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Robust purchasing and information asymmetry in supply chains with a price-only contract

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This article proves that information can be a double-edged sword in supply chains. A simple supply chain is studied that consists of one supplier and one retailer, interacting via a wholesale price contract, where one firm knows the probabilistic distribution of demand and the other only knows the mean and variance. The firm with limited distributional knowledge applies simple robust optimization techniques. It is proved that a firm's informational advantage is not necessarily beneficial and can lead to a *reduction* of the firm's profit, demonstrating the detriment of information. It is shown how the direction of asymmetry, demand variability, and product economics affect both firms' profits. These results also provide an understanding of how asymmetric information impacts the double-marginalization effect for the cumulative profits of the supply chain in certain cases reducing the effect. The symmetric incomplete informational case, where both firms only know the mean and variance of demand, is also studied and it is shown that it is possible that both firms can benefit from their collective lack of information. Throughout this article, practical guidelines where a supplier or retailer is motivated to share, hide, or seek information are identified.

Keywords: Supply chain management, value of information, asymmetric information

1. Introduction

The complexity and geographic breadth of modern supply chains makes it unrealistic to assume that all participating firms have the same information. In this article, we consider a simple two-stage supply chain, consisting of a supplier (she) and a retailer (he), where each firm has varying degrees of information about the stochastic final customer demand. A significant portion of our article considers the situation where one firm has a precise probabilistic distribution to characterize demand, while the other only knows the mean and variance of demand. We consider this asymmetry in both directions, allowing both firms to have the informational advantage. We also study the symmetric case where both retailer and supplier only know the mean and variance of the demand.

We model the retailer as a price taker, who simply orders an appropriate quantity in response to the wholesale price proposed by the supplier. In contrast, we model a supplier who considers the buying behavior of the retailer in deciding what wholesale price to offer. Furthermore, we study both the case where the supplier correctly assesses the informational state and buying strategy of the retailer, as well as the case where the supplier incorrectly characterizes the retailer.

In this article we design a framework, utilizing robust optimization, to study how asymmetric and incomplete demand information affects individual firm and cumulative supply chain profits under a price-only contract. Most notably, we show that an informational advantage does not necessarily lead to a profit increase, with respect to the case where both firms have full information. Conversely, even if neither firm has full information, it is possible that both firms' profits increase. We show how the direction of informational asymmetry, demand variability, and product economics affect both firms' profits, with an emphasis on identifying environments where a supplier or retailer is motivated to share, hide, or seek information. Consequently, our conclusions have implications for supply chain negotiations.

Our research is also relevant to the coordination of a supply chain. Coordinating contracts have received substantial attention in recent years, both by researchers and in business school curricula. Indeed, for a simple supplier–retailer supply chain, it is well known that decentralized decision making, coupled with wholesale price contracts,

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results in double marginalization. Despite the introduction and substantial study of a number of coordinating contracts (e.g., Cachon (2003)), it has been observed in practice that, despite their theoretical limitations, wholesale price contracts are prevalent. A number of possible explanations have been postulated: for example, the simplicity of wholesale price contracts makes them appealing, and the additional complexity and administrative burden of coordinating contracts have reduced their adoption in practice. Our results provide another explanation, namely, that there exist environments where asymmetric and incomplete information result in a *reduction* of the double-marginalization effect, increasing the appeal of wholesale price contracts. Finally, note that our results apply only to a *price-only contract*; we conjecture that information asymmetry is always detrimental for a coordinating contract (e.g., buy-back, revenue-sharing, etc.), since under perfect information they maximize the cumulative supply chain profits, whereas price-only contracts do not.

Next, we provide a literature review to survey the relevant research and to clearly position our article.

1.1. *Literature review*

Our article focuses on the study of asymmetric information in a supply chain, the resulting differences in power among the relevant firms, and implications for the system. For a further discussion of power in the retail segment, see Ailawadi (2001). Messinger and Narasimhan (1995) discussed related issues in the grocery business and Bloom and Perry (2001) considered Wal-Mart's power. Indeed, some firms are quite protective of their information and do not share with market research firms, as noted in Jiang *et al.* (2011). These authors studied newsvendor competition among many retailers under asymmetric information, via robust optimization; we similarly apply a robust optimization approach, under asymmetric information, except in a contractual setting between a supplier and retailer.

A stream of research, closely linked with our article, is concerned with the sharing of information in a supply chain. Li (2002) studied a situation where multiple retailers have an informational advantage over a single supplier and identified environments where the retailers are motivated to either share or hide their information; however, this research did not consider the cases where the supplier has the informational advantage or where both firms are disadvantaged, as we do in this article. Cachon and Fisher (2000) considered the sharing of demand and inventory information, where one supplier interacts with multiple retailers, and provided an upper bound on the value of the information for stationary demand. Lee *et al.* (2000) showed that a manufacturer benefits when a retailer shares point-of-sale demand data under non-stationary demand; Raghunathan (2001) built upon the model in Lee *et al.* (2000) and showed that intelligent forecasting on the part of the manufacturer eliminates this benefit, and the need for sharing information is removed. Our article discusses similar motivations for sharing or seeking information. Chen (1998) studied a serial network where the value of centralized demand information was determined. Gavirneni *et al.* (1999) considered the value to the supplier of knowing different characteristics of the retailer, such as the inventory policy being implemented. Ha and Tong (2008) considered two supply chains, each with a single supplier and retailer, where each supply chain had a different cost for sharing information. Using a game-theoretic model, these authors showed that equilibrium information sharing depends on the structure of the contract between the retailer and supplier. Cachon and Lariviere (2001) studied intricacies in sharing demand forecasts in a supply chain. For further results, Chen (2003) provides an excellent survey of information sharing in supply chains.

Asymmetric information is another key concept in our paper. Most related to our article is Kalkanci and Erhun (2012), which studied decentralized assembly systems with two suppliers and a single manufacturer and is focused on information asymmetry and sequential contracting; i.e., there is a leader supplier and a follower supplier. They showed that the follower supplier can benefit from the information asymmetry and would suffer with additional information. We provide similar results for the supplier in our model (without requiring a second supplier), as well as for the retailer. Taylor and Xiao (2010) considered a single supplier and retailer, where the latter has superior demand information, and showed that the supplier's profit function is convex in the retailer's forecasting accuracy. We study a similar scenario, except we focus on the inherent demand variability, not forecasting accuracy, and we also consider the case where the supplier has superior demand information. Ozer and Wei (2006) studied a variety of supply chain contracts under asymmetric demand forecasts and showed that the degree of asymmetry and risk-adjusted profit margins dictate the contract that should be selected. Akan *et al.* (2011) studied service contracts under asymmetric demand information and derived optimal contracts that replicated the full information solution. Ha (2001) studied the effect of asymmetric cost information from the supplier's point of view. Asymmetric information can also lead to manipulation of the environment. Some buyers utilize so-called phantom orders to induce higher capacities in their suppliers, as discussed in Lee *et al.* (1997). Terwiesch *et al.* (2005) provided an empirical study of forecast sharing in the semiconductor industry, where the impact of frequency and magnitude of forecast revisions, as well as inflated forecasts, were evaluated. Cohen *et al.* (2003) studied a semiconductor supply chain interaction where a manufacturer is biased to provide inaccurate forecasts to a supplier. More related references can be found in Chapter 10 of Cachon (2003).

Most related to our article is the recent work of Kalkanci *et al.* (2011), which strived to provide a rationale for why simpler contracts are popular in practice, despite their theoretical limitations. Their work also employed asymmetric information in a two-tier supply chain, but instead of a theoretical analysis, they considered behavioral experiments where one player is human and the other is a computerized newsvendor model. Their results show that simple contracts, such as wholesale price contracts, are sufficient under asymmetric information in a behavioral setting.

1.2. *Contributions*

In this article we design a framework to study how asymmetric and incomplete demand information affect individual firm and cumulative supply chain profits, when simple robust optimization techniques are applied. We prove theoretically, and show numerically, that an informational advantage does not necessarily lead to a profit advantage under a price-only contract. We compare the incomplete informational states with a benchmark state where both firms have full distributional knowledge.

We first analytically show that, if the retailer has an informational advantage, then the supplier always loses profit, but the retailer *may or may not* benefit from the information asymmetry; we supplement these theoretical results with simulation studies, for normally distributed demand, that characterize the environments that lead to increased retailer's profits. Second, if the supplier has an informational advantage but incorrectly assesses the retailer's state, we analytically show that the retailer always loses profit but the supplier *may or may not* benefit; we similarly identify the environmental characteristics, via simulation studies, that lead to improved supplier's profits. If the advantaged supplier correctly assesses the state of the retailer, it is possible that both firms benefit, both lose, or one firm (either) benefits and the other suffers from the information asymmetry; we utilize simulation studies to understand how demand variability and product economics influence the outcome. These last results also apply to the case where neither firm has distributional knowledge. Finally, we compare the profits for a decentralized supply chain with the profits for the optimal centralized supply chain and show, via simulation studies, that the lack of information can result in the capture of a very high proportion of the possible profit (e.g., $> 95\%$). We provide further details and discussion in the appropriate sections.

1.3. *Outline*

In Section 2 we detail our Stackelberg game models that combine standard and distribution-free newsvendor models. In Section 3 we analyze the case where the retailer has the informational advantage. In Section 4 we consider the situation where the supplier has the advantage, a portion of which also applies to the case where neither firm knows the distribution of demand. Section-specific computational studies are included in Sections 3.4, 4.1.3, and 4.2.3. In Section 5 we provide comparisons of the profits for various decentralized supply chains with those of the optimal centralized supply chain. Concluding thoughts are provided in Section 6. All proofs appear in the Appendix.

2. Models and methodology

Our article analyzes the behavior of the retailer's profit, supplier's profit, and the total profit of the supply chain under a wholesale price contract where the retailer's and supplier's knowledge of the final customer demand differs. We choose the wholesale price contract as it is prevalent in practice and simple to analyze, allowing us to focus on understanding the impact of different information among the firms. The retailer sells a single product to final customers at unit revenue r and salvages leftover units at unit value v. The retailer purchases its stock from the supplier, via a one-time wholesale price contract, at a unit cost of w. The supplier produces at a unit cost of c, which we assume is information private to the supplier. To avoid trivial cases, it is assumed that $r > w > c > v$. The supplier chooses the wholesale price w and then the retailer chooses an ordering quantity q, via a Stackelberg game where the supplier is the leader.

The retailer sells to stochastic final customer demand $D \geq 0$, a random variable, which is characterized by a distribution function F, with mean μ and standard deviation σ. We assume that F^{-1} exists. Each firm will either know (i) the full distribution F; or (ii) only the mean μ and standard deviation σ. We formalize these informational scenarios with the following notation: R_F indicates that the retailer knows F and $R_{\mu,\sigma}$ indicates that the retailer knows μ and σ, but not the full distribution F. S_F and $S_{\mu,\sigma}$ are defined similarly for the supplier. Therefore, we have four informational states:

$$(R_F, S_F),\ (R_{\mu,\sigma}, S_F),\ (R_F, S_{\mu,\sigma}),\ (R_{\mu,\sigma}, S_{\mu,\sigma}). \qquad (1)$$

We next model the retailer's and supplier's behaviors under these states.

2.1. *Retailer's behavior*

The supplier's costs are private information, unknown to the retailer, which limits any strategic or learning aspects for the retailer. Therefore, we model the retailer as a price-taker, who simply chooses an order quantity once the wholesale price is proposed by the supplier. Note that under the wholesale price contract, the retailer absorbs all demand variability and his profit is a random variable. Therefore, the retailer's performance is traditionally measured via his *expected* profit. Under R_F, the retailer, applying a newsvendor model, maximizes his expected profit, which has the solution

$$q_{nv}(w) = \arg\max_{q\geq 0}\; r\,E_F[\min\{q, D\}] + v\,E_F[\max\{q-D, 0\}] - wq = F^{-1}\left(\frac{r-w}{r-v}\right). \quad (2)$$

Under $R_{\mu,\sigma}$ the retailer has limited distributional information and his expected profit is not well defined. However, we wish to remain consistent with the retailer choosing an order quantity via an expected profit measurement, as under R_F. Therefore, we need a specific distribution. Rather than arbitrarily choosing a distribution under $R_{\mu,\sigma}$ with mean μ and standard deviation σ, and motivated by the prevalence of robust optimization in the modern literature, we model the retailer's behavior using a classic robust variant of the newsvendor model. The retailer maximizes his minimum expected profit over all distributions G, corresponding to non-negative random variables with the given mean and standard deviation, which has the formulation

$$\begin{aligned} q_{mm}(w) = \arg\max_q\; \min_G\; & r\,E_G[\min\{q, D\}] + v\,E_G[\max\{q-D, 0\}] - wq \\ \text{s.t.} \int_0^\infty dG(x) &= 1 \\ \int_0^\infty x\,dG(x) &= \mu \\ \int_0^\infty x^2 dG(x) &= \sigma^2 + \mu^2. \end{aligned} \quad (3)$$

The worst-case distribution, which is used to determine $q_{mm}(w)$, was shown in Scarf (1958) to be a two-point distribution with mass $\sigma^2/(\mu^2+\sigma^2)$ at zero and mass $\mu^2/(\mu^2+\sigma^2)$ at $\mu+\sigma^2/\mu$. Note that the worst-case distribution is only for determining the order quantity; the real expected cost is evaluated using the true distribution F (full details follow in Section 2.3).

Letting $\rho = \sigma/\mu$ denote the coefficient of variation, Scarf (1958) showed that the optimal ordering quantity for Problem (3) is

$$q_{mm}(w) = \begin{cases} 0, & \dfrac{r-w}{w-v} < \rho^2, \\ \mu + \dfrac{\sigma}{2}\left(\sqrt{\dfrac{r-w}{w-v}} - \sqrt{\dfrac{w-v}{r-w}}\right), & \dfrac{r-w}{w-v} \geq \rho^2. \end{cases} \quad (4)$$

It has been pointed out in the literature that the order quantities in Equation (4) are too conservative, an assessment with which we agree, due to the retailer not ordering at all when $(r-w)/(w-v) < \rho^2$. However, our article focuses only on the latter case, where $(r-w)/(w-v) \geq \rho^2$, which induces an order quantity not much different than the order quantities under well-known distributions. For example, setting $r = 100$, $v = 0$, $\mu = 1000$, and $\rho = 0.25$, we compare $q_{mm}(w)$, defined in Equation (4), with $q_{nv}(w)$, defined in Equation (2), for F being either the normal and uniform distributions, as a function of $w \in [0, 100]$; see Fig. 1. Note that with these parameters, the normal distribution is non-negative with probability > 0.99997 and is essentially indistinguishable from a truncated normal distribution, which is contained in the feasible region of Problem (3). Except for the orders of zero when $w \in [95, 100]$, the three order quantities are quite similar, and it is on this region that our article focuses. In particular, in Section 3.1, we constrain the supplier to choose a wholesale price that induces a positive order quantity from the retailer, reducing the conservatism of the retailer's model. The analytical benefit of this model is that the order quantity has a closed form, allowing a more tractable analysis than other comparable retailer strategies that do not (for example, the minimum-regret strategy of Perakis and Roels (2008) does not have a closed form when the mean and standard deviation are known).

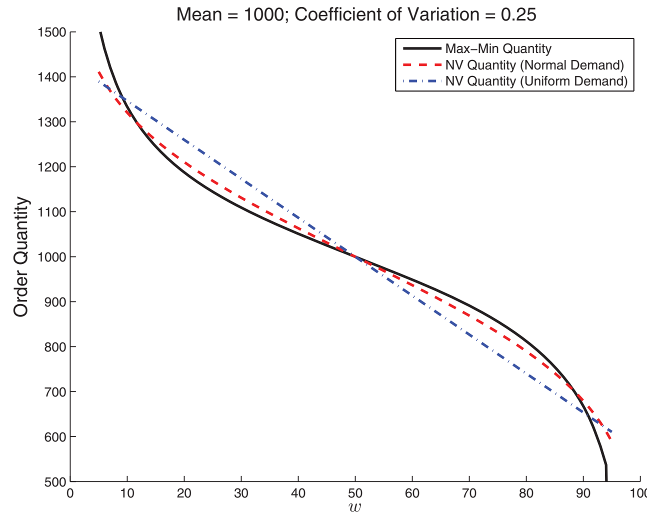

Fig. 1. The differences between $q_{mm}(w)$ and $q_{nv}(w)$ for F being either the normal or uniform distribution.

2.2. *Supplier's behavior*

We next detail the supplier's behavior, which depends on whether or not she correctly assesses the informational state of the retailer.

If the supplier knew that the retailer will order according to $q_{nv}(w)$, as defined in Equation (2), the supplier would then solve

$$w_{nv} = \arg\max_w\; (w-c)q_{nv}(w) \quad (5)$$

to maximize her (deterministic) profit. Lariviere and Porteus (2001) showed that, as long as F has an increasing generalized failure rate (i.e., $xf(x)/(1-F(x))$ is increasing), w_{nv} is unique (though usually without a closed-form expression). These authors also showed that this assumption is mild, being satisfied by most common distributions.

Table 1. Trade terms when the supplier *correctly* assesses the informational state of retailer

	R_F: *Retailer knows* F	$R_{\mu,\sigma}$: *Retailer only knows* μ *and* σ
S_F: Supplier knows F	$q_{nv}(w_{nv})$	$q_{mm}(w_{mm})$
$S_{\mu,\sigma}$: Supplier only knows μ and σ	n/a	$q_{mm}(w_{mm})$

Table 2. Trade terms when the supplier *incorrectly* assesses the informational state of retailer

	R_F: *Retailer knows* F	$R_{\mu,\sigma}$: *Retailer only knows* μ *and* σ
S_F: Supplier knows F	$q_{nv}(w_{mm})$	$q_{mm}(w_{nv})$
$S_{\mu,\sigma}$: Supplier only knows μ and σ	$q_{nv}(w_{mm})$	n/a

Alternatively, if the supplier knew that the retailer will order according to $q_{mm}(w)$, as defined in Equation (4), the supplier would then solve

$$w_{mm} = \arg\max_{w}\ (w - c)q_{mm}(w) \tag{6}$$

to maximize her profit. In Section 3 we analyze Problem (6) and show that w_{mm} is unique as well.

In practice, the supplier may or may not correctly assess the informational state of the retailer. We consider both cases. To maintain the tractability of our model, we assume that the supplier knows that R_F and $R_{\mu,\sigma}$ are the only informational states possible for the retailer. The following list discusses each of the information states in Equation (1) in turn. These possible trade terms are then summarized in Table 1 for correct supplier assessments of the retailer's information and in Table 2 for incorrect assessments.

1. Under (R_F, S_F), the supplier has a belief about the informational state of the retailer, which can be correct or incorrect. If the supplier is correct, she knows that the retailer is in state R_F; alternatively, if the supplier's belief is incorrect, she mistakenly assumes that the retailer is in $R_{\mu,\sigma}$. In the former case, the supplier correctly assumes that the retailer orders according to $q_{nv}(w)$ and thus offers w_{nv} to the retailer, who subsequently orders $q_{nv}(w_{nv})$; this case serves as our benchmark, which is discussed in the next subsection. In the latter case, the supplier incorrectly assumes that the retailer orders according to $q_{mm}(w)$ and offers the retailer w_{mm}, who then orders $q_{nv}(w_{mm})$.
2. Under ($R_{\mu,\sigma}$, S_F), the supplier has a belief about the informational state of the retailer, which can be correct or incorrect. If the supplier is correct, she knows that the retailer is in state $R_{\mu,\sigma}$; if the supplier's belief is incorrect, she mistakenly assumes that the retailer is in R_F. In the former case, the supplier correctly assumes that the retailer orders according to $q_{mm}(w)$ and thus offers w_{mm} to the retailer, who subsequently orders $q_{mm}(w_{mm})$. In the latter case, the supplier incorrectly assumes that the retailer orders according to $q_{nv}(w)$ and offers the retailer w_{nv}, who then orders $q_{mm}(w_{nv})$.
3. Under (R_F, $S_{\mu,\sigma}$), the supplier has a belief about the informational state of the retailer, which can be correct or incorrect. If the supplier is incorrect, she mistakenly assumes that the retailer is in $R_{\mu,\sigma}$ and offers the retailer w_{mm}, and the retailer orders $q_{nv}(w_{mm})$. Alternatively, the supplier's belief can be correct, and she knows that the retailer is in R_F; unfortunately, since the supplier does not know F, she also does not know $q_{nv}(w)$ and does not have a well-defined model to help her choose an appropriate wholesale price. Therefore, this last scenario is outside the scope of our article, which we list as "n/a" in Table 1, and we do not discuss it further.
4. Under ($R_{\mu,\sigma}$, $S_{\mu,\sigma}$), the only meaningful model is where the supplier correctly assumes that the retailer is in $R_{\mu,\sigma}$, which leads to a wholesale price w_{mm} and an order quantity $q_{mm}(w_{mm})$. If the supplier incorrectly assumes that the retailer is in R_F, there is not much she can do, since she does not know F; we list this undefined model as "n/a" in Table 2.

Our article models both the information possessed by the supplier as well as the supplier's (possibly incorrect) belief about the retailer's state. It is the combination of this information and belief, and the correctness of the belief, that drives the firms' interaction. This is why certain terms of trade appear twice in the above tables ($q_{mm}(w_{mm})$ in Table 1 and $q_{nv}(w_{mm})$ in Table 2), since a supplier might not utilize all of the information she possesses due to her beliefs. Other combinations preclude the supplier from acting on her beliefs since she does not have the requisite information; these are the "n/a"s in Tables 1 and 2. Figure 2 plots the different terms of trade in Tables 1 and 2 as a function of the profit margin of the supply chain $(r - c)/r$ and coefficient of variation $\rho \in \{0.01, 0.11, 0.22, 0.33\}$; these behaviors are derived in subsequent sections but are presented here as a preview.

2.3. *Benchmarking*

In each of the four informational states given in Equation (1), the supplier will choose a wholesale price, which depends on her assessment of the retailer's informational state, and the retailer will order a corresponding quantity, which we write generically as w and $q(w)$, respectively. This will result in state-dependent profits for the retailer, supplier, and supply chain, according to

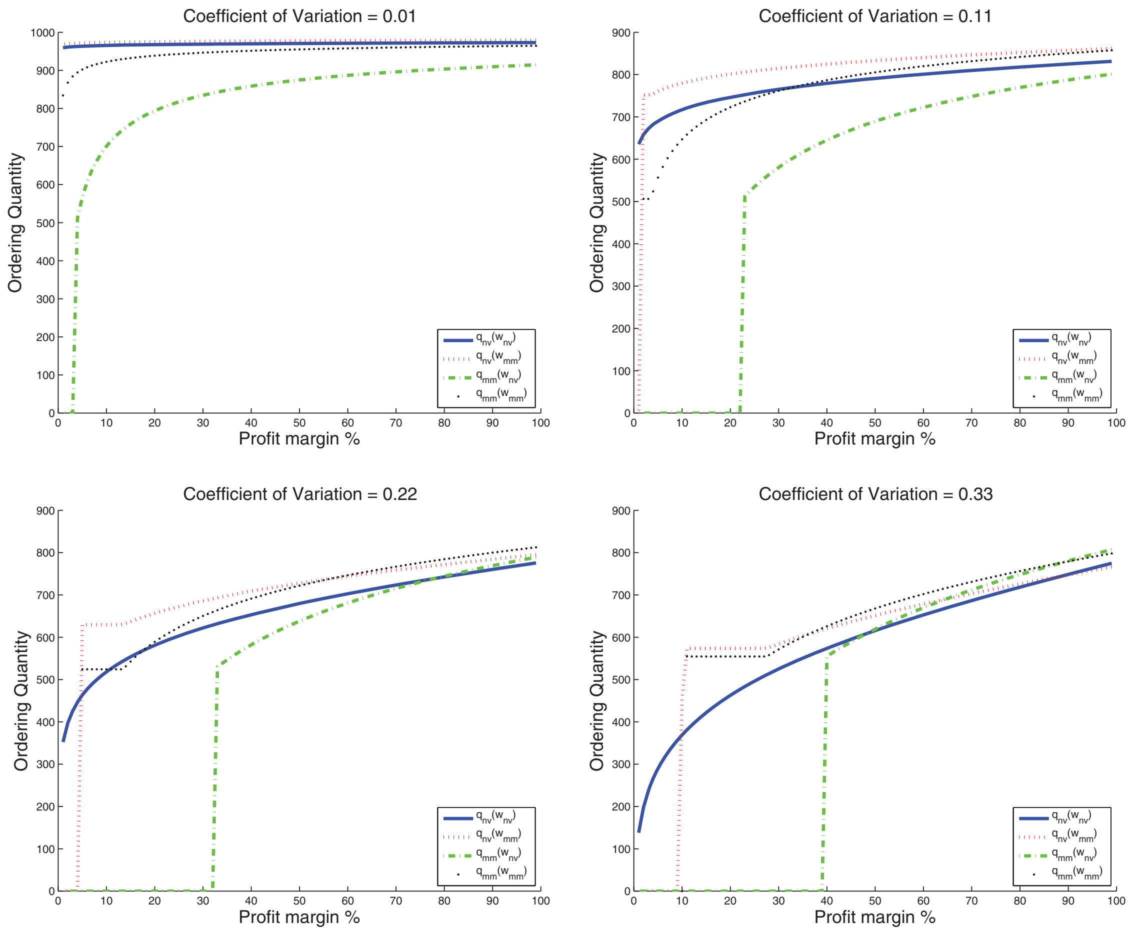

Fig. 2. The terms of trade $q_{nv}(w_{nv})$, $q_{nv}(w_{mm})$, $q_{mm}(w_{nv})$, $q_{mm}(w_{mm})$ for various levels of uncertainty and margin.

$$\Pi_r = r\,E_F[\min\{q(w),\, D\}] + v\,E_F[\max\{q(w) - D,\, 0\}] - wq(w), \tag{7}$$

$$\Pi_s = (w - c)q(w), \tag{8}$$

$$\begin{aligned}\Pi_{sc} &= \Pi_r + \Pi_s \\ &= r\,E_F[\min\{q(w),\, D\}] + v\,E_F[\max\{q(w) - D,\, 0\}] - cq(w),\end{aligned} \tag{9}$$

respectively. The (R_F, S_F) case, where the supplier correctly assesses the informational state of the retailer, serves as our benchmark to study the impact of information availability and asymmetry on individual firm and supply chain profits. In this benchmark, the quantity ordered by the retailer is $q_{nv}(w_{nv})$. The benchmark profits are Equations (7) to (9) evaluated at $w = w_{nv}$ and $q(w) = q_{nv}(w_{nv})$. We compare these profits with those obtained under the other trade terms, as listed in Tables 1 and 2, to determine the impact of information availability and asymmetry in a supply chain.

As can be seen from Tables 1 and 2, the only non-benchmark order quantities are $q_{nv}(w_{mm})$, $q_{mm}(w_{nv})$, and $q_{mm}(w_{mm})$. A large portion of our article consists in comparing these three ordering quantities with $q_{nv}(w_{nv})$ and determining the resulting implications for the retailer's profit, supplier's profit, and total profit of the supply chain. In particular, evaluating Equations (7) to (9) at these wholesale prices and ordering quantities provides the corresponding state-dependent profits, which we can compare with the profits arising in the benchmark case.

Note that it is well documented in the literature (originally by Spengler (1950)) that the benchmark ordering quantity $q_{nv}(w_{nv})$ induces double marginalization, or suboptimal supply chain profits. Indeed, an order quantity of

$$q^* = F^{-1}\left(\frac{r-c}{r-v}\right) \tag{10}$$

induces the maximum expected profit for the entire supply chain, which can be achieved by setting $w = c$ in the decentralized (R_F, S_F) case, i.e., $q_{nv}(c) = q^*$. However, this eliminates all supplier's profit and the retailer's profit is equal to the profit of the supply chain, clearly an infeasible option in the decentralized case, the focus of our article. However, the notion of an optimal ordering quantity of the supply chain, q^*, is useful for subsequent analyses and is formalized here in Equation (10).

3. Asymmetric distributional information in favor of the retailer: $(R_F, S_{\mu,\sigma})$

In this section we consider the case where the retailer has full knowledge of the demand distribution F and the supplier only knows the mean μ and standard deviation σ of the demand. A retailer usually has more access to the final customer and consequently has more data to analyze customer demands, than the supplier. In other words, we consider the $(R_F, S_{\mu,\sigma})$ case and compare it with the benchmark scenario under (R_F, S_F). Under the $(R_F, S_{\mu,\sigma})$ scenario, we focus our study on the case where the supplier mistakenly believes the retailer is under $R_{\mu,\sigma}$, rather than the reality R_F. Alternatively, if the supplier correctly assesses the retailer to be in R_F, the supplier is unable to do much else, since she does not know F, and we do not discuss this situation further; this is the "n/a" in Table 1. Since in both the benchmark (R_F, S_F) and the $(R_F, S_{\mu,\sigma})$ cases the retailer applies $q_{nv}(w)$, the study (in this section) simplifies to comparing w_{nv} with w_{mm}. Finally, note that the analysis in this section also applies to the (R_F, S_F) informational state, where the supplier mistakenly thinks that the retailer is in $R_{\mu,\sigma}$, and the order quantity is also $q_{nv}(w_{mm})$; see Table 2.

In Section 3.1, we analyze Problem (6) and show that the supplier's profit function is strictly concave, which implies that w_{mm} is unique. We next study, in Section 3.2, the effect that the supplier's lack of information has on the total profit of the supply chain, and, in Section 3.3, we analyze the effect on the individual firms.

3.1. *The supplier's decision under limited information*

We now describe the optimization problem the supplier faces to choose an optimal wholesale price, given the information available. First, using the structure of the conjectured retailer ordering quantity (4), the supplier believes that the retailer will order only if the wholesale price is not greater than an upper bound:

$$\frac{r-w}{w-v} \geq \rho^2 \qquad \text{or, equivalently,}$$

$$w \leq \left(\frac{1}{1+\rho^2}\right) r + \left(\frac{\rho^2}{1+\rho^2}\right) v.$$

Next, if the retailer orders, he will order

$$q_{mm}(w) = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{r-w}{w-v}} - \sqrt{\frac{w-v}{r-w}}\right),$$

and the supplier faces the following problem to maximize her profit:

$$\Pi^*_{S_{\mu,\sigma}} = \max_w \, (w-c)\left(\mu + \frac{\sigma}{2}\left(\sqrt{\frac{r-w}{w-v}} - \sqrt{\frac{w-v}{r-w}}\right)\right),$$
$$\text{s.t.}\;\; c \leq w \leq \left(\frac{1}{1+\rho^2}\right) r + \left(\frac{\rho^2}{1+\rho^2}\right) v. \qquad (11)$$

Note that if

$$c > \left(\frac{1}{1+\rho^2}\right) r + \left(\frac{\rho^2}{1+\rho^2}\right) v,$$

the supplier's problem is not feasible. In other words, if the cost of producing a product is high with respect to a variability-weighted average of revenue and salvage values, the supplier will (mistakenly) conclude that the retailer will not be interested in ordering at supplier-feasible wholesale prices. Since this condition only depends on economic primitives, and not any signal from the retailer, the supplier has little evidence to conclude that her belief about the retailer's informational state is wrong. For the remainder of this section we assume that

$$c \leq \left(\frac{1}{1+\rho^2}\right) r + \left(\frac{\rho^2}{1+\rho^2}\right) v.$$

We next discuss the behavior of feasible instances of Problem (11). As the coefficient of variation ρ decreases, resulting in a less restrictive distribution, the upper bound $(1/(1+\rho^2))r + (\rho^2/(1+\rho^2))v$ *increases* (i.e., the feasible space is enlarged). For simplicity we let $v = 0$, and we discuss a few specific demand distributions. Many nonnegative demand distributions are positively skewed, such as the exponential distribution, which has $\rho = 1$; this value of ρ results in a somewhat restrictive upper bound on the wholesale price $w \leq 0.5r$, since the natural upper bound is $w \leq r$. Note that, from the practical point of view, the value $\rho = 1$ is effectively the largest we should consider; products with standard deviations larger than their means are undesirable for many reasons. We next consider distributions with $\rho < 1$, such as the Erlang distribution. For concreteness, let $\rho = 1/2$, which results in the less-restrictive bound $w \leq 0.8r$. Finally, a common model of stochastic demand is the normal distribution, which, to ensure nonnegativity of the random variable with high probability, requires $\mu - \Gamma\sigma \geq 0$ for some value of Γ, where usually $\Gamma \geq 3$; this implies that $\rho \leq 1/3$. Considering a normal distribution with a non-extreme value of ρ, say $\rho = 1/4$, results in the bound $w \leq 0.9412r$, which is reasonably close to the standard assumption that $w \leq r$. In summary, our model becomes less restrictive (i.e., less conservative) as the coefficient of variation ρ decreases and is applicable for all demand distributions except those with very high coefficients of variation.

We next show that the supplier's problem is well behaved. Let

$$\Pi_{S_{\mu,\sigma}}(w) = (w-c)\left(\mu + \frac{\sigma}{2}\left(\sqrt{\frac{r-w}{w-v}} - \sqrt{\frac{w-v}{r-w}}\right)\right)$$

denote the supplier's profit belief, as a function of the wholesale price w. The next result shows that for feasible values of w, the supplier's Problem (11) will have a unique wholesale price solution.

Lemma 1. $\Pi_{S_{\mu,\sigma}}(w)$ *is strictly concave in the wholesale price* w *for* $c \leq w \leq r$.

Using Lemma 1, we can characterize the supplier's optimal wholesale price w_{mm} under incomplete information, as shown in our next result.

Lemma 2. *The supplier's optimal wholesale price* w_{mm} *is the solution to*

$$q_{mm}(w_{mm}) = \frac{\sigma}{4}\frac{(w_{mm}-c)(r-v)^2}{((w_{mm}-v)(r-w_{mm}))^{\frac{3}{2}}},$$

if a solution exists. Otherwise, $w_{mm} = (1/(1+\rho^2))r + (\rho^2/(1+\rho^2))v$.

Lemma 2 shows that the supplier has a unique optimal wholesale price that maximizes her conjectured profit when she believes the retailer acts according to the distribution-free newsvendor problem modeled in Equation (4). This price induces a specific form of $q_{mm}(w)$ that depends on the economics of the situation (r, c, v) as well as the demand moment information (μ, σ).

In the next subsection we analyze the effect of this optimal wholesale price on the entire supply chain. We derive theoretical conditions where this wholesale price increases the profit of the supply chain, reducing the double-marginalization effect (compared with the benchmark (R_F, S_F) scenario). As a result, we also have conditions that characterize the amplification of the double-marginalization effect. Both situations have clear managerial implications, which we discuss in turn.

3.2. *The effect of retailer advantage on supply chain performance*

In this subsection we study the economic and demand conditions where the supplier's lack of distributional information reduces the double-marginalization effect, which increases the supply chain profit. This provides a rigorous rationale for the observation that wholesale price contracts are quite popular in practice, despite their theoretical shortcomings. A lack of information in practice can improve the suboptimality of wholesale price contracts, reducing the appeal of more complicated coordinating contracts (such as a buy-back contract). Our next theorem considers the case where the supplier's optimal wholesale price under full information is large, with respect to the unit revenue r.

Theorem 1. *If*

$$w_{nv} > \left(\frac{1}{1+\rho^2}\right)r + \left(\frac{\rho^2}{1+\rho^2}\right)v,$$

then the supplier's lack of knowledge of F increases the total profit of the supply chain.

The condition

$$w_{nv} > \left(\frac{1}{1+\rho^2}\right)r + \left(\frac{\rho^2}{1+\rho^2}\right)v$$

represents a relatively low-margin environment at optimality for the retailer, which is relaxed as demand uncertainty increases. The increase in the profit of the supply chain occurs because the lack of demand information induces the supplier to choose a lower wholesale price than what would have been chosen under complete information. As is intuitively clear, this lower wholesale price increases the total profit of the supply chain. Consequently, if a retailer expects that a supplier will charge a relatively high wholesale price under complete demand information, then the retailer has no incentive to share information. We follow this theorem with an example for uniformly distributed demand, which allows a closed-form expression for w_{nv}.

Example 1. Let D be uniformly distributed on $[a, b]$, where $a = \mu - \sqrt{3}\sigma$ and $b = \mu + \sqrt{3}\sigma$, so that the mean and standard deviation of the demand are μ and σ, respectively. It is straightforward to see that

$$q_{nv}(w) = F^{-1}\left(\frac{r-w}{r-v}\right) = a + (b-a)\left(\frac{r-w}{r-v}\right).$$

Elementary calculus shows that

$$\begin{aligned} w_{nv} &= \arg\max_{c \leq w \leq r}(w-c)\left(a + (b-a)\left(\frac{r-w}{r-v}\right)\right) \\ &= \frac{r+c}{2} + \frac{a}{b-a}\frac{r-v}{2}. \end{aligned}$$

For additional simplicity, let $\rho = 1/\sqrt{3}$, which reduces $a = 0$. Then, Theorem 1 states that if $c > (r+v)/2$, then the supplier's lack of information increases the supply chain profit. This supports the low-margin discussion above.

Next, we see that even if the supplier does not charge a relatively high wholesale price under complete information, conditions exist where the supplier's lack of information again increases the supply chain profit. ◄

Theorem 2. *If*

$$w_{nv} \leq \left(\frac{1}{1+\rho^2}\right)r + \left(\frac{\rho^2}{1+\rho^2}\right)v$$

and, further, the following two conditions are satisfied:

$$(r - w_{nv}) < (w_{nv} - v) + \frac{(w_{nv}-c)(r-v)^2}{2(w_{nv}-v)(r-w_{nv})}, \quad (12)$$

and

$$\frac{4((w_{nv}-v)(r-w_{nv}))^{\frac{3}{2}}}{(w_{nv}-c)(r-v)^2-2(r-2w_{nv}+v)(w_{nv}-v)(r-w_{nv})} < \rho, \quad (13)$$

then the supplier's lack of knowledge of F increases the profit of the supply chain. Alternatively, if

$$w_{nv} \leq \left(\frac{1}{1+\rho^2}\right) r + \left(\frac{\rho^2}{1+\rho^2}\right) v,$$

and either Equation (12) *or Equation* (13) *is violated, then the profit for the supply chain decreases.*

This theorem shows that even if the supplier were to charge a relatively low wholesale price under full information, which does not satisfy the condition of Theorem 1, there still exist combinations of economic and demand parameters that induce the supplier, with incomplete information, to choose a lower than supplier-optimal wholesale price, which improves the performance of the wholesale price contract for the supply chain. Our theorem clearly contains many subtleties (e.g., w_{nv} depends on F), so we shortly refine our understanding in the context of numerical studies in Section 3.4; for now, we continue with Example 1 to illustrate Theorem 2 for a uniformly distributed demand on $[0, \mu + \sqrt{3}\sigma]$. Our main point is that we are providing theoretical and computational evidence to support a clear managerial message: information can be a double-edged sword in the sense that less information can either increase or decrease the profit of the supply chain.

Example 2. Let D be uniformly distributed on $[0, \mu + \sqrt{3}\sigma]$, where $\rho = 1/\sqrt{3}$ and $w_{nv} = (r + c)/2$. For additional simplicity, let $v = 0$. The three necessary conditions of Theorem 2 simplify to $c \leq r/2$, $0 < c + r^2/(r + c)$, and $\sqrt{r^2 - c^2}(r + c)/(r^2 + rc + c^2) < 1/\sqrt{3}$. Although the first condition is easy to grasp and the second is trivially true, note that even for this greatly simplified problem, the third condition requires further analysis. Letting $f(r) = \sqrt{r^2 - c^2}(r + c)/(r^2 + rc + c^2)$, it is straightforward to see that the derivative $f'(r) = 3rc^2(r + c)/(\sqrt{r^2 - c^2}(r^2 + rc + c^2)^2)$, which is strictly positive for any $r > c$. The first condition of Theorem 2 requires $r \geq 2c$, which implies $f(r) \geq f(2c) = 3\sqrt{3}/7$, which contradicts the third requirement that $f(r) < 1/\sqrt{3}$. Therefore, in this case, the supplier's lack of information decreases the profit for the supply chain. ◀

Unfortunately, we found similar analyses to that of Example 2 for other distributions to be intractable. We instead rely on subsequent simulation studies, in Section 3.4, to build intuition for normally distributed demand. We demonstrate that, in contrast with Example 2, the supplier's lack of information can result in an increase in the profit for the supply chain under normally distributed demand.

3.3. *The effect of retailer advantage on individual firm performance*

In this subsection, we refine our analysis to investigate the effect of the supplier's lack of information on the individual firms. We first consider the supplier. Recall that the retailer, being in state R_F, applies $q_{nv}(w)$ as defined in Equation (2). As previously mentioned, Lariviere and Porteus (2001) showed that, as long as F has an increasing generalized failure rate, the supplier's profit function $(w - c)q_{nv}(w)$ is unimodal with a unique maximizer at w_{nv} (see Equation (5)). Therefore, any deviation from w_{nv} results in suboptimal profits for the supplier; under the $S_{\mu,\sigma}$ informational state, the supplier chooses w_{mm} as the wholesale price, reducing her profits. We have proven the following corollary, which clearly motivates the supplier to seek the information that she does not have.

Corollary 1. *The supplier's lack of knowledge of F decreases her profit.*

We next show that the retailer's profit experiences the same effect as the supply chain; in particular, we prove that there exist environments where the retailer's informational edge increases his profits. However, what is interesting about this result is its second part: a retailer's informational advantage can reduce his profits, a somewhat counterintuitive result. In the next subsection, we provide computational studies to refine our understanding of the environmental characteristics that lead to a motivation for the retailer to either share or hide information.

Corollary 2. *If either of the following two conditions are satisfied*

1. $w_{nv} > \left(\frac{1}{1+\rho^2}\right) r + \left(\frac{\rho^2}{1+\rho^2}\right) v$
or
2. $(r - w_{nv}) < (w_{nv} - v) + \frac{(w_{nv}-c)(r-v)^2}{2(w_{nv}-v)(r-w_{nv})}$ *and* $\frac{4((w_{nv}-v)(r-w_{nv}))^{\frac{3}{2}}}{(w_{nv}-c)(r-v)^2-2(r-2w_{nv}+v)(w_{nv}-v)(r-w_{nv})} < \rho,$

then the supplier's lack of knowledge of F increases the retailer's profit. Otherwise, the retailer's profit decreases.

3.4. *Computational results*

In this section we provide numerical studies that help identify environmental characteristics that indicate whether or not the supplier's lack of information is detrimental to the retailer (it is always detrimental to the supplier, per Corollary 1). These studies will assist us in identifying those environments where the retailer should share his information with the supplier, to increase his profits, as well as identifying those environments where the retailer is motivated to conceal his information, to preserve his higher profits.

Our experimental design is as follows. We consider a unit revenue $r = 100$ and a unit salvage value $v = 0$, and we vary the unit cost c to study different economic

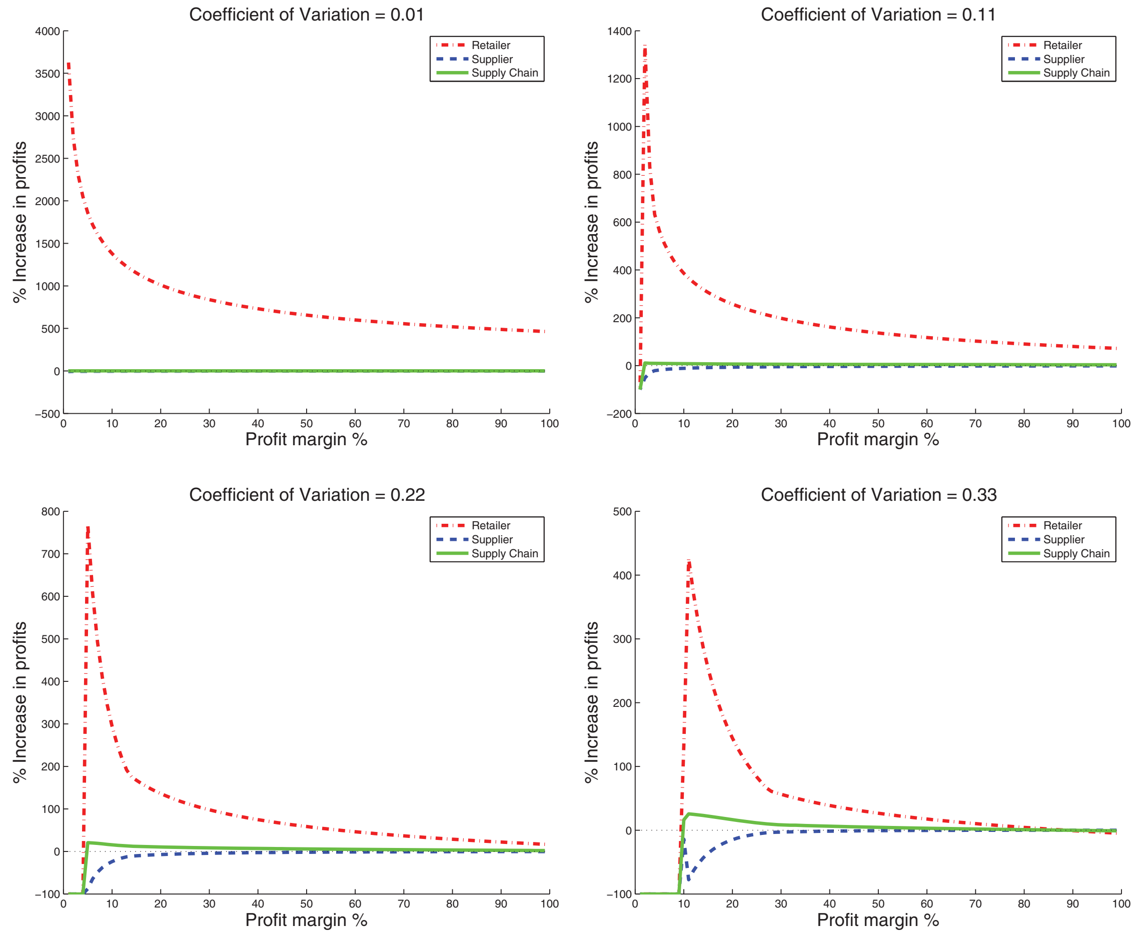

Fig. 3. The percentage increase in profits for various levels of uncertainty and margin under $(R_F, S_{\mu,\sigma})$. Discontinuities at low profit margins are due to the supplier's profit-maximization problem under $S_{\mu,\sigma}$ becoming infeasible.

environments. In particular, we present our results in terms of the supply chain profit margin $(r - c)/r$, which we vary from 1 to 99%. We model uncertain demand as a normal random variable, under different uncertainty environments. Scarf (1958) also utilized a normal distribution in numerically analyzing Problem (3), which approximated non-negative Poisson-distributed demand. We set the mean of demand $\mu = 1000$ and considered coefficients of variation $\rho \in \{0.01, 0.11, 0.22, 0.33\}$, representing extremely low, low, medium, and high uncertainty environments, respectively. We chose a maximum coefficient of variation of 0.33, to allow a three-standard deviation realization below the mean to remain non-negative under a normal distribution of demand; in other words, we have $\mu - 3\sigma \geq 0$, which is satisfied by 99.7% of normal random variable realizations. For each coefficient of variation, in Fig. 3 we plot the percentage change in retailer's profit, supplier's profit, and total profit for the supply chain, as a function of the system's profit margin, with respect to the (R_F, S_F) case where both supplier and retailer have full correct information. In these plots, the horizontal scales are fixed from zero to 100%, but the vertical scales vary to better display the results.

We can see from Fig. 3 that the retailer's profit *usually* increases due to the informational advantage in state $(R_F, S_{\mu,\sigma})$, as compared with (R_F, S_F). We also note that as the coefficient of variation ρ increases (moving from plot

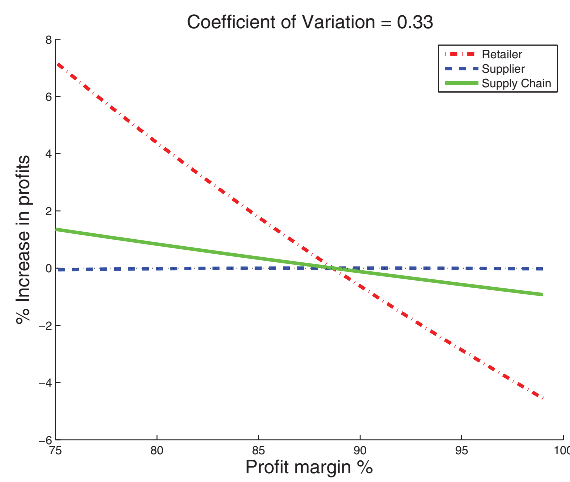

Fig. 4. A closeup view of the retailer losing profit in the $(R_F, S_{\mu,\sigma})$ case.

to plot in Fig. 3) or the system profit margin $(r-c)/r$ increases (moving along the horizontal axis in a given plot), this retailer benefit *decreases*. Furthermore, these trends continue to a point where the retailer's informational advantage translates into an overall *reduction* of his profit, in an environment characterized by high uncertainty ($\rho = 0.33$) and high system profit margin ($(r-c)/r \geq 90\%$); a closeup view of this behavior is presented in Fig. 4. Note that the optimal newsvendor profit is increasing in the mean of the demand and decreasing in the standard deviation (see Hochbaum and Wagner (2015) for a proof in a similar contracting context); therefore, the fact that our retailer's profit is decreasing in the coefficient of variation is not surprising. In contrast, the supplier always loses profit, verifying Corollary 1. However, she experiences a complementary effect to that of the retailer: as the coefficient of variation ρ increases or the system profit margin $(r-c)/r$ increases, the supplier's losses are reduced (but never eliminated).

The reasons for these behaviors can be better understood by examining the supplier's profit-maximization problems. We first consider the definition of w_{nv} in Equation (5), which was studied in depth by Lariviere and Porteus (2001). They showed that increasing ρ drives down w_{nv} (Lemma 1 in their paper). Also, the optimality condition for deriving w_{nv}, in terms of $q = q_{nv} = F^{-1}(1 - w_{nv}/r)$ with $v = 0$, is given in Theorem 1 of their paper as

$$(1 - F(q))\left(1 - \frac{qf(q)}{1 - F(q)}\right) = \frac{c}{r}.$$

Assuming that F has an increasing generalized failure rate, the left-hand side is decreasing in q. Therefore, increasing the system's profit margin $(r-c)/r$ is equivalent to decreasing c/r, thus resulting in an increasing optimal value of q_{nv} or a decreasing optimal value of w_{nv}. Therefore, increasing ρ and increasing $(r-c)/r$ both drive down w_{nv}.

Next, we examine the determination of w_{mm}, namely, Problem (11), reproduced here for $v = 0$:

$$w_{mm} = \arg\max_{w} \,(w - c)\left(\mu + \frac{\sigma}{2}\left(\sqrt{\frac{r-w}{w}} - \sqrt{\frac{w}{r-w}}\right)\right),$$
$$\text{s.t.} \quad c \leq w \leq \left(\frac{1}{1+\rho^2}\right) r. \tag{14}$$

First, an increasing value of ρ reduces the upper bound on feasible wholesale prices. If w_{mm} is an endpoint solution, namely $w_{mm} = (1/(1+\rho^2))r$, then it is reduced as well (cf. Lemma 2); if not, then the effect on w_{mm} is undetermined. Second, a high value of $(r-c)/r$ implies that $c \ll r$, ensuring the feasibility of Problem (14). Furthermore, increasing $(r-c)/r$ also increases the relative weight of the $w-c$ parameter in the objective function, leading to an *increase* in w_{mm}.

Putting these analyses together, increasing ρ and $(r-c)/r$ results in a monotonic decrease in w_{nv} and both increases and decreases in w_{mm}. Clearly, there is more downward pressure on w_{nv} and, if we continue to increase ρ and $(r-c)/r$, at some point w_{nv} will pass below w_{mm}, resulting in a loss of profit for *both* the retailer and supplier.

Finally, note that when $(r-c)/r$ is low and ρ is high, all profits drop to zero, since the supplier will not participate in the supply chain because her profit-maximization problem is infeasible. This is represented by the discontinuities in Fig. 3 and motivates the retailer to share information.

4. Asymmetric distributional information in favor of the supplier: ($R_{\mu,\sigma}$, S_F)

In this section we consider the case where the supplier has full knowledge of the demand distribution F and the retailer only knows the mean μ and standard deviation σ of the demand. This modeling choice would be appropriate when a supplier focuses on a single product and knows its demand patterns well and a partnering retailer sells a large variety of products and does not focus on any given product. As an example, consider a retailer who shares Point-Of-Sale (POS) data with the supplier. A retailer can easily, using built-in functions in most spreadsheet software, estimate the mean and standard deviation of demand from POS data (assuming stockouts are negligible). However, since the retailer sells a large variety of products, he does not necessarily have the motivation to estimate the distribution of demand for all products, since this is a more difficult task. In contrast, since the supplier only sells a single product through the retailer, she is motivated to fully analyze the data to better understand her final customer demand; thus, the supplier is much more likely to create high-quality forecasts (i.e., the distribution F), which could potentially improve the product's flow through the supply chain. In summary, although both firms have access to the POS data, only the supplier is motivated to expend the effort to estimate a distribution, which results in an informational advantage. Notationally, we consider the $(R_{\mu,\sigma}, S_F)$ case and compare it with the benchmark full-information scenario (R_F, S_F).

We first consider, in Section 4.1, the situation where the supplier incorrectly assesses the retailer to be in R_F rather than the reality $R_{\mu,\sigma}$. The supplier offers a wholesale price of w_{nv}, to which the retailer responds by ordering $q_{mm}(w_{nv})$. Since in both the $(R_{\mu,\sigma}, S_F)$ and (R_F, S_F) cases the supplier proposes w_{nv}, the study simplifies to comparing $q_{nv}(w_{nv})$ with $q_{mm}(w_{nv})$.

We then consider, in Section 4.2, the situation where the supplier correctly assesses the retailer to be in $R_{\mu,\sigma}$ and offers a wholesale price of w_{mm}, to which the retailer responds by ordering $q_{mm}(w_{mm})$. We consider this case last, as it is the most complex (both the wholesale price and the ordering curve change simultaneously, with respect to the benchmark case). Finally, note that this analysis is also applicable to the $(R_{\mu,\sigma}, S_{\mu,\sigma})$ case, where the supplier correctly assesses the retailer to be in $R_{\mu,\sigma}$ (see Table 1); if the supplier were to incorrectly assess the retailer to be in R_F, then there is no well-defined model for the supplier to determine an appropriate wholesale price, since she does not know F (the "n/a" in Table 2).

4.1. *Incorrect supplier's assessment of the retailer's information state*

In this subsection, we study the $(R_{\mu,\sigma}, S_F)$ case, where the supplier mistakenly assumes the retailer is in R_F. The supplier proposes w_{nv} and the retailer responds by ordering $q_{mm}(w_{nv})$. As a first step in our analysis, we derive a probabilistic distribution G that, if used as a model for random demand, would induce the equivalence $q_{nv}(w) = q_{mm}(w)$ for all $w \in [c, r]$. We first define this distribution and then prove its important properties.

Definition 1. Let

$$G(d) = \begin{cases} \dfrac{\rho^2}{1+\rho^2}, & 0 \le d < \mu + \dfrac{\sigma}{2}\left(\rho - \dfrac{1}{\rho}\right) \\[2ex] \dfrac{z^2}{1+z^2}, & d \ge \mu + \dfrac{\sigma}{2}\left(\rho - \dfrac{1}{\rho}\right) \end{cases},$$

where $\rho = \sigma/\mu$ and

$$z = \frac{(d-\mu)}{\sigma} + \sqrt{\frac{(d-\mu)^2}{\sigma^2} + 1}.$$

Lemma 3. *If G is the distribution of demand, then $q_{nv}(w) = q_{mm}(w)$, for all $w \in [c, r]$.*

Note that F, rather than G, is the true distribution of the demand. However, comparisons between F and G, and accompanying market size interpretations, will allow us to understand the impact of the informational asymmetry when the supplier has the advantage. In Section 4.1.1 we first study the effect on the profit of the supply chain and in Section 4.1.2 we examine the individual firms. We conclude this section with computational results, to enhance our understanding of these effects, in Section 4.1.3.

4.1.1. *The effect of supplier advantage on supply chain performance*

We begin by considering the profit of the supply chain as a function of q, or

$$\Pi_{sc}(q) = r\,E_F[\min\{q, D\}] + v\,E_F[\max\{q - D, 0\}] - cq. \tag{15}$$

In the following theorem, we show that using the concavity properties of Equation (15), there exists a range of ordering quantities that are superior to $q_{nv}(w_{nv})$ and are attainable under $(R_{\mu,\sigma}, S_F)$. We then combine this knowledge with the distributions F and G, representating two markets, to characterize the environments where the retailer's lack of information improves the profit for the supply chain. Recall that the optimal order quantity for the supply chain was derived in Equation (10) as $q^* = F^{-1}((r-c)/(r-v))$ and note that, by Lemma 3, $G^{-1}((r-w)/(r-v)) = q_{mm}(w)$ for all $w \in [c, r]$.

Theorem 3. *There exists $\tilde{q} > q^*$ such that, if*

$$\underbrace{F^{-1}\left(\frac{r - w_{nv}}{r - v}\right)}_{q_{nv}(w_{nv})} < \underbrace{G^{-1}\left(\frac{r - w_{nv}}{r - v}\right)}_{q_{mm}(w_{nv})} < \tilde{q},$$

then the retailer's lack of knowledge of F increases the profit for the supply chain. Otherwise the supply chain profit decreases.

Although a closed-form expression for $\tilde{q}$ is intractable to derive, Theorem 3 does provide some intuition about the effect of the retailer's lack of information. We can consider F and G as representating two different markets, with G modeling a benchmark market. Let $y = (r - w_{nv})/(r - v)$ denote the critical fractile driving the order quantities under F and G. If a y-proportion of demand occurs at or below a demand threshold under F that is lower than the threshold under G, then we say this submarket under F is smaller than that under G. This results in the retailer's lack of information (unless $q_{mm} > \tilde{q}$) improving the supply chain performance. More loosely stated, the retailer's lack of information is beneficial for the supply chain in small markets. We revisit this result and enhance our understanding of it via computational studies in Section 4.1.3. For now, we continue illustrating our theorems via an example for uniformly distributed demand.

Example 3. Let D be uniformly distributed on $[0, \mu + \sqrt{3}\sigma]$, where $v = 0$, $\rho = 1/\sqrt{3}$, and $w_{nv} = (r + c)/2$. From Example 1, $F^{-1}(1 - w_{nv}/r) = (\mu + \sqrt{3}\sigma)(1 - c/r)/2$. We then calculate

$$\begin{aligned} q_{mm}(w_{nv}) &= G^{-1}\left(\frac{r - w_{nv}}{r - v}\right) \\ &= \mu + \frac{\sigma}{2}\left(\sqrt{\frac{r-c}{r+c}} - \sqrt{\frac{r+c}{r-c}}\right). \end{aligned}$$

Noting that $\mu = \sigma\sqrt{3}$, the first inequality of Theorem 3 can be simplified to

$$0 < \frac{\sqrt{3}c}{r} + \frac{1}{2}\left(\sqrt{\frac{r-c}{r+c}} - \sqrt{\frac{r+c}{r-c}}\right).$$

Even in this very simple case, the conditions of Theorem 3 are complex, so we resort to computational studies, in Section 4.1.3, to gain insight into when the retailer's lack of knowledge increases the level of profit for the supply chain. ◀

4.1.2. *The effect of supplier advantage on an individual firm's performance*

In this subsection, we refine our analysis to investigate the effect of the retailer's lack of information on the individual firms. We first consider the retailer. By using a similar analysis to that in the proof of Theorem 3, we can see that the retailer's profit function is concave and maximized at $q_{nv}(w_{nv})$. Under the $R_{\mu,\sigma}$ informational state, the retailer instead applies $q_{mm}(w_{nv})$. Since, in general, $q_{mm}(w_{nv}) \neq q_{nv}(w_{nv})$, the retailer's profit decreases. This is formalized in the following corollary.

Corollary 3. *The retailer's lack of knowledge of F decreases his profit.*

We next discuss the change in supplier's profits. Recall that in the (R_F, S_F) environment, the supplier's profit is $(w_{nv} - c)q_{nv}(w_{nv})$, whereas in the $(R_{\mu,\sigma}, S_F)$ environment, the supplier's profit is $(w_{nv} - c)q_{mm}(w_{nv})$. Therefore, the supplier's informational advantage translates into more profit if and only if $q_{nv}(w_{nv}) < q_{mm}(w_{nv})$. This observation is formalized in the next corollary in terms of the distributions F and G, allowing an interpretation in terms of submarket size.

Corollary 4. *If*

$$\underbrace{F^{-1}\left(\frac{r - w_{nv}}{r - v}\right)}_{q_{nv}(w_{nv})} < \underbrace{G^{-1}\left(\frac{r - w_{nv}}{r - v}\right)}_{q_{mm}(w_{nv})},$$

then the retailer's lack of knowledge of F increases the supplier's profit. Otherwise, the supplier's profit decreases.

Note that if we assume $\tilde{q} > q_{mm}(w_{nv})$, the conditions of Theorem 3 and Corollary 4 are identical. In other words, the supplier's and supply chain's profits move in tandem as a function of the informational asymmetry. The additional condition in Theorem 3, in terms of $\tilde{q}$, ensures that the increase in supplier's profit is larger than the decrease in the retailer's profit, resulting in a net gain for the supply chain. We can also interpret Corollary 4 in terms of market size. Using arguments identical to those above, smaller submarkets allow the supplier to benefit from her informational advantage. In contrast, the supplier's informational advantage in larger submarkets, beyond a size determined by the benchmark market defined by G, actually results in a loss in supplier's profit. It is in these larger markets that the supplier is motivated to share her information with the retailer.

4.1.3. *Computational results*

In this section we provide numerical studies that help identify environmental characteristics that indicate whether or not the retailer's lack of information is detrimental to the supplier (it is always detrimental to the retailer, per Corollary 3). These studies will assist us in identifying those environments where the supplier should share her information with the retailer, to increase her profits, as well as identifying those environments where the supplier is motivated to conceal her information.

Our experimental design is identical to that described in Section 3.4. For each coefficient of variation, in Fig. 5 we plot the percentage change in retailer's, supplier's, and supply chain's total profit, as a function of the system's profit margin, with respect to the (R_F, S_F) case where both supplier and retailer have full information. In these plots, the horizontal scales are fixed from zero to 100%, but the vertical scales vary to better display the results.

In Fig. 5 we see that the supplier's profit *decreases* in a large variety of scenarios, despite her informational advantage in state $(R_{\mu,\sigma}, S_F)$, as compared with (R_F, S_F). This is at least partially due to the supplier's mistaken characterization of the retailer's informational state. However, despite this mistake, the supplier can still benefit from the retailer's deviation from the optimal order quantity due to limited information. It is when the coefficient of variation ρ is either 0.22 or 0.33, and the profit margin $(r - c)/r$ is larger than a ρ-dependent threshold, that the supplier's informational advantage translates into increased supplier's profits (see bottom two plots in Fig. 5). We also note that as ρ increases or the margin $(r - c)/r$ increases, the supplier's profits increase with respect to those attained in the (R_F, S_F) case. Note that these behaviors are the exact opposite to those found in Section 3.4 for the retailer, who had the informational advantage in that section, coupled with a supplier who mischaracterizes the retailer's informational state. Therefore, the influence of variability and system margin strongly depend on which firm has the informational advantage. Increasing these parameters benefits the supplier when she has the advantage; in contrast, if the retailer has the advantage, increasing these parameters will decrease his profits.

We also see that the retailer loses profit in all environments, with respect to the (R_F, S_F) case, verifying Corollary 3. He experiences a similar effect to that of the supplier: as ρ increases or the margin $(r - c)/r$ increases, the retailer's losses are reduced (but never eliminated). Note that this is again the opposite of the effect observed in Section 3.4, since in that section the retailer's and supplier's profits moved in opposite directions by changing these parameters.

We next explain these observations. The supplier's profit expressions, in the $(R_{\mu,\sigma}, S_F)$ and (R_F, S_F) environments, are $(w_{nv} - c)q_{mm}(w_{nv})$ and $(w_{nv} - c)q_{nv}(w_{nv})$, respectively. Consequently, we only need to compare $q_{mm}(w_{nv})$ and $q_{nv}(w_{nv})$. Recall that in Section 3.4 we showed that increasing ρ and increasing $(r - c)/r$ both drive down w_{nv}, which in turns drives up both $q_{mm}(w_{nv})$ and $q_{nv}(w_{nv})$. Therefore, $q_{mm}(w_{nv})$ increasing at a faster rate than $q_{nv}(w_{nv})$ must drive the improved supplier's benefit from increased ρ and $(r - c)/r$.

We next consider the retailer, who also (usually) benefits from increased ρ and $(r - c)/r$. This can be explained by examining the computational results in deeper detail. For the environments where we observe these improvements, we learn that $q_{mm}(w_{nv}) < q_{nv}(w_{nv})$. Recalling that the retailer's profit function is concave with a maximizer at

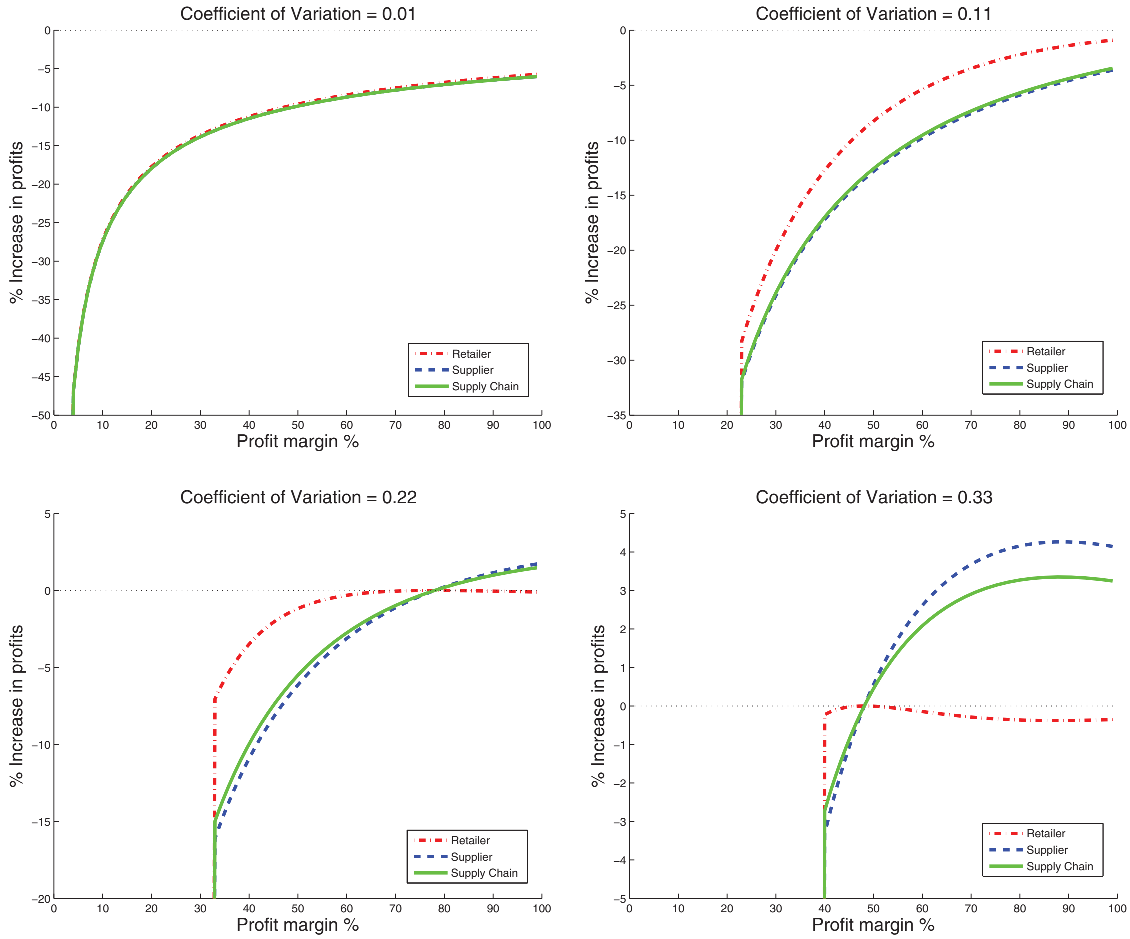

Fig. 5. The percentage increase in profits for various levels of uncertainty and margin under $(R_{\mu,\sigma}, S_F)$ for an *incorrect* supplier's assessment of the retailer's information. Discontinuities at low profit margins are due to the retailer's profit-maximization problem under $R_{\mu,\sigma}$ becoming infeasible.

$q_{nv}(w_{nv})$, since $q_{mm}(w_{nv})$ is increasing faster than $q_{nv}(w_{nv})$, we can conclude that $q_{mm}(w_{nv})$ gets closer to the optimal solution, resulting in an improved retailer's performance. The one exception to this reasoning occurs when $\rho = 0.33$. Increasing $(r - c)/r$ beyond 50% results in a deterioration of the retailer's profits; this corresponds exactly to the case where $q_{mm}(w_{nv}) > q_{nv}(w_{nv})$, and a faster rate of increase for $q_{mm}(w_{nv})$ results in this order quantity moving away from the optimal solution.

Finally, note that when $(r - c)/r$ is low and ρ is high, all profits drop to zero, since the retailer will not participate in the supply chain because his profit-maximization problem is infeasible. This is represented by the discontinuities in Fig. 5 and motivates the supplier to share information.

4.2. *Correct supplier's assessment of the retailer's information state*

In this subsection, we again study the $(R_{\mu,\sigma}, S_F)$ case, except now the supplier *correctly* assumes that the retailer is in $R_{\mu,\sigma}$. The supplier proposes w_{mm} and the retailer responds by ordering $q_{mm}(w_{mm})$. In contrast with previous sections, the current analysis must vary *both* the functional form of the ordering quantity (q_{mm} versus q_{nv}) as well as the wholesale price (w_{mm} versus w_{nv}); in other words, we need to compare $q_{mm}(w_{mm})$ with $q_{nv}(w_{nv})$. Furthermore, we must also combine this comparison analysis with a more-explicit profit analysis, in order to obtain a more complete understanding of this case. Therefore, the analysis in this section is more complex than in previous sections. Fortunately, we can heavily leverage our previous results.

We first study the effect on cumulative supply chain profits in Section 4.2.1. Subsequently, in Section 4.2.2, we analyze the impact on the retailer's and supplier's profits. We conclude with computational studies in Section 4.2.3, which enhance our understanding of the theoretical results. We also note that this analysis is applicable to the $(R_{\mu,\sigma}, S_{\mu,\sigma})$ case, where the supplier correctly assesses the informational state of the retailer; i.e., $q_{mm}(w_{mm})$ is ordered by the retailer.

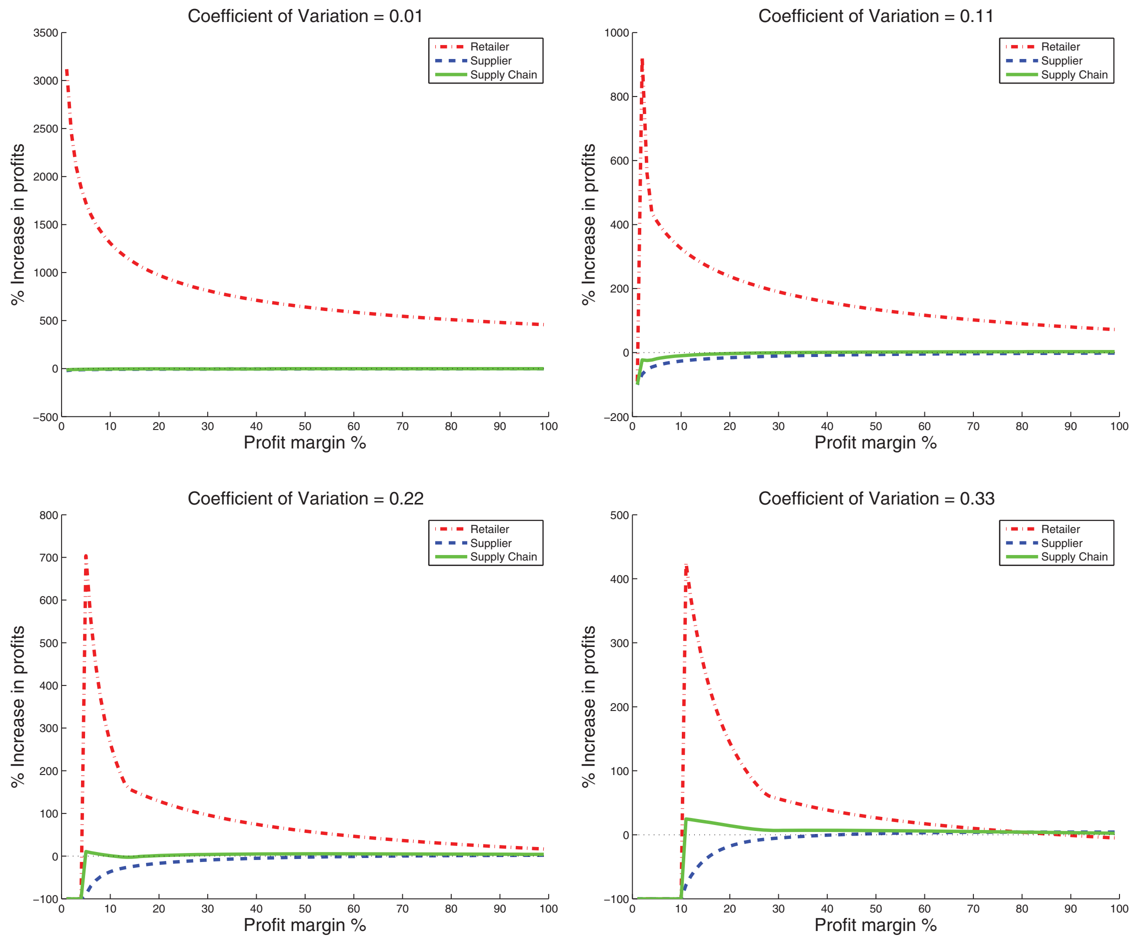

Fig. 6. The percentage increase in profits for various levels of uncertainty and margin under $(R_{\mu,\sigma}, S_F)$ for a *correct* supplier's assessment of the retailer's information. Discontinuities at low profit margins are due to the supplier's and retailer's profit-maximization problems under $S_{\mu,\sigma}$ and $R_{\mu,\sigma}$, respectively, becoming infeasible.

4.2.1. *The effect of supplier advantage on the supply chain's performance*

Using reasoning similar to that of the proof of Theorem 3, the supply chain's profits increase under $(R_{\mu,\sigma}, S_F)$ if $q_{nv}(w_{nv}) < q_{mm}(w_{mm}) < \tilde{q}$, for some $\tilde{q} > q^*$. We formalize this result in the following theorem, in terms of the distributions F and G (see Definition 1), to allow a market size interpretation.

Theorem 4. *There exists $\tilde{q} > q^*$ such that, if*

$$\underbrace{F^{-1}\left(\frac{r - w_{nv}}{r - v}\right)}_{q_{nv}(w_{nv})} < \underbrace{G^{-1}\left(\frac{r - w_{mm}}{r - v}\right)}_{q_{mm}(w_{mm})} < \tilde{q},$$

then the retailer's lack of knowledge of F increases the supply chain's profit. Otherwise, the supply chain's profit decreases.

Recall the discussion of submarkets in the context of Theorem 3: If the submarket defined by F is smaller than a benchmark submarket defined by G, the lack of information increases the supply chain's profits. That discussion also applies here, with one exception: Theorem 4 differs in terms of how the benchmark submarket is defined. In particular, if $w_{mm} < w_{nv}$, then the submarket defined by G is larger than the submarket associated with Theorem 3; conversely, if $w_{mm} > w_{nv}$, then the G submarket is smaller. Therefore, if the lack of information reduces the wholesale price under $(R_{\mu,\sigma}, S_F)$, then the allowable size of the submarket of F, which results in improved profits for the supply chain, *increases*. The opposite effect, namely, decreasing the allowable size of the submarket of F, occurs if wholesale price increases under $(R_{\mu,\sigma}, S_F)$. Note that the conditions for whether or not $w_{mm} < w_{nv}$ can be found in Corollary 2.

4.2.2. *The effect of supplier advantage on an individual firm's performance*

We first consider the supplier. If $(w_{nv} - c)q_{nv}(w_{nv}) < (w_{mm} - c)q_{mm}(w_{mm})$, then the supplier's profit increases under $(R_{\mu,\sigma}, S_F)$. This statement can be written in terms of the distributions F and G, to better compare them with previous results.

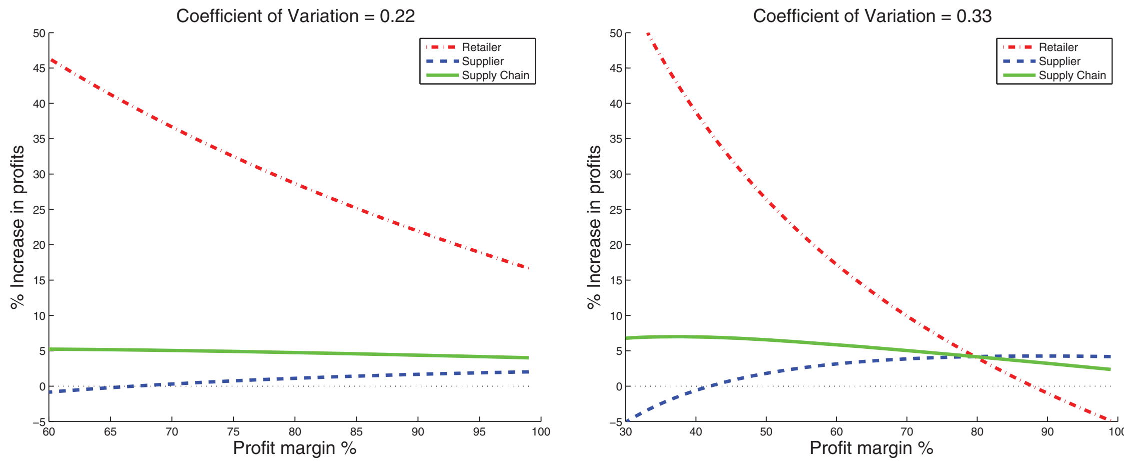

Fig. 7. Expanded views of the bottom two plots of Fig. 6.

Corollary 5. *If*

$$(w_{nv} - c)F^{-1}\left(\frac{r - w_{nv}}{r - v}\right) < (w_{mm} - c)G^{-1}\left(\frac{r - w_{mm}}{r - v}\right),$$

then the retailer's lack of knowledge of F increases the supplier's profit. Otherwise, the supplier's profit decreases.

Corollary 5 is essentially a tautology, but we present it in this form to relate it to our previous results and for completeness. Assuming that $\tilde{q}$ is large enough, if $w_{mm} < w_{nv}$, then the condition for improved profits for the supplier is more restrictive than that for the supply chain, as given in Theorem 4. In contrast, if $w_{mm} > w_{nv}$, the supplier faces a less-restrictive condition that the entire supply chain. This is not entirely surprising, as a higher wholesale price w_{mm} under $(R_{\mu,\sigma}, S_F)$ indeed benefits the supplier but not necessarily the entire supply chain.

We also notice a similarity between Corollary 5 and Corollary 4, where the latter considers the change in supplier's profits when she mistakenly assumes the retailer is in R_F. We show in the computational results of Section 4.2.3 that the supplier's performances for correct and incorrect characterizations of the retailer's informational states are indeed very similar (but not identical). Therefore, we conclude that the assessment abilities of the supplier do not matter much if the retailer lacks information.

We next consider the impact on the retailer's profit and provide two theoretical results, depending on whether or not $w_{mm} < w_{nv}$; note that conditions that determine the relative order of these wholesale prices can be found in Corollary 2. In the first corollary, we see that if the lack of information drives up the wholesale price, then the retailer's profit is reduced. The second, more complex, corollary considers the case where the lack of information drives down the wholesale price and identifies conditions where the retailer's profit increases.

Corollary 6. *If $w_{mm} > w_{nv}$, then the retailer's lack of knowledge of F decreases his profit.*

In order to present our second corollary, recall that $q_{nv}(w_{mm}) = F^{-1}((r - w_{mm})/(r - v))$ denotes the profit-maximizing quantity chosen by the retailer in the $(R_F, S_{\mu,\sigma})$ case (see Section 3).

Corollary 7. *If $w_{mm} < w_{nv}$, then there exists $\bar{q} > q_{nv}(w_{mm})$ such that, if*

$$\underbrace{F^{-1}\left(\frac{r - w_{nv}}{r - v}\right)}_{q_{nv}(w_{nv})} < \underbrace{G^{-1}\left(\frac{r - w_{mm}}{r - v}\right)}_{q_{mm}(w_{mm})} < \bar{q},$$

then the retailer's lack of knowledge of F increases his profit. Otherwise, his profit decreases.

Corollaries 6 and 7 imply an impact on the retailer similar to that in the $(R_F, S_{\mu,\sigma})$ case. Corollary 6 states that, if $w_{mm} > w_{nv}$, then the retailer's profits decrease due to the lack of information in the $(R_{\mu,\sigma}, S_F)$ case, which is identical to the impact under $(R_F, S_{\mu,\sigma})$. Conversely, Corollary 7 states that, if $w_{mm} < w_{nv}$, then the retailer's profit *might* increase under $(R_{\mu,\sigma}, S_F)$, whereas it *will* increase under $(R_F, S_{\mu,\sigma})$ (cf. Corollary 2). The additional condition, beyond requiring $w_{mm} < w_{nv}$, that induces an increase in his profits under $(R_{\mu,\sigma}, S_F)$ is very similar to the stand-alone condition for the supply chain, as given in Theorem 4 (the only difference is replacing $\tilde{q}$ with $\bar{q}$).

4.2.3. *Computational results*

In this section we provide numerical studies that help identify environmental characteristics that indicate whether or not the retailer's lack of information is detrimental to the firm's profits, when the supplier *correctly* assesses the retailer's lack of information.

Our experimental design is identical to that described in Section 3.4. For each coefficient of variation, in Fig. 6 we plot the percentage change in retailer's, supplier's, and supply chain's total profit, as a function of the system's profit margin, with respect to the (R_F, S_F) case where both

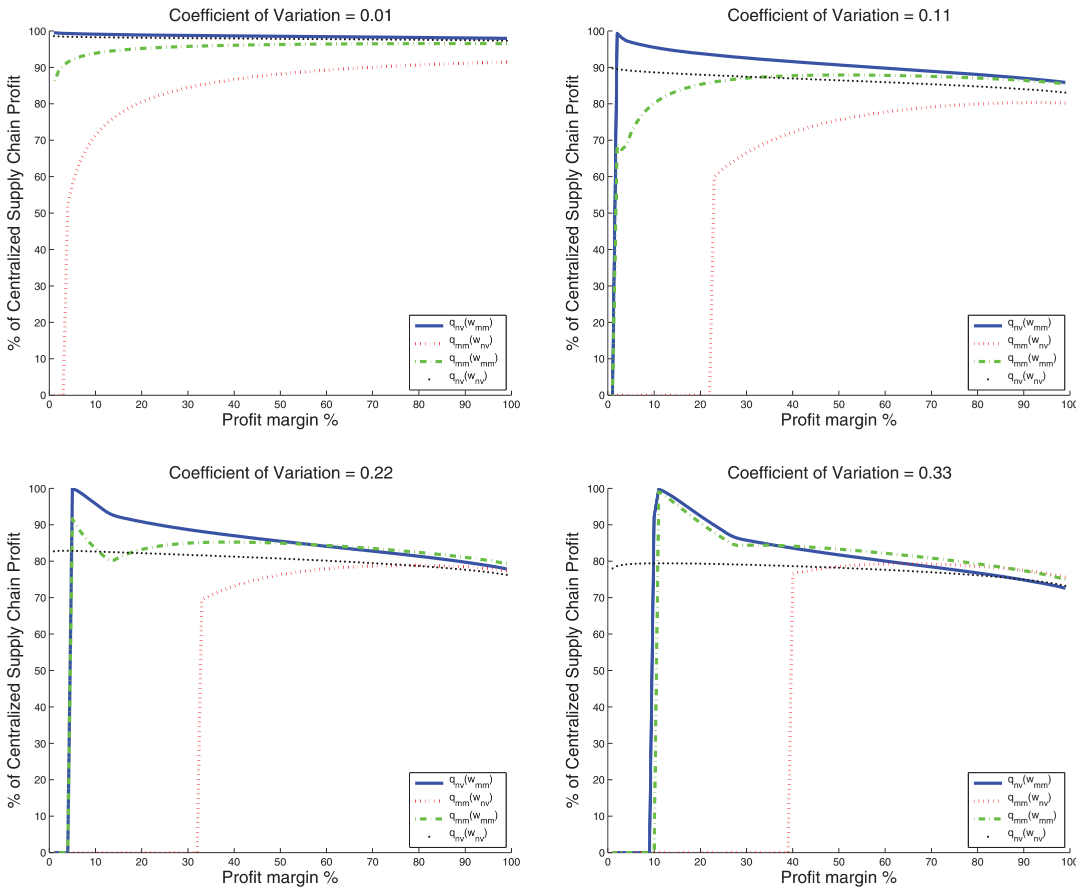

Fig. 8. The percentage of the optimal profit of the optimal centralized supply chain captured by the decentralized supply chains under the different terms of trade for various levels of profit margin and uncertainty.

supplier and retailer have full information. In these plots, the horizontal scales are fixed from zero to 100%, but the vertical scales vary to better display the results.

We first notice that Figs. 3 and 6 closely resemble each other. This is driven by very similar retailer impacts in the $(R_{\mu,\sigma}, S_F)$ and $(R_F, S_{\mu,\sigma})$ cases, which justifies our discussion following Corollaries 6 and 7. However, closer scrutiny of the supplier's performance reveals important differences. In Fig. 3, the supplier always loses profit, as proven in Corollary 1. However, in Fig. 6, the supplier's profit can increase due to the lack of information when $\rho \in \{0.22, 0.33\}$. Furthermore, *both* firms can benefit at the same time from the retailer's lack of information. To characterize the environments where this happens, we must examine the supplier and retailer individually. Note that the supplier's performance improves under $(R_{\mu,\sigma}, S_F)$ when ρ increases and $(r-c)/r$ increases. Conversely, the retailer's performance deteriorates under $(R_{\mu,\sigma}, S_F)$ when ρ increases and $(r-c)/r$ increases. Therefore, for a given value of ρ, there *might* exist a range of $(r-c)/r$ values where both firms have higher profits than the (R_F, S_F) case. Also, note that when both firms benefit, it is possible that the retailer benefits more than the supplier, from a percentage increase perspective, as well as *vice versa*. Figure 7 shows expanded views of the appropriate portions of the bottom two plots of Fig. 6 to clearly display these results.

We finally note that this analysis is also applicable to the $(R_{\mu,\sigma}, S_{\mu,\sigma})$ case, where the supplier correctly assesses the informational state of the retailer ($q_{mm}(w_{mm})$ is ordered by the retailer). This means that, despite the fact that both firms lack distributional knowledge of the demand, it is possible that both firms can obtain higher profits from this lack of information, with respect to the full informational case (R_F, S_F). We next provide an explanation of this observation. Although it is perhaps counterintuitive that less information results in better performance, it is the Stackelberg game interaction that allows this possibility. Recall that, under the full-informational case (R_F, S_F), double marginalization is present due to the firms' incentives being misaligned with that of the supply chain. Under the $(R_{\mu,\sigma}, S_{\mu,\sigma})$ case, the firms' incentives are still misaligned, but *differently* than in the (R_F, S_F) case, due to the lack of information. We have shown that these misdirected incentives under $(R_{\mu,\sigma}, S_{\mu,\sigma})$ can be corrective, collectively resulting in increased profit for both the supply chain and the individual firms. As discussed above,

Fig. 7 displays the environments where these appealing scenarios occur, namely, for medium- to high-uncertainty environments: $\rho \in \{0.22, 0.33\}$. Therefore, in our model, both firms knowing the full distribution of demand under relatively high uncertainty is inferior to both firms knowing only the mean and standard deviation (for medium to high profit margins).

5. Comparison with profits for on optimal centralized supply chain

In this section we explicitly compare the profits for a decentralized supply chain, under the four informational scenarios $(R_F, S_{\mu,\sigma})$, $(R_{\mu,\sigma}, S_F)$, $(R_{\mu,\sigma}, S_{\mu,\sigma})$, (R_F, S_F), with the profits for the optimal *centralized* supply chain, which are driven by the order quantity $q^* = F^{-1}((r - c)/(r - v))$; cf. the discussion surrounding Equation (10). Note that our benchmark is no longer the full-informational (R_F, S_F) case, which is a decentralized supply chain, but rather a centralized system (without any double marginalization). In Fig. 8 we plot the ratios of the profits of the decentralized supply chain to the profits of the centralized supply chain, as a function of the profit margin $(r - c)/r$ for $\rho \in \{0.01, 0.11, 0.22, 0.33\}$, for all terms of trade.

Whenever the curves corresponding to the $q_{nv}(w_{mm})$, $q_{mm}(w_{nv})$, or $q_{mm}(w_{mm})$ terms of trade are above the curve for $q_{nv}(w_{nv})$, the lack of information increases the total profit of the supply chain; these results are consistent with previous graphs. However, we can now see more clearly that the improvement can be rather substantial, when measured with respect to the centralized supply chain. For example, consider the $(R_F, S_{\mu,\sigma})$ informational case (the solid lines corresponding to $q_{nv}(w_{mm})$) for $\rho = 0.22$ (bottom left plot) when the profit margin is 10%: the decentralized supply chain captures 95.8% of the total possible profit, despite the supplier not having full information. In contrast, the dotted curve for the (R_F, S_F) case indicates that if all firms had full information, only 82.7% of the total possible profit is captured. Similar examples can be found throughout Fig. 8, providing further evidence for the appeal of wholesale price contracts under limited information.

6. Conclusions

In this article we considered a supply chain consisting of a retailer and supplier, which interact via a wholesale price contract and collectively face uncertain final customer demand. We design a framework to study how asymmetric and incomplete demand information affects the profits of an individual firm and the cumulative profit of the supply chain. We showed that in an asymmetric informational state, where one firm has an informational advantage, the disadvantaged firm usually, but not always, loses profit. We also showed that an informational advantage does not necessarily lead to an increase in profit and can instead reduce profit. Practically speaking, our research identifies environments where a supplier or retailer is motivated to share, hide, or seek information, insights that are valuable to managers at various stages in a supply chain. In particular, with respect to the full-information case, we showed the following situations can occur.

If the retailer has full distributional knowledge about demand and the supplier only knows the mean and variance:

1. The retailer's profit will increase due to the informational advantage, *except* when demand variability is high and product margin is high, in which case the retailer is motivated to share his information with the supplier. More generally, the retailer's profit decreases as demand variability increases and/or product margin increases.
2. The supplier's profit will always decrease. However, this decrease is mitigated as demand variability increases and/or product margin increases.
3. We show that this analysis also applies to the case where both firms have full distributional knowledge, but the supplier mistakenly assumes that the retailer only knows the mean and standard deviation.

If the supplier has full distributional knowledge about demand, the retailer only knows the mean and variance, and the supplier *incorrectly* assumes that the retailer has full distributional knownledge:

1. The supplier's profit will decrease due to the informational advantage, *except* when demand variability is high and product margin is high, in which case the supplier is motivated to hide her information from the retailer. More generally, the supplier's profit increases as demand variability increases and/or product margin increases.
2. The retailer's profit will always decrease, an effect that is mitigated when demand variability increases and/or product margin increases.

If the supplier has full distributional knowledge about demand, the retailer only knows the mean and variance, and the supplier *correctly* assesses the retailer's informational state:

1. The retailer's profit will increase, except when demand variability is high and product margin is high, as in the first case.
2. The supplier's profit will decrease, except when demand variability is high and product margin is high, as in the second case.
3. It is possible that neither, either or both of the firms benefit from the retailer's lack of information.
4. We show that this analysis is also applicable to the case where both the supplier and retailer only know the mean and standard deviation of the demand.

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Appendix

Proofs

Let

$$\Pi_s(w) = (w - c)q_w \quad \text{and}$$
$$\Pi_r(w) = (r - w)q_w - (r - v)\int_0^{q_w} F(x)dx$$

denote the supplier's and retailer's profits, respectively, as a function of the wholesale price w, where $q_w \triangleq q(w)$ is the retailer's generic response to the supplier's price w (in contrast to the optimal newsvendor response). The retailer's profit expression is obtained from Equation (7), using integration by parts and noting that $F(0) = 0$. The total profit of the supply chain as a function of w, denoted $\Pi_{sc}(w)$, is simply the sum of the retailer's and supplier's profits:

$$\begin{aligned}\Pi_{sc}(w) &= \Pi_s(w) + \Pi_r(w)\\ &= (r - c)q_w - (r - v)\int_0^{q_w} F(x)dx. \end{aligned}\tag{A1}$$

The next lemma will be useful in our subsequent proofs.

Lemma A1. *The derivatives of the total profit of the supply chain and the retailer's and supplier's profits are*

$$\frac{\partial \Pi_{sc}(w)}{\partial w} = \frac{\partial q_w}{\partial w}\left[(r - c) - (r - v)F(q_w)\right],$$
$$\frac{\partial \Pi_r(w)}{\partial w} = \frac{\partial q_w}{\partial w}\left[(r - w) - (r - v)F(q_w)\right] - q_w,$$
$$\frac{\partial \Pi_s(w)}{\partial w} = (w - c)\frac{\partial q_w}{\partial w} + q_w,$$

respectively.

Proof of Lemma A1. Using the Leibniz derivative rule, the derivative of the total profit of the supply chain satisfies

$$\frac{\partial \Pi_{sc}}{\partial w} = (r-c)\frac{\partial q_w}{\partial w} - (r-v)\frac{\partial q_w}{\partial w}F(q_w) = \frac{\partial q_w}{\partial w}\left[(r-c)-(r-v)F(q_w)\right],$$

and the derivative of the retailer's profit satisfies

$$\frac{\partial \Pi_r}{\partial w} = (r-w)\frac{\partial q_w}{\partial w} - q_w - (r-v)\frac{\partial q_w}{\partial w}F(q_w) = \frac{\partial q_w}{\partial w}\left[(r-w)-(r-v)F(q_w)\right] - q_w.$$

Finally, the derivative of the supplier's profit function is the difference between those of the supply chain and retailer, namely,

$$\frac{\partial \Pi_s}{\partial w} = (w-c)\frac{\partial q_w}{\partial w} + q_w.$$

This completes the proof. ■

Proof of Lemma 1. Define

$$S(w) = \sqrt{\frac{r-w}{w-v}} - \sqrt{\frac{w-v}{r-w}}.$$

$\Pi_{S_{\mu,\sigma}}(w)$ is strictly concave iff $\Pi''_{S_{\mu,\sigma}}(w) < 0$. If $\Pi_{S_{\mu,\sigma}}(w) = f(w)g(w)$, then $\Pi'_{S_{\mu,\sigma}} = fg' + f'g$ and $\Pi''_{S_{\mu,\sigma}} = fg'' + 2f'g' + f''g$; assigning $f(w) = w-c$ and $g(w) = \mu + (\sigma/2)S(w)$, we see that $f' = 1$, $f'' = 0$, $g' = (\sigma/2)S'$ and $g'' = (\sigma/2)S''$. Therefore,

$$\Pi''_{S_{\mu,\sigma}} = (w-c)\frac{\sigma}{2}S''(w) + \sigma S'(w).$$

Noting that

$$\frac{\partial}{\partial w}\left(\frac{r-w}{w-v}\right)^{\frac{1}{2}} = -\frac{1}{2}\frac{(r-v)(r-w)}{((r-w)(w-v))^{\frac{3}{2}}} \quad \text{and}$$

$$\frac{\partial}{\partial w}\left(\frac{w-v}{r-w}\right)^{\frac{1}{2}} = \frac{1}{2}\frac{(r-v)(w-v)}{((r-w)(w-v))^{\frac{3}{2}}},$$

we have that

$$S'(w) = -\frac{1}{2}\frac{(r-v)^2}{((w-v)(r-w))^{\frac{3}{2}}} \quad \text{and}$$

$$S''(w) = \frac{3}{4}\frac{(r-v)^2(r-2w+v)}{((w-v)(r-w))^{\frac{5}{2}}}.$$

We want $\Pi''_{S_{\mu,\sigma}} = (w-c)\frac{\sigma}{2}S''(w) + \sigma S'(w) < 0$, which is equivalent to

$$\frac{\frac{3}{4}(w-c)(r-v)^2(r-2w+v) - (r-v)^2(w-v)(r-w)}{((w-v)(r-w))^{\frac{5}{2}}} < 0.$$

Since the denominator is positive, examining the numerator, we require

$$\frac{3}{4}(w-c)(r-2w+v) < (w-v)(r-w),$$

which is easily seen by

$$\begin{aligned}(w-c)\frac{3}{4}(r-2w+v) &= (w-c)\frac{3}{4}(r-w-(w-v)) \\ &< (w-v)((r-w)-(w-v)) \\ &< (w-v)(r-w).\end{aligned}$$

This completes the proof. ■

Proof of Lemma 2. Define

$$w_{UB} \triangleq \left(\frac{1}{1+\rho^2}\right)r + \left(\frac{\rho^2}{1+\rho^2}\right)v,$$

to denote the upper bound on feasible wholesale prices. Lemma 1 shows that a unique solution exists to the supplier's Problem (11). Using the notation of Lemma 1's proof, we can write that

$$\frac{\partial \Pi_{S_{\mu,\sigma}}(w)}{\partial w} = (w-c)\frac{\sigma}{2}S'(w) + \mu + \frac{\sigma}{2}S(w) = (w-c)\frac{\sigma}{2}S'(w) + q_{mm}(w).$$

Consequently, $\partial\Pi_{S_{\mu,\sigma}}(w=c)/\partial w = q_{mm}(c) > 0$ and the optimal wholesale price can occur either (i) where the derivative is equal to zero or (ii) at an endpoint of the feasibility interval $[c, w_{UB}]$. If $\partial\Pi_s(w=w_{UB})/\partial w > 0$, the supplier's profit is still increasing at the upper limit of feasible wholesale prices, and the supplier chooses w_{UB} as the optimal wholesale price. Alternatively, if $\partial\Pi_s(w=w_{UB})/\partial w \le 0$, there exists a wholesale price $w_{mm} \in [c, w_{UB}]$ where the derivative is zero and the profit is maximized; the derivative is zero precisely where

$$q_{mm}(w) = -(w-c)\frac{\sigma}{2}S'(w) = (w-c)\frac{\sigma}{4}\frac{(r-v)^2}{((w-v)(r-w))^{\frac{3}{2}}}.$$

This completes this proof. ■

Proof of Theorem 1. If $w_{nv} > w_{UB}$, then $w_{nv} > w_{mm}$ since, by definition, $w_{mm} \le w_{UB}$. The retailer, having full information, will react according to the newsvendor model, ordering $q(w) = F^{-1}((r-w)/(r-v))$ for $w \in \{w_{mm}, w_{nv}\}$. By Lemma A1, since $\partial q(w)/\partial w = -1/(r-v)f(q(w))$, the derivative of the supply chain's profit is

$$\frac{\partial \Pi_{sc}(w)}{\partial w} = -\frac{(w-c)}{(r-v)f(q(w))} < 0.$$

Therefore, the profit of the supply chain is decreasing in the wholesale price and the supplier's choice of w_{mm} increases the total profit of the supply chain, compared with w_{nv}, since $w_{mm} < w_{nv}$. This completes the proof. ■

Proof of Theorem 2. First note that $w_{nv} \in [c, w_{UB}]$. Lemmas 1 and 2, and their proofs, show that, if the supplier's

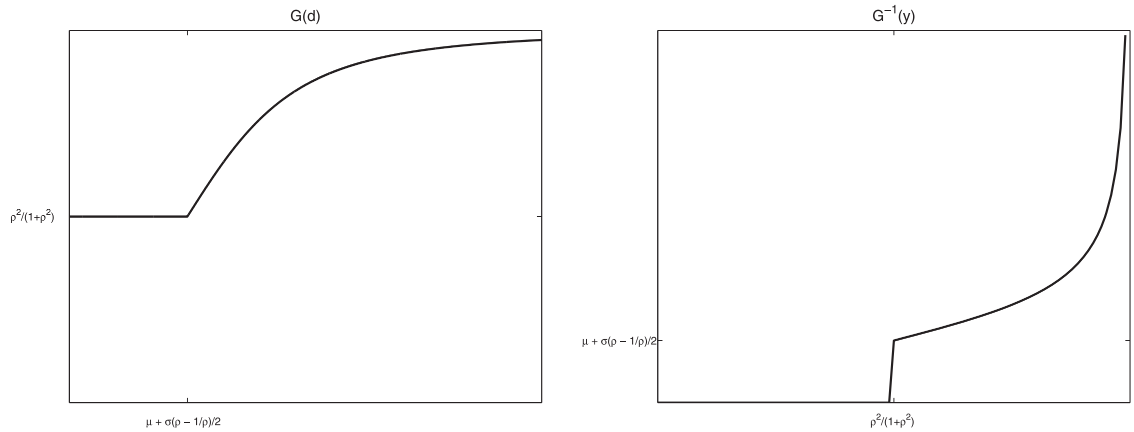

Fig. A1. Graphical representation of $G(d)$ and its inverse $G^{-1}(y) = q_{mm}(w)$ where $y = (r - w)/(r - v)$.

derivative of profit evaluated at w_{nv},

$$(w_{nv} - c)\frac{\sigma}{2}S'(w_{nv}) + q_{mm}(w_{nv}), \tag{A2}$$

is negative, then the supplier's optimal (under incomplete information) wholesale price w_{mm} is strictly less than w_{nv}, which increases the total profit of the supply chain. Expanding and rearranging the negativity condition on Equation (A2), we get

$$\begin{aligned} &-(w_{nv} - c)\frac{\sigma}{2}\frac{1}{2}\frac{(r-v)^2}{((w_{nv} - v)(r - w_{nv}))^{\frac{3}{2}}} + \mu \\ &+\frac{\sigma}{2}\frac{(r - 2w_{nv} + v)}{\sqrt{(w_{nv} - v)(r - w_{nv})}} < 0 \\ \Leftrightarrow\ &\frac{1}{\rho} < \frac{1}{4}\frac{(w_{nv}-c)(r-v)^2 - 2(r-2w_{nv}+v)(w_{nv}-v)(r-w_{nv})}{((w_{nv}-v)(r-w_{nv}))^{\frac{3}{2}}}. \end{aligned}$$

If the numerator of the right-hand side is negative, the inequality can never hold; consequently, the positivity of the numerator is a necessary condition for the total profit of the supply chain to increase; this positivity is represented in Condition (12). Finally, the inequality can be arranged to give the lower bound on the coefficient of variation, given in Condition (13). This completes the proof. ■

Proof of Corollary 2. Recall that the retailer, who has full knowledge of the distribution F, will apply the newsvendor solution $q_{nv}(w) = F^{-1}((r - w)/(r - v))$ for any supplier's wholesale price w. By Lemma A1 and the newsvendor behavior $\partial q_{nv}(w)/\partial w = -1/(r - v) f(q_{nv}(w))$, we conclude that $\partial \Pi_r(w)/\partial w = -q_{nv}(w)$. Therefore, the retailer's profit experiences the same effect as the entire supply chain and increases if $w_{mm} < w_{nv}$. This completes the proof. ■

Proof of Lemma 3. We need to a find a distribution G where $q_{nv}(w) = q_{mm}(w)$ for all $w \in [c, r]$ or, equivalently,

$$G^{-1}\left(\frac{r-w}{r-v}\right) = \begin{cases} 0, & \frac{r-w}{w-v} < \rho^2 \\ \mu + \frac{\sigma}{2}\left(\sqrt{\frac{r-w}{w-v}} - \sqrt{\frac{w-v}{r-w}}\right), & \frac{r-w}{w-v} \geq \rho^2 \end{cases}.$$

We apply the change of variable $y = (r - w)/(r - v)$, so that our search simplifies to finding G such that

$$G^{-1}(y) = \begin{cases} 0, & y < \frac{\rho^2}{1+\rho^2} \\ \mu + \frac{\sigma}{2}\left(\sqrt{\frac{y}{1-y}} - \sqrt{\frac{1-y}{y}}\right), & y \geq \frac{\rho^2}{1+\rho^2} \end{cases}.$$

Note that there is a point of discontinuity at $y = \rho^2/(1 + \rho^2)$, and the value of the inverse is

$$G^{-1}\left(\frac{\rho^2}{1+\rho^2}\right) = \mu + \frac{\sigma}{2}\left(\rho - \frac{1}{\rho}\right),$$

which defines the discontinuity threshold for $G(d)$.

Next, for the range where $y \geq \rho^2/(1 + \rho^2)$, we simply solve for y to derive the original function $y = G(d)$ for $d \geq \mu + (\sigma/2)(\rho - (1/\rho))$. Since $d = G^{-1}(y)$, we have that

$$\begin{aligned} d = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{y}{1-y}} - \sqrt{\frac{1-y}{y}}\right) &\Leftrightarrow 2\left(\frac{d-\mu}{\sigma}\right) \\ &= \sqrt{\frac{y}{1-y}} - \sqrt{\frac{1-y}{y}} \\ &\Leftrightarrow \alpha = z - \frac{1}{z} \\ &\Leftrightarrow 0 = z^2 - \alpha z - 1, \end{aligned}$$

where the third equivalence is due to the changes of variable $\alpha = 2(d - \mu)/\sigma$ and $z = \sqrt{y/(1 - y)}$. The roots of the quadratic are $z = (\alpha \pm \sqrt{\alpha^2+4})/2$. Since z is defined as a square root, it must be positive and we discard the negative root and conclude that

$$z = \frac{\alpha + \sqrt{\alpha^2+4}}{2} = \left(\frac{d-\mu}{\sigma}\right) + \sqrt{\left(\frac{d-\mu}{\sigma}\right)^2 + 1}.$$

Solving for y gives $y = z^2/(1+z^2)$, which establishes the structure of $G(d)$ for $d \geq \mu + (\sigma/2)(\rho + (1/\rho))$. For $y < \rho^2/(1+\rho^2)$, or $d < \mu + (\sigma/2)(\rho + (1/\rho))$, $G(d)$ is constant at the value $\rho^2/(1+\rho^2)$. Figure A1 gives a graphical representation of this proof.

Proof of Theorem 3. Equation (15) can be written, using integration by parts and noting that $F(0) = 0$, as

$$\Pi_{sc}(q) = (r-c)q - (r-v)\int_0^q F(x)dx.$$

The second derivative of $\Pi_{sc}(q)$ is $-(r-v)f(q)$, establishing the concavity of the profit function. The optimal ordering quantity for the supply chain $q^* = F^{-1}((r-c)/(r-v))$ (given in Equation (10)) maximizes the profit of the supply chain.

The ordering quantity under (R_F, S_F), namely, $q_{nv}(w_{nv})$, is suboptimal for the supply chain (i.e., it induces double marginalization). Clearly, any $q \in (q_{nv}(w_{nv}), q^*]$ induces a higher profit for the supply chain than $q_{nv}(w_{nv})$, due to the concavity of the profit function. For $q > q^*$, the profit of the supply chain decreases and there exists a point $\tilde{q}$ where the profit passes below that for $q_{nv}(w_{nv})$. Therefore, for $q \in (q_{nv}(w_{nv}), \tilde{q}]$, the profit of the supply chain is higher than the profit induced under $q_{nv}(w_{nv})$. Finally, applying the definitions of $q_{nv}(w_{nv})$ and G, the proof is complete.

Proof of Corollary 6. Let

$$\Pi_r(q(w)) = (r-w)q(w) - (r-v)\int_0^{q(w)} F(x)dx \tag{A3}$$

denote the retailer's expected profit when the wholesale price is w and the ordering quantity $q(w)$ is a function of w. We next show that, if $w_{mm} > w_{nv}$, then the profit under $(R_{\mu,\sigma}, S_{\mu,\sigma})$ is less than that under (R_F, S_F); mathematically,

$$\begin{aligned}
\Pi_r(q_{mm}(w_{mm})) &= (r-w_{mm})q_{mm}(w_{mm}) \\
&\quad -(r-v)\int_0^{q_{mm}(w_{mm})} F(x)dx \\
&< (r-w_{nv})q_{mm}(w_{mm}) - (r-v)\int_0^{q_{mm}(w_{mm})} \\
&\quad \times F(x)dx \qquad (\text{since } w_{nv} < w_{mm}) \\
&\leq (r-w_{nv})q_{nv}(w_{nv}) - (r-v)\int_0^{q_{nv}(w_{nv})} \\
&\quad \times F(x)dx \qquad (\text{by optimality of } q_{nv}(w_{nv})) \\
&= \Pi_r(q_{nv}(w_{nv})),
\end{aligned}$$

and the proof is complete.

Proof of Corollary 7. Note that $w_{mm} < w_{nv}$ implies $q_{nv}(w_{mm}) > q_{nv}(w_{nv})$. Now, consider the retailer's profit, with $w = w_{mm}$, as a function of q, namely,

$$\Pi_r(q) = (r-w_{mm})q - (r-v)\int_0^q F(x)dx.$$

This function is concave in q, with a maximizer at $q_{nv}(w_{mm})$. As in the proof of Theorem 3, there exists a $\bar{q} > q_{nv}(w_{mm})$, which induces the same profit as $q_{nv}(w_{nv})$ ($\bar{q}$ plays the role of $\tilde{q}$ and $q_{nv}(w_{nv})$ plays the role of q^*). Therefore, if $q \in (q_{nv}(w_{nv}), \bar{q})$, then $\Pi_r(q) > \Pi_r(q_{nv}(w_{nv}))$; conversely, if $q < q_{nv}(w_{nv})$ or $q > \bar{q}$, we have that $\Pi_r(q) < \Pi_r(q_{nv}(w_{nv}))$. Setting $q = q_{mm}(w_{mm})$, and substituting in the distributional forms of the order quantities, completes the proof of the corollary.

Biography

Michael R. Wagner is an assistant professor of operations management at the Michael G. Foster School of Business at the University of Washington. His research interests are in decision making under uncertainty (stochastic, online, and robust optimization, as well as hybrids thereof). In particular, he investigates the value of information in a variety of application domains, such as supply chain management, inventory control, resource allocation, and logistics.