ME 354 MECHANICS OF MATERIALS LABORATORY
STRESSES IN STRAIGHT AND CURVED BEAMS

January 2008 NJS

OBJECTIVES

The objectives of this laboratory exercise are to introduce an experimental stress analysis technique known as "photoelasticity", and to apply this technique to measure the stresses induced in a curved beam. In order to use the method of photoelasticity, we must first measure a material property called the "material fringe value", denoted f_s. The f_s will be measured for the polymer PSM-1. There are several ways of measuring f_s. The technique used for calibration in this lab is based on a *straight beam* subjected to four-point bending. Once f_s has been measured, then the stresses induced at several points in a *curved beam* will be measured using photoelasticity. The experimental measurements will be compared to predicted stress levels obtained using analytical solutions as well as numerical solutions from finite element analysis (FEA).

EQUIPMENT USED

- A total of two test specimens will be used: one straight beam and one curved beam (made from PSM-1).
- A four-point loading fixture with load pan and calibrated masses (used to load the straight beam)
- A line-loading fixture with load pan and calibrated masses (used to load the curved beam)
- A circular polariscope equipped with a monochromatic (green) light source
- Video camera, B/W monitor and a video printer

EXPERIMENTAL PROCEDURES

STRAIGHT BEAM TESTS

i) Install the straight beam in the four-point loading fixture (see Figure 1a)

ii) Attach the load pan (Note: the combined mass of the pan and fixture is ~0.980kg)

iii) Apply two 10 kg masses, one at a time, to the load pan.

iv) Use the video camera/printer to record the fringe pattern observed with a "dark field" (i.e., the image obtained when the polarizer and analyzer axes are crossed)

v) Use the video camera/printer to record the fringe pattern observed with a "light field" (i.e., the image obtained when the polarizer and analyzer axes are parallel)

CURVED BEAM TEST

i) Install the curved beam in the line loading fixture (see Figure 1b)

ii) Attach the load pan (Note: The combined mass of the pan and the fixture is ~0.454 kg)

iii) Apply *one* 2 kg mass to the load pan. (Note; *Do not apply more than 2kg at any time)*

iv) Use the video camera/printer to record the fringe pattern observed with a "dark field" (i.e., the image obtained when the polarizer and analyzer axes are crossed)

v) Use the video camera/printer to record the fringe pattern observed with a "light field" (i.e., the image obtained when the polarizer and analyzer axes are parallel)

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WORKSHEET

NAME____________________________________DATE___________________

1) Preliminaries: Specimen Dimensions

Complete the following table (Refer to Figure 1-3).

Table 1: Dimensions and Loading for the Straight and Curved Photoelastic Beams

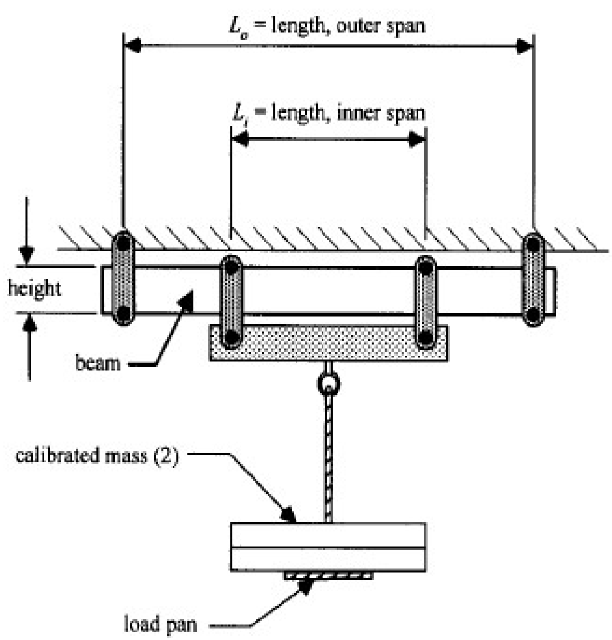
Straight Beam

Total Applied Force F_s $=(M_{weight}+M_{fixture}+M_{pan})g$ (N)	
Outer span L_o (mm)	
Inner span L_i (mm)	
Height (straight), h (mm)	
Thickness, b (mm)	
Momment of Inertia $I=bh^3/12$ (mm^4)	

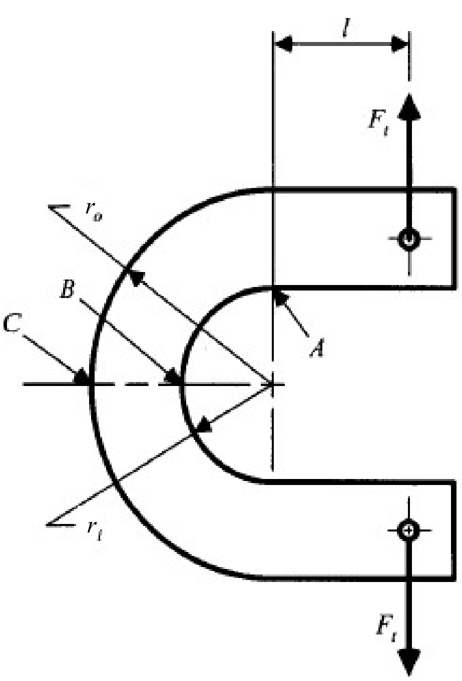
Curved Beam

Total Applied Force F_c $=(M_{weight}+M_{fixture}+M_{pan})g$ (N)	
Outer radius r_o (mm)	
Inner radius r_i (mm)	
Height $h_c = r_o - r_i$ (mm)	
Thickness, b (mm)	
Average radius $r_{ave} = \bar{r} = (r_o + r_i)/2$ (mm)	
Straight leg length l (mm)	

Note: the calibration and test force must include the mass of the fixture and pan, and the added mass in kg. Gravitational constant is $g = 9.816$ kg m/s^2.



a) Straight Beam



b) Curved Beam

Figure 1 Nomenclature for the a) Straight and b) Curved Beams

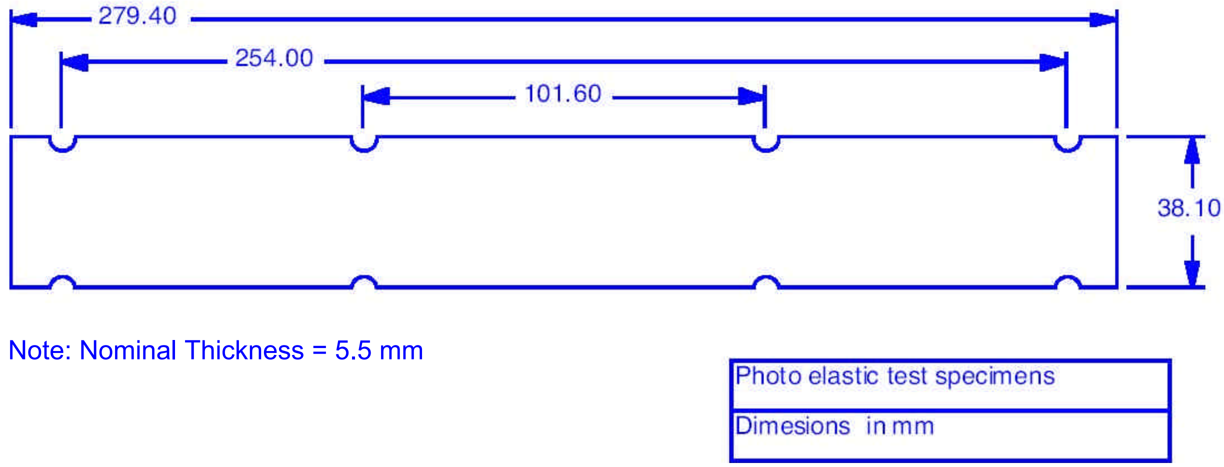


Figure 2 Sketch of the Straight Beam showing Dimensions

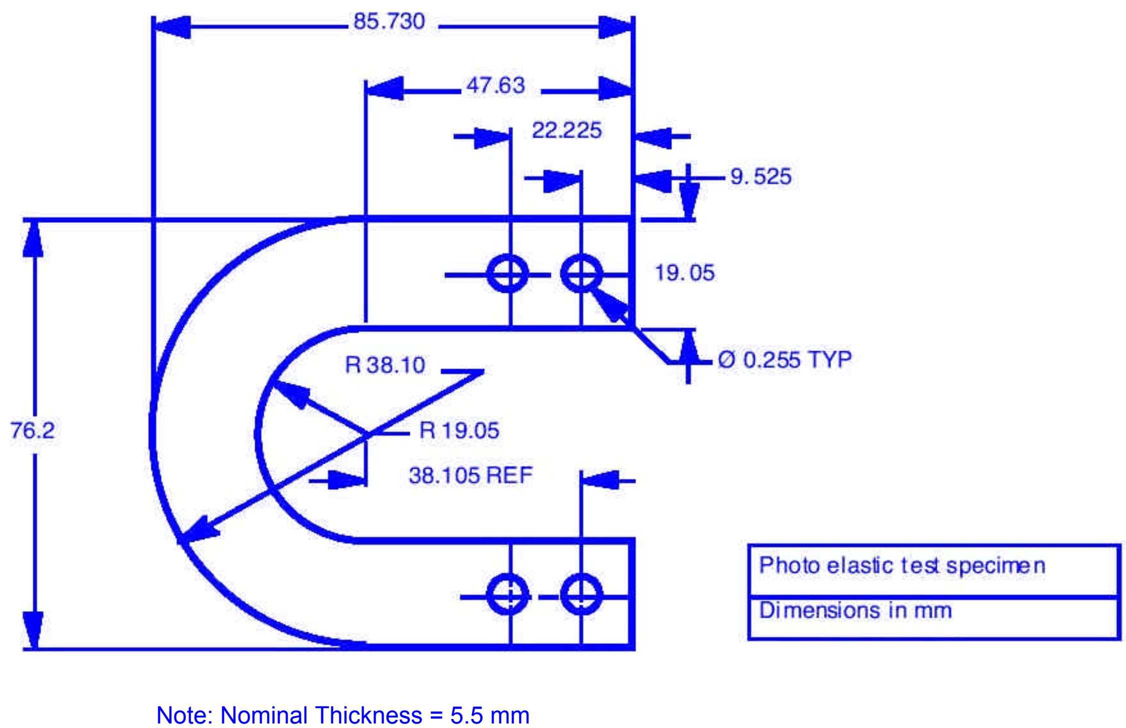


Figure 3 Sketch of the Curved Beam showing Dimensions

2) Determination of the Material Fringe Value f_s

Interpretation of 2-D photoelastic fringe patterns is based on the stress-optic law:

$$(s_1 - s_2) = \frac{Nf_s}{b}$$

where s_1 and s_2 are the in-plane principle stresses, N is the fringe order, b is the thickness of the photoelastic model, and f_s is the material fringe value.

In this lab, the fringe patterns observed in straight beams subjected to pure bending are used to determine f_s. Recall the stresses induced in a straight beam with a constant cross section are given by (using the *x*-*y*-*z* coordinate system shown in Figure 4):

$s_z = t_{xz} = t_{yz} = 0$	(Out of plane stresses are assumed to be zero (plane stress assumption))
$s_y = 0$	(normal stresses transverse to the beam are ignored)
$s_x = \dfrac{-My}{I}$	(axial stresses are given by the “flexure formula”)
$t_{xy} = \dfrac{VQ}{Ib}$	(shear stresses and given by the “shear formula”)

A free-body, shear force, and bending moment diagram for a straight beam loaded in four point flexure loading is shown in Figure 4. Note that this loading arrangement places the central region of the beam in “pure bending”. That is, over the central region, the shear force is zero (V = 0) and the bending moment is constant ($M = F_s\,(L_o\text{-}L_i)/4$). Consequently in this inner region, the principle stresses are:

$$s_1 = s_x = \frac{My}{I} = \frac{F_s(L_o - L_i)y}{4I}, \quad s_2 = 0, \quad s_3 = 0$$

Substituting these results into the stress optics law and rearranging (slightly), we find:

$$Z = (f_s)N$$

where:

$$Z = \frac{F_s(L_o - L_i)y}{4I}b$$

Hence f_s can be determined by plotting the quantity Z versus the fringe order, N and fitting a straight line to this data using a linear regression.

If the data behaves “perfectly”, then the slope given by the linear regression will equal f_s and the intercept = 0. Nowadays, linear regression can be performed using scientific hand calculators and computer software (such as EXCEL). Equivalently, by completing Table 2 you will, in effect, be performing a linear regression of the data obtained for the straight beam.

Perform the following:

a) Enter the required data in columns 1 and 2 of Table 2 by scaling the recorded images using the dark and light field polariscope settings (also indicate whether a dark or light field was used in column 5)

b) Complete all calculations indicated in Table 2 thereby determining f_s.

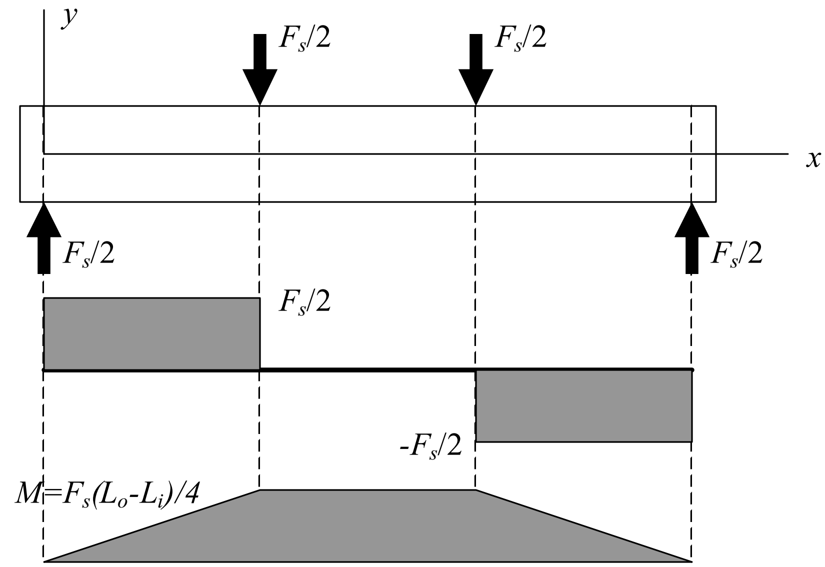


Figure 4 Free Body, Shear Force and Bending Moment Diagrams for Four-Point Flexure Loading

Table 2: Linear Regression of Straight Beam PSM-1 Data

Distance y (mm)	Fringe Order* N	N^2	$Z = \frac{F_s(L_o - L_i)y}{4I}b$	NZ	Dark/Light Field?
$\sum($ $)$					

*Define the fringes located at negative y positions as "negative" fringes

The total number of data points: n = ________________

The slope (i.e., the material fringe value) is given by:

$$f_s = \frac{\sum N \sum Z - n \sum NZ}{\left(\sum N\right)^2 - n\left(\sum N^2\right)} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Note: the "handbook" value for the fringe value for the material used here is around 7 kN/m fringe.

The intercept is given by:

$$\text{intercept} = \frac{\sum Z - f_s \sum N}{n} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

3) Experimentally Measured Stresses in the Curved Beam using Photoelasticity

At the free edge of selected location of the curved beam ("A" "B" and "C" in Figure 1b), the stress states are uniaxial and the stress-optic law can be used to calculate the normal stress using the relation between the fringe order at the free edge, the stress optic coefficient for the material, and the specimen thickness.

Fill in the table with the values for the loaded test specimen that are used in the following calculations.

Fringe value counted at point "A", N_A	
Fringe value counted at point "B", N_B	
Fringe value counted at point "C", N_C	
Thickness, b (mm)	

At point "A", the normal stress: $s_A = \dfrac{f_s N_A}{b} =$ _______MPa.

At point "B", the normal stress: $s_B = \dfrac{f_s N_B}{b} =$ _______MPa.

At point "C", the normal stress: $s_C = \dfrac{f_s N_C}{b} =$ _______MPa.

4) Analytically Determined Stress at Point "A" Using the Straight Beam Relations

For the straight part of the beam, a straight beam flexure analysis may be used. The applied bending Moment at point "A" is determined as the applied test force F_c=_________N multiplied by the length of the straight leg l =_________mm such that $M_A = F_c\, l$ =_________N mm.

The moment of inertia is calculated from the height of the beam h_c and the thickness of the beam, b such

that $I_A = \dfrac{bh_c^3}{12}$ =_________mm^4.

The distance from the neutral axis to the point "A" is $c = h_c/2$ =_________mm.

The normal stress at "A" for a straight beam assumption is $s_A^{straight} = \dfrac{M_A c}{I_A}$ =____________MPa.

(Confirm that the normal stress at "A" should be tensile (positive stress))

5) Analytically Determined Stresses Using the Curved Beam Relations

For the curved part of the beam (in this case points "B" and "C") The analytical calculation must take into account the initial curvature of the beam.

a) At the line in the curve connecting "B" and "C", the radius of the neutral axis for the rectangular cross section can be calculated from the outer radius, r_o and the inner radius r_i such that:

$R = \dfrac{r_o - r_i}{\ln(r_o / r_i)}$ =____________mm.

The eccentricity, e, can be calculated from the average radius (or centroid) $\bar{r}$ and the radius of the neutral axis such that $e = \bar{r} - R$ =____________mm.

The cross sectional area is calculated from the thickness, b, and the curve height, h_c such that $A = b\, h_c$ = ____________mm^2.

(Note that the distance from the Neutral axis to the point of interest is $y = r - R$).

b) At point “B”, $r = r_i$; therefore, $y_B = r_i - R$ = ___________mm.

The bending moment at point “B” is determined as the applied test force F_c=_________N multiplied by the length of the straight leg l =_________mm plus the average radius $\bar{r} = (r_o + r_i)/2$ =_________mm such that $M_B = F_c(\bar{r} + l)$ =_________N mm.

The normal stress in the curved beam at “B” is $s_B^{curved} = \dfrac{-M_B y_B}{Ae(y_B + R)}$ =___________MPa.

c) At point “C”, $r = r_o$; therefore, $y_C = r_o - R$ = ___________mm.

The bending moment at point “C” is the same as at point “B” such that:

$M_C = M_B$ = ___________N mm.

The normal stress in the curved beam at “C” is $s_C^{curved} = \dfrac{-M_C y_C}{Ae(y_C + R)}$ =___________MPa.

6) Additional Axial Normal Stress Component

Because the bending moment at “B-C” is produced by a transverse force (that is, not a pure bending moment), the total stress at “B-C” has two components: a tensile axial (in the loading direction) stress and a tensile/compressive bending stress (computed above).

a) The tensile axial stress is calculated from the applied test force F_c=_________N and the cross sectional area $A = b\,h_c$ = ___________mm^2.

The axial stress is $s^{axial} = \dfrac{F_c}{A}$ =_________MPa.

(Make sure that this axial normal stress is tensile (i.e. positive stress)

7) Comparison of the Total Normal Stress (Bending and Axial) at “B” and “C”

a) At “B”, the total calculated stress using the curved beam assumption is:

$s_B^{total(curved)} = s^{axial} + s_B^{curved}$ = ____________MPa.

Percent difference between the actual photoelastically measured stress and the calculated stress is:

$100\dfrac{s_B^{total(curved)} - s_B}{s_B}$ = ____________%.

b) At “B”, the numerically determined (from the finite element analysis (FEA) solution of Figure 5) normal stress is:

s_B^{FEA} = ____________MPa.

Percent difference between the actual photoelastically measured stress and the numerically determined stress is: $100\dfrac{s_B^{FEA} - s_B}{s_B}$ = ____________%.

c) At “C”, the total calculated stress using the curved beam assumption is:

$s_C^{total(curved)} = s^{axial} + s_C^{curved}$ = ____________MPa.

Percent difference between the actual photoelastically measured stress and the calculated stress is:

$100\dfrac{s_C^{total(curved)} - s_C}{s_C}$ = ____________%.

d) At “C”, the numerically determined (from FEA) normal stress is:

s_C^{FEA} = ____________MPa.

Percent difference between the actual photoelastically measured stress and the numerically determined stress is: $100\dfrac{s_C^{FEA} - s_C}{s_C}$ = ____________%.

8) Comparison of the Normal Stress (bending) at "A"

a) At "A", the total calculated stress using the straight beam assumption is:

$s_A^{straight} =$ ______________ MPa.

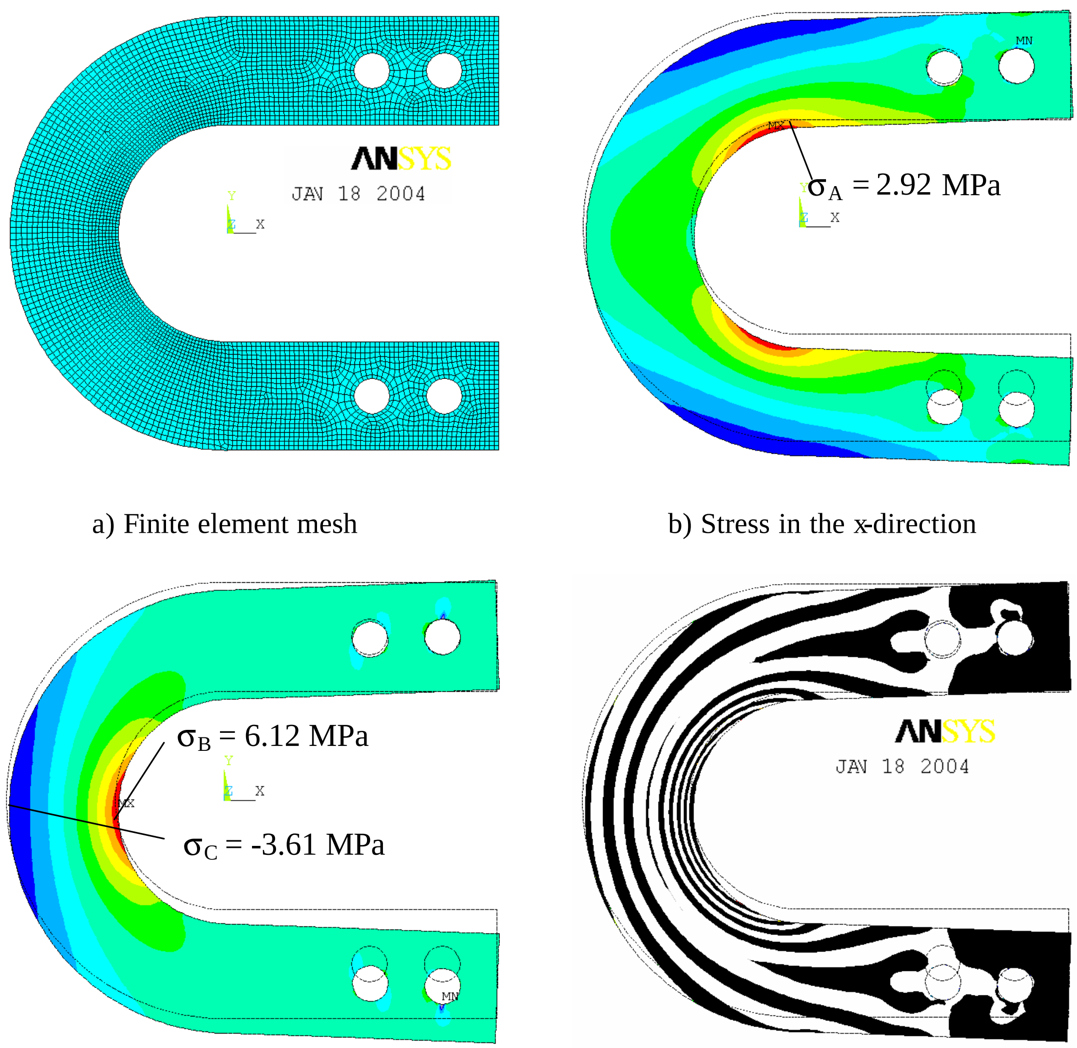
Percent difference between the actual photoelastically measured stress and the calculated stress is:

$$100\frac{s_A^{straight} - s_A}{s_A} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \%.$$

b) At "A", the numerically determined (from FEA) normal stress is:

$s_A^{FEA} =$ ______________ MPa.

Percent difference between the actual photoelastically measured stress and the numerically determined stress is: $100\frac{s_A^{FEA} - s_A}{s_A} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \%.$



a) Finite element mesh

b) Stress in the x-direction

c) Stress in the y-direction

d) Simulated fringe pattern

Figure 5 Finite Element Analysis Solution to the Curved Beam Bending Problem