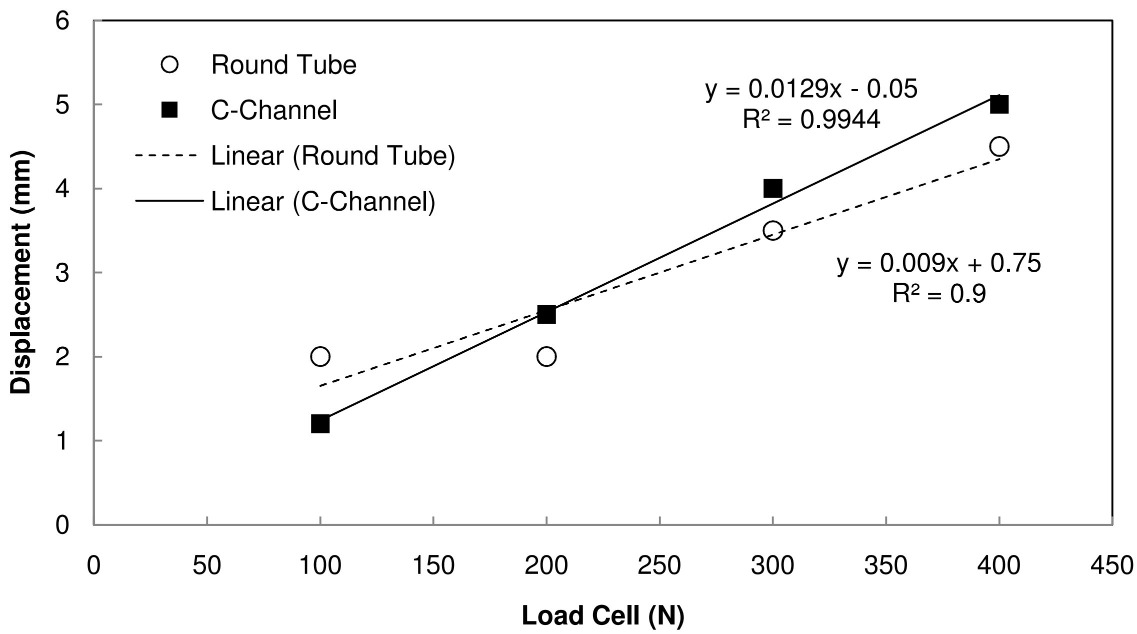

Figure 2. Displacement vs. load for each beam design. *(Right-click and select "Edit Data" to change values)*

Table 1. Measured vs. Predicted Displacement for Each Beam Design

Round Tube			***C-Channel***		
δ_{meas} **(mm)**	δ_{theory} **(mm)**	***Percent Error***	δ_{meas} **(mm)**	δ_{theory} **(mm)**	***Percent Error***

C.2 Strains

The measured strains for each strain gauge are listed in Table 2 to 8 for each beam design. To predict the stress and strain at each gauge location, it was assumed that shear effects are negligible and the only stresses are due to bending (Appendix E). The bending stress at each strain gauge was calculated based upon the bending moment at that location and the distance from the centroidal neutral axis. A rotation of the coordinate system was then used to predict the strain at an angle θ that corresponds to the orientation of each strain gauge. This transformation equation is given by

$$\varepsilon_\theta = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta \qquad (1)$$

where ε_x,ε_x,and γ_{xy} are the predicted strains. The comparison of strains for the Round Tube is tabulated in Tables 2-5 and for the C-Channel in Tables 6-8.

Table 2. Measured vs. Predicted Strains at Strain Gauge 1 for Round tube

SG 1a (θ = _______)			***SG 1b (θ = _______)***		
ε_{meas} (με)	ε_θ (με)	***Percent Error***	ε_{meas} (με)	ε_θ (με)	***Percent Error***

Table 3. Measured vs. Predicted Strains at Strain Gauge 2 for Round tube

SG 2a (θ = _______)			***SG 2b (θ = _______)***		
ε_{meas} (με)	ε_θ (με)	***Percent Error***	ε_{meas} (με)	ε_θ (με)	***Percent Error***

Table 4. Measured vs. Predicted Strains at Strain Gauge 3 for Round tube

SG 3a (θ = _______)			***SG 3b (θ = _______)***		
ε_{meas} (με)	ε_θ (με)	***Percent Error***	ε_{meas} (με)	ε_θ (με)	***Percent Error***

Table 5. Measured vs. Predicted Strains at Strain Gauge 4 for Round tube

SG 4a (θ = _______)			***SG 4b (θ = _______)***		
ε_{meas} (με)	ε_θ (με)	***Percent Error***	ε_{meas} (με)	ε_θ (με)	***Percent Error***

Table 6. Measured vs. Predicted Strains at Strain Gauge 1 for C-Channel

SG1a (θ = _______)			***SG 1b (θ = _______)***			***SG 1c (θ = _______)***		
ε_{meas} (με)	ε_{θ} (με)	***Percent Error***	ε_{meas} (με)	ε_{θ} (με)	***Percent Error***	ε_{meas} (με)	ε_{θ} (με)	***Percent Error***

Table 7. Measured vs. Predicted Strains at Strain Gauge 2 for C-Channel

SG2a (θ = _______)			***SGb (θ = _______)***			***SG2c (θ = _______)***		
ε_{meas} (με)	ε_{θ} (με)	***Percent Error***	ε_{meas} (με)	ε_{θ} (με)	***Percent Error***	ε_{meas} (με)	ε_{θ} (με)	***Percent Error***

Table 8. Measured vs. Predicted Strains at Strain Gauges 3 and 4 for C-Channel

SG3 (θ = _______)			***SG4 (θ = _______)***		
ε_{meas} (με)	ε_{θ} (με)	***Percent Error***	ε_{meas} (με)	ε_{θ} (με)	***Percent Error***

D. Discussion:

Discuss the comparison between the predicted and measured deflections in Table 1. Comment on the linearity of the data in Figure 2 and the R^2 value. Discuss the comparison between the predicted and measured strains in Tables 3-8. Assess the validity of the working hypothesis. Explain possible sources of error in the measurements and how it would affect outcome the data. Indicate which beam design is most likely to fail first and at what location do we expect the onset failure to occur. Make a recommendation about which design to pick based upon deflection (flexibility) vs. failure (strain).

Appendix A: Dimensions and Material Properties

The dimensions of the Round tube and C-Channel are given in Tables A.1 and A.2, respectively. The given material properties of aluminum at given in Table A.3

Table A.1. Round tube dimensions

L	
L_A	
X	
$Y2$	
$Y3$	

Table A.2. C-Channel dimensions

L	
L_A	
$X1$	
$X2$	
$X3\&4$	
$Y3$	
$Y4$	

Table A.3. Material Properties of Aluminum

Elastic Modulus, E (MPa)	69000
Yield Strength (0.2%offset), σ_o (MPa)	275
Poisson's ratio, υ	0.33
Ultimate Tensile Strength, σ_u (MPa)	324
Percent Elongation, %el (50.8mm gage length)	12

Appendix B: Free Body Diagram & Applied Load

The applied load P for each beam design is found from solving the equations of static equilibrium and the force reading at the load cell, R_{LC}.

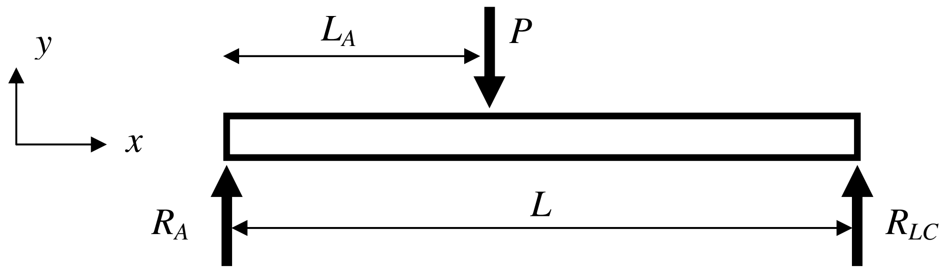

Figure B.1 Free-body diagram of 3-pt loaded beam.

To find the applied load, the sum of the moments about A gives

$$\sum M_A = 0 = -PL_A + R_{LC}L \tag{B.1}$$

To find the reaction force at A, the sum of the forces in the y-direction gives

$$\sum F_y = 0 = R_A - P + R_{LC} \tag{B.2}$$

The values for each load cell measurement for the round tube and C-channel are given in Table B.1 and B.2, respectively.

Table B.1. Applied Loads for Round tube

R_{LC} (N)	P (N)	R_A (N)

Table B.2. Applied Loads for C-channel

R_{LC} (N)	P (N)	R_A (N)

Appendix C: Centroid and Moment of Inertia

Calculation of the centroid and moment of inertia requires the use of the cross-sectional dimensions of each beam. The values are contained in Table C.1 and C.2.

Table C.1. Dimensions of the Round tube

Dimension	**Length (mm)**
Outer Diameter, D_0	
Inner Diameter, D_i	

Table C.2. Dimensions of the C-channel

Dimension	**Length (mm)**
Width, w	
Height, h	
Thickness, t	

C.1 Round Tube

The *y*-position of the centroid from the bottom of the round tube is

$$\bar{y} = \frac{D_0}{2} \tag{C.1}$$

and has a value of __________________ mm.

The moment of inertia about the *z*-axis is

$$I_z = \frac{\pi}{64}\left(D_0^4 - D_i^4\right) \tag{C.2}$$

and has a value of __________________ mm^4.

C.1 C-Channel

The centroid and moment of inertia requires that the C-channel be calculated as if three rectangular sections are in union with each other. There is one top section with area (*w-2t*)(t) and two identical flange sections on each side with area *ht*. The *y*-position of the centroid from the bottom of the C-channel is

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{\left(\frac{1}{2}h\right)(ht) + \left(\frac{1}{2}h\right)(ht) + \left(h - \frac{1}{2}t\right)\left((w-2t)(t)\right)}{ht + ht + (w-2t)(t)} \tag{C.3}$$

where y_i is y-axis position of each sections centroid and A_i is area of each section. The centroid of the C-channel is located _________________ mm from the bottom.

The moment of inertia is calculated using the parallel-axis theorem and is

$$\begin{aligned} I_z &= \sum \left(I_i^4 + A_i d_i^2\right) \\ &= \left(\tfrac{1}{12}th^3 + ht\left(\bar{y} - \tfrac{1}{2}h\right)^2\right) + \left(\tfrac{1}{12}th^3 + ht\left(\bar{y} - \tfrac{1}{2}h\right)^2\right) + \left(\tfrac{1}{12}(w-2t)t^3 + (w-2t)t\left(\bar{y} - \left(h - \tfrac{1}{2}t\right)\right)^2\right) \end{aligned} \tag{C.4}$$

where I_i is the moment of inertia about the z-axis for each rectangular section and d_i is the distance in the y-direction from centroid of the C-channel to the local centroid of each section. The moment of inertia f the C-channel is _________________ mm^4.

Appendix D: Beam Deflection Calculation

The maximum deflection at each simply supported beam can be found from Euler-Bernouli beam theory and is given by

$$\delta_{theory} = -PL_A^{\,2}\left(L - L_A\right)^2 \Big/ 3EI_zL \qquad \text{(D.1)}$$

For each load *P*, the predicted displacements are given in Table D.1 and D.2, respectively.

Table D.1. Displacement for Round tube

R_{LC} (N)	δ_{theory} (mm)

Table D.2. Displacement for C-channel

R_{LC} (N)	δ_{theory} (mm)

Appendix E: Predicted Stress and Strain in Beams

If we neglect the effect of shear forces, then the stresses at each strain gage are only due to bending:

$$\sigma_x = \frac{My}{I_z}, \sigma_y = 0, \tau_{xy} = 0 \tag{E.1}$$

where M is the moment induced by the applied load P at a given point along the length of the beam ($M = R_A x$ for $0 \le x \le L_A$ and $M = R_{LC}(L-x)$ for $L_A \le x \le L$) and y is the distance in the y-direction between the location of the strain gage and the centroid of beam $(y = \bar{y} - y_i)$. The normal and shear strains are given by Hooke's law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \tag{E.2}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \tag{E.3}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \tag{E.4}$$

E.1 Round Tube

The predicted stresses and strains at each strain gage on the Round tube are listed in Tables E.1 to E.4

Table E.1. Stress and Strain at Strain Gage 1 for Round tube.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)

Table E.2. Stress and Strain at Strain Gage 2 for Round tube.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)

Table E.3. Stress and Strain at Strain Gage 3 for Round tube.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)

Table E.4. Stress and Strain at Strain Gage 4 for Round tube.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)

E.2 C-Channel

The predicted stresses and strains at each strain gage on the C-Channel are listed in Tables E.5 to E.8

Table E.5. Stress and Strain at Strain Gage 1 for C-Channel.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)

Table E.6. Stress and Strain at Strain Gage 2 for C-Channel.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)

Table E.7. Stress and Strain at Strain Gage 3 for C-Channel.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)

Table E.8. Stress and Strain at Strain Gage 4 for C-Channel.

M (N mm)	y (mm)	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	ε_x (με)	ε_y (με)	γ_{xy} (με)