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Incentive Contracts in Serial Stochastic Projects

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In this paper we propose an incentive payment contract for stochastic projects defined by a series of stages or tasks that are outsourced to independent subcontractors. Projects defined by sequentially completed independent stages are common in new product development and other high-risk projects. Our goal is to maximize the client's expected discounted profit. Our proposed contract reflects the convex time-cost trade-off that is well known in the project scheduling literature. We show that this type of contract dominates a fixed price contract with respect to expected client's profit and schedule performance, regardless of payment timing considerations. Using a piecewise linear approximation, we show that our contract is a generalization of an incentive/disincentive contract that is frequently used in practice. We show how our contract can be used to find the optimal due date and penalties/bonuses in an incentive/disincentive contract. We compare this contract with several variations and discuss implications for both the client and subcontractors.

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1. Introduction

The importance and complexity of strategic projects have increased greatly in recent years; these projects include many new product development (NPD) projects as well as infrastructure and information technology projects. The failure of many of these projects to meet their stated goals appears to be widespread; Flyvbjerg et al. (2002) reported that 90% of 258 transportation infrastructure projects they studied in Europe, North America, and 10 developing nations exceeded their estimated cost by an average of 28%. With respect to NPD projects, Tatikonda and Rosenthal (2000) reported that the "average company ... had achieved the objectives for past development projects only to a low or moderate extent" (p. 78). Empirical evidence gathered by the Standish Group (2009) indicates that less than 35% of recent information technology projects could be classified as successful. As a result, researchers and practitioners have examined the allocation of incentives and risks between clients and subcontractors in various types of contracts (Dayanand and Padman 2001) in an attempt to increase the likelihood of project success.

Empirical evidence comparing contract type and project outcome is limited but generally supports the conclusion that incentive contracts can have a significant and positive impact on project outcomes (Meng and Gallagher 2012). Several case studies (Bubshait 2003, Berends 2000) support these conclusions. Many state transportation departments use incentive/disincentive (I/D) contracts, including the Washington State Department of Transportation (WSDOT) that reports it has used I/D contracts with favorable outcomes to reward subcontractors for early completion of a project phase and/or penalize a subcontractor for late completion or failure to meet quality standards (Walker 2010).

To better understand how to improve outcomes in real-world projects, we have been studying the design and implications of various contracts and propose a new contract that we denote an incentive payment contract. This contract generalizes several types of contracts observed in practice today and allows tractable analysis. Our analysis is based on a project that consists of $n \geq 1$ sequential stages where the duration of each stage is characterized by a nonnegative random variable. Many organizations structure large risky projects as a series of sequential subprojects or stages with numerous review points (or "stage gates") between stages (Santiago and Vakili 2005) and outsource many or all of these stages to subcontractors. In this way, organizations can focus on managing the overall project and leave specialized functions to experienced subcontractors.

Following previous research (Kwon et al. 2010b), we model this process as a Stackelberg game and assume that the client subcontracts the work at each stage to an independent subcontractor and sets the contract terms (i.e., payment amounts and timing) with each subcontractor at the start of the project. Each subcontractor subsequently determines the work rate that maximizes her respective expected discounted profit. The client receives a fixed payment when the project is completed and sets the contract terms to maximize his expected discounted profit.

The contract that we propose assumes that the client sets values $p_i > 0$ and $\beta_i \geq 0$ for the ith subcontractor at the start of the project; each subcontractor receives a payment equal to $\rho_i(t_i) = p_i e^{-\beta_i t_i}$, where t_i represents the realized duration of stage i. The exponential form of these contracts reflects both the inverse relationship between direct costs and task durations that is generally accepted in most project management literature (Klastorin 2010) as well as the convexity of this time-cost trade-off (Elmaghraby 1977). In this way, the contract sets payment terms that reflect the nondecreasing marginal costs associated with reducing stage duration. (A similar mechanism was suggested by Bernstein and Federgruen 2005 for coordinating decentralized supply chains.)

The variable β_i set by the client represents an incentive that impacts each subcontractor's work rate and performance. Furthermore, we assume that each subcontractor has an opportunity cost $O_i \geq 0$ that represents alternative investment opportunities (that, in turn, also reflects general economic conditions). Given values $p_i > 0$ and $\beta_i \geq 0$, the subcontractor sets the work rate to maximize her expected profit; however, if this expected profit is less than O_i, the subcontractor would decide to not participate in this project. Knowing the value of O_i, we show how this opportunity cost can influence the contract terms offered by the client.

We also examine the timing of subcontractor payments as part of the contract definition; for example, the client can pay a subcontractor when she completes her work or when the entire project is completed. The former payment mechanism is similar to most payment schemes in current practice (Dayanand and Padman 2001, Meng and Gallagher 2012) and includes payments made at defined milestones or fixed intervals. Alternatively, the client can pay all subcontractors when the entire project is completed; Kwon et al. (2010b) labeled this type of contract as a "delayed payment contract." In contrast to the results reported by Kwon et al. (2010b) for a project when all tasks can be performed simultaneously (i.e., a parallel precedence network), we show that there is no difference at equilibrium between delayed and nondelayed payments for all firms in a sequential project (a result that holds for a variety of modeling assumptions).

In this paper we describe our proposed incentive payment contract and several variations:

- A fixed payment contract that occurs when $\beta_i = 0$. In this case, the subcontractor is paid an amount p_i that is independent of the stage duration. Fixed payment contracts are widely used in practice.
- A delayed payment contract when the client pays each subcontractor when the project is completed.
- A hybrid contract that consists of the proposed incentive payment contract with a guaranteed minimum payment that is independent of stage duration.
- A dynamic incentive payment contract when subcontractors determine their respective work rates after observing the realized duration of preceding stages.

In all cases, we assume that the client sets the payments p_i and values of β_i that maximize his expected discounted profit subject to subcontractor participation constraints; the subcontractors respond by setting their work rates that maximize their expected discounted profits. We compare the various contract types with respect to the expected project makespan and discounted profits for the client and subcontractors. Given the general form of the incentive payment, $\rho_i(t_i) = p_i e^{-\beta_i t_i}$, we can analytically compare alternative contracts. For example, we show that the incentive payment contract with $\beta_i > 0$ always dominates a fixed price contract ($\beta_i = 0$) with respect to a client's expected discounted profit and expected project makespan. We describe implications for other contract definitions as well.

In addition, we show that the general form of the incentive payment contract generalizes many I/D contracts that include penalties for tardiness as well as rewards for early completion (Shr and Chen 2004). Specifically, we show how a client can derive an optimal I/D contract (including due dates, tardiness penalties, and earliness rewards) that approximates the equilibrium incentive payment contract but would be easier to implement. To our knowledge, this is the first work that analytically compares fixed price and incentive contracts in a project environment and presents a structured methodology for setting due dates, tardiness penalties, and earliness bonuses in an I/D contract.

1.1. Literature Review

Incentive contracts have been widely studied (Weitzman 1980), generally in the context of principal–agent theory. Our work is also related to previous research on managing innovative development processes (for an overview, see Shane and Ulrich 2004). However, specific research that combines this area with project management literature has been limited. Dayanand and Padman (2001) considered the problem of setting payment amounts and timing to maximize the discounted client's profit; whereas their analysis considered general project networks, their models were limited to deterministic task durations. In the case of stochastic projects, Buss and Rosenblatt (1997) considered the problem of finding optimal task start times in projects with parallel precedence networks when task durations are exponential with a goal to maximize the expected discounted profit (a fixed revenue is earned when the project is completed). Unlike our analysis, however, their work did not consider payments to subcontractors or the impact of these payments on subcontractors' levels of effort. With respect to the stochastic time-cost trade-off problem in project management, Elmaghraby (2005) considered a two-stage serial project with exponential durations and time-cost trade-offs. Klastorin and Mitchell (2007) presented an effective methodology for the stochastic compression problem that also assumed exponential task duration times. Papers that applied robust optimization to the stochastic time-cost trade-off problem included Goh and Hall (2013), who developed a satisficing time-cost trade-off model, and Cohen et al. (2007), who minimized total cost.

When projects are subdivided and outsourced, Gutierrez and Paul (2000) considered the problem of a subcontractor who faces the choice of partitioning a project into multiple subprojects or outsourcing the project as a whole to a single subcontractor. They showed that partitioning the project and outsourcing each subproject is preferred in a serial project, if the mean completion times of the subcontractors are ordered or the subcontractors are consistent across the project. Kwon et al. (2011) examined the impact of different sourcing decisions of project tasks with exponential completion times on operation profits. Their models analyze the trade-off between efficiency (outsourced tasks) and control (tasks performed in-house) under both parallel and serial project networks. Their results showed that when the revenue of the organization is relatively small (large), it is beneficial for the organization to keep the project in-house (outsource). Bayiz and Corbett (2005) presented a model based on the assumption that a linear incentive contract was used to coordinate the relative efforts of all subcontractors under asymmetric information. Their work suggested that contracts that increase payments to subcontractors if their relative tasks are completed before a given due date are weakly superior to fixed-price contracts in terms of a shorter expected project makespan and higher expected profits for the client. Kwon et al. (2010a) applied the concepts of supply chain coordination to project management. Assuming that duration times are exponentially distributed and the cost of a subcontractor is a quadratic function of the work rate, they showed that time-based and cost sharing contracts can achieve optimal channel coordination when there is a single subcontractor. Our research extends their work by considering the timing as well as the magnitude of payments in the presence of continuous discounting.

Chen and Lee (2013) showed that the delivery-schedule-based contracts are able to coordinate the decentralized supply chain in a project management context. In their work, they assumed that payments, penalty rates, and bonus rates are exogenously given; their main focus is on the subcontractor's optimal decision about the targeted material delivery schedule, as well as the subcontractor's optimal decision about her own production schedule. Our paper complements theirs by showing how to determine the payments as well as the penalty and bonus rates (in our paper, we use the term "incentive factors") to each of the nonhomogeneous suppliers.

To the best of our knowledge, Kwon et al. (2010b) were the first to examine delayed payment contracts in the context of projects with parallel tasks. Their model uses an "imputed" continuous-time discount rate to capture the view that both suppliers and manufacturers discount the value of future payments. Their work also assumes that all subprojects are of equal difficulty, so all subcontractors' total cost per unit time are equal (thus the work rate is the same for all subcontractors). Unlike their work, we focus on sequential, rather than parallel, projects and include nonhomogeneous subcontractors; furthermore, we incorporate the payment amounts as decision variables (rather than exogenous parameters) and introduce "incentive payment" contracts. In contrast to the results found by Kwon et al. (2010b) that delayed payment projects may be more profitable for a client under some conditions, we find that delayed payment contracts can never be more profitable in serial stochastic projects; this result differs from Kwon et al. (2010b) primarily because of the differences in network topology.

1.2. Contributions and Overview

In this paper we propose an incentive payment contract for a serial stochastic project and analyze its expected profitability (for both the client and subcontractors) and makespan by modeling this contract as a Stackelberg game. We compare this contract to several variations, including a fixed price contract (when subcontractors are guaranteed a fixed payment regardless of stage duration), a hybrid contract (that places a lower bound on the subcontractor payments), a dynamic contract (when subcontractors negotiate with the client only after observing the performance of previous subcontractors), and two payment timing options. Our analysis indicates a number of significant implications, including the nonintuitive result that the incentive payment contract is equivalent with respect to expected client and subcontractor profits and project makespan for both the delayed and nondelayed payment options (a result that differs from the results found by Kwon et al. 2010b when tasks are performed in parallel). We show that our results extend to fixed price contracts (a special case of the incentive payment contract), hybrid, and dynamic contracts. We also show how certain contractual arrangements can encourage undesired results (e.g., delays and lower expected profits).

Our work shows that "pay for increased effort" contracts are always superior to fixed price contracts in terms of shorter expected project duration and higher expected profits for the client. Whereas we prove this result analytically when task durations are exponential, we show numerically that this result also holds when task durations follow both normal and gamma distributions. Recognizing that a contract based on a nonlinear function might be difficult to implement in practice, we show how a client can use the equilibrium solution to the (exponential) contract to define a comparable incentive/disincentive (I/D) contract (i.e., define due dates, tardiness penalties, and earliness bonuses) by using a piecewise linear approximation.

The rest of the paper is organized as follows. In §2, we define our proposed incentive payment contract and related notation. Modeling the contracting process as a Stackelberg game, we show how risk-neutral subcontractors would react to the terms presented by a client to maximize their respective expected discounted profit. We also define several variations of the basic incentive payment contract that are commonly used in practice and show how subcontractors would respond in these cases. In this section, we also show how a client would set contract parameters for each subcontractor to maximize his expected discounted profit. In the third section we compare these contracts with respect to client and subcontractor profitability and project makespan, and we consider the case when subcontractors may have better alternatives (i.e., greater opportunity costs) elsewhere and do not want to participate in this project as a result. In §4, we show how the solution for an incentive payment contract can be used to define an I/D contract (i.e., the due date, tardiness penalties, and earliness bonus) that is frequently used in practice. The final section summarizes the contributions of this work and indicates extensions that we are presently exploring. All proofs and technical details are provided in the online supplement (available as supplemental material at http://dx.doi.org/10.1287/msom.2015.0528).

2. Incentive Payment Contract and Stackelberg Game Defined

We assume that the client is managing a complex project that is defined by a series of sequential stages that are outsourced to independent subcontractors. Without loss of generality, we assume that the subcontractors are sequentially indexed from the start of the project; that is, the subcontractors proceed in order $i = 1, \ldots, n$. Following Kamien and Schwartz (1972), Buss and Rosenblatt (1997), Kwon et al. (2010b), and others, we assume that the client receives a fixed payment Q when the project is completed. We assume that the duration t_i of stage i is exponentially distributed with density $f(t) = r_i e^{-r_i t} dt$ where the parameter $r_i > 0$ defines the work rate set by the ith subcontractor. The exponential completion time assumption has been widely used in previous project management research (e.g., Buss and Rosenblatt 1997, Tavares 2002, Klastorin and Mitchell 2007, Kwon et al. 2010b). We assume that the subcontractors incur an operating cost $k(r)$ per unit time; generalizing Kwon et al. (2010b), we let $k_i(r_i) = K_i + k_i r_i^2$ with $K_i \geq 0$ and $k_i > 0$, although we could use any nondecreasing convex function of the work rate r. Assuming a positive discount rate $\alpha > 0$, subcontractor i's discounted cost at time 0 is defined by $e^{-\alpha \sum_{j=1}^{i-1} t_j} \int_0^{t_i} (K_i + k_i r_i^2) e^{-\alpha t} dt$, where stage i starts at time $\sum_{j=1}^{i-1} t_j$. Our primary analytical results are based on the assumption that $K_i = 0$ although we study the impact of positive $K_i > 0$ numerically. Furthermore, we assume that the parameters K_i and k_i are common knowledge for all participants in the Stackelberg game, which is reasonable in many scenarios (e.g., the client and subcontractors interact repeatedly).

In this game, the client initially sets $p_i > 0$ and $\beta_i \geq 0$ for all subcontractors to maximize his discounted profit, where each ith subcontractor receives an amount $p_i e^{-\beta_i t_i}$ when stage i (or the project) is completed. Given the values of β_i and p_i, each subcontractor determines her equilibrium work rate (and expected stage duration) that maximizes her discounted profit.

In our analysis, we assume that each subcontractor will only accept the contract terms from the client if she can earn a discounted profit that equals or exceeds her opportunity cost $O_i \geq 0$, where we define the opportunity costs as a function H_i of the expected task duration $E[t_i]$, $O_i = H_i(E[t_i])$ where $E[t_i] = r_i^{-1}$. Opportunity costs could indicate varying economic environments; for example, if $H_i(E[t_i]) = a_i + b_i E[t_i]$, small values of $a_i > 0$ and b_i could indicate a difficult economic environment (where subcontractors have limited choices), whereas larger values of a_i and b_i may indicate multiple alternatives made possible by a strong or improving economic environment. Alternatively, an opportunity cost could indicate the reputation and quality standards of the subcontractor (e.g., a highly respected subcontractor would have more alternatives and could charge a higher rent). Opportunity costs could also reflect expected indirect and overhead costs; for example, a greater value of b_i may indicate the case where a subcontractor expects her task to incur a substantial time commitment with a diminished appeal of project participation. (The case where overhead and indirect costs are based on realized task durations is addressed in §5 of this paper.)

The notation used in the remainder of the paper is summarized below.

Client Decision Variables:
p_i payment amount to subcontractor i
β_i incentive factor for subcontractor i

Subcontractor Decision Variable:
r_i work rate chosen by subcontractor i

Parameters:
K_i subcontractor i's fixed resource cost per time period
k_i subcontractor i's variable resource cost parameter
α (continuous) discount rate
O_i subcontractor i's opportunity cost

2.1. Basic Incentive Payment Contract Defined

In the basic incentive payment contract, the client sets $p_i > 0$ and $\beta_i \geq 0$ at time 0 and pays an amount $\rho_i(t_i) = p_i e^{-\beta_i t_i}$ at the conclusion of the ith subcontractor's stage (when stage duration t_i is realized). We denote this as contract $\mathbb{I}$; the expected discounted profit for a subcontractor in this case is given by

$$E[\pi_i^{\mathbb{I}}] = E\left[(p_i e^{-\beta_i t_i}) e^{-\alpha \sum_{j=1}^{i} t_j} - e^{-\alpha \sum_{j=1}^{i-1} t_j} \int_0^{t_i} (K_i + k_i r_i^2) e^{-\alpha t}\, dt\right]$$
$$= \left(\frac{p_i r_i}{\alpha + r_i + \beta_i} - \frac{(K_i + k_i r_i^2)}{\alpha + r_i}\right) \prod_{j=1}^{i-1} \frac{r_j}{\alpha + r_j}. \qquad (1)$$

The expected profit defined by (1) indicates how changes in other subcontractors' work rates impact the expected profit of subcontractor i. Specifically, when the work rates of predecessor subcontractors k (for $k < i$) increase, the profits of the ith subcontractor increase. With respect to successor subcontractors, changes in their work rates have no impact on subcontractor i's expected work rates or expected payoff.

The subcontractor's expected profit defined by (1) is concave with respect to r_i; this observation leads to the result in Proposition 1.

Proposition 1. *Given contract $\mathbb{I}$, the equilibrium work rate, r_i^*, for each subcontractor is given by the unique positive solution to the following equation*:

$$r_i^* = \sqrt{\alpha^2 + \frac{p_i(\alpha + \beta_i)(\alpha + r_i^*)^2}{k_i(\alpha + \beta_i + r_i^*)^2} + \frac{K_i}{k_i}} - \alpha. \qquad (2)$$

Subcontractor i will participate in contract $\mathbb{I}$ if and only if $(p_i r_i^*/(\alpha + r_i^* + \beta_i) - (K_i + k_i r_i^{*2})/(\alpha + r_i^*))(\prod_{j=1}^{i-1} r_j^*/(\alpha + r_j^*)) \geq O_i$.

For positive $K_i > 0$, we cannot guarantee that the subcontractor will earn positive profit; however, when $K_i = 0$, we can show that $E[\pi_i^{\mathbb{I}}] > 0$ for the equilibrium work rate r_i^* found by solving (2), as stated in the following proposition.

Proposition 2. *If $K_i = 0$, given values of $p_i > 0$ and $\beta_i \geq 0$, $E[\pi_i^{\mathbb{I}}] > 0$ when the work rate for the ith subcontractor r_i^* is determined by the unique solution to* (2).

This proposition implies that a subcontractor would always participate in the project for any $p_i > 0$ and $\beta_i \geq 0$ set by the client when opportunity costs $O_i = 0$. (In §3 we discuss the case when $O_i > 0$ and $K_i > 0$.) For the remainder of this section, we assume that $O_i = K_i = 0\ \forall\, i$ so that all subcontractors participate.

We can characterize the relationship between the subcontractor's equilibrium work rate and the value of β_i assuming a fixed payment p_i. Using (2), we can show, using implicit differentiation and algebraic manipulation, that

$$\frac{\partial r_i^*}{\partial \beta_i} = \frac{(\alpha + r_i^*)[(r_i^* - (\alpha + \beta_i))/(\alpha + \beta_i + r_i^*)]}{2(\alpha + \beta_i)[\alpha^2/r_i^*(2\alpha + r_i^*) + (\alpha + r_i^*)/(\alpha + \beta_i + r_i^*)]};$$

since the denominator is positive, the sign of $\partial r_i^*/\partial \beta_i$ depends on the term $r_i^* - (\alpha + \beta_i)$. This observation allows us to provide the following result, which links the effect of β_i with the value of the payment chosen.

Corollary 1. *The following conditions hold if $K_i = 0$:*

- *If $p_i < 3\alpha k_i$, there does not exist any $\beta_i \geq 0$ where $r_i^* = \alpha + \beta_i$, which implies $\partial r_i^*/\partial \beta_i < 0$, for $\beta_i \geq 0$.*
- *If $p_i = 3\alpha k_i$ then only $\beta_i = 0$ satisfies $r_i^* = \alpha + \beta_i$, which implies $\partial r_i^*/\partial \beta_i < 0$, for $\beta_i > 0$.*
- *If $p_i > 3\alpha k_i$, there exists a unique value $\beta_i = \hat{\beta}_i > 0$ that satisfies $r_i^* = \alpha + \hat{\beta}_i$, which implies $\partial r_i^*/\partial \beta_i > 0$, for $0 \leq \beta_i < \hat{\beta}_i$ and $\partial r_i^*/\partial \beta_i < 0$, for $\beta_i > \hat{\beta}_i$.*

These results indicate that the relationship between the incentive factor β_i and a subcontractor's equilibrium work rate r_i^* (and expected duration) depends on the value of p_i. Specifically, if $p_i \leq 3\alpha k_i$, increasing β_i will result in a subcontractor reducing her work rate (and increasing her expected stage duration). This occurs when p_i is relatively small (representing an unappealing project); in this case, increasing β_i will further reduce the motivation for the subcontractor to participate in the project. On the other hand, if $p_i > 3\alpha k_i$, increasing β_i will result in a subcontractor increasing her work rate (and reducing the expected duration) as long as $\beta_i < \hat{\beta}$. The relationship between p_i and β_i values is further discussed in §2.2 that considers the equilibrium value of β_i that maximizes the client's expected discounted profit.

We can also analyze the relationship between β_i and a subcontractor's expected discounted profit. Our results are indicated in Corollary 2, which shows subcontractor i's expected profit will decrease as β_i increases, given that the subcontractor sets an equilibrium work rate defined by (2).

Corollary 2. *If $K_i = 0$ for a subcontractor operating with contract $\mathbb{I}$, then $\partial E[\pi_i^{\mathbb{I}}]/\partial\beta_i < 0$, where $E[\pi_i^{\mathbb{I}}]$ is defined by (1) and the equilibrium work rate is defined by (2).*

2.2. Maximizing Expected Client Payoff

In this section we continue to assume that the subcontractor opportunity costs $O_i = 0$ and $K_i = 0$ for all i; all subcontractors will participate for all values of p_i and β_i since they are guaranteed to earn a positive profit (Proposition 2). Using contract $\mathbb{I}$, the client (knowing subcontractor cost parameters k_i) sets β_i (where $\beta_i \geq 0$) at time 0 and pays $p_i e^{-\beta_i t_i}$ to the ith subcontractor when her stage is completed at time $\sum_{k=1}^{i} t_k$. As previously indicated, each subcontractor sets her equilibrium work rate using (2). When the project is completed at time $T = \sum_{k=1}^{n} t_k$, the client receives a fixed payment Q.

Under contract $\mathbb{I}$, the expected discounted profit earned by the client is equal to

$$E[\Pi_C^{\mathbb{I}}] = \left[Q e^{-\alpha T} - \sum_{i=1}^{n} (p_i e^{-\beta_i t_i}) e^{-\alpha \sum_{j=1}^{i} t_j} \right]$$
$$= Q \prod_{j=1}^{n} \frac{r_j}{\alpha + r_j} - \sum_{i=1}^{n} \frac{p_i r_i}{\alpha + \beta_i + r_i} \prod_{j=1}^{i-1} \frac{r_j}{\alpha + r_j}, \tag{3}$$

where r_j is defined by (2) and Proposition 1. Given values of r_i, values of p_i can be defined by inverting Equation (2); that is,

$$p_i = \frac{(r_i^2 + 2\alpha r_i)(\alpha + \beta_i + r_i)^2 k_i}{(\alpha + \beta_i)(\alpha + r_i)^2}. \tag{4}$$

Using (3) and (4), the problem for the client is then to find the equilibrium values of r_i^* and β_i^* that maximize his expected discounted profit,

$$E[\Pi_C^{\mathbb{I}}] = Q \prod_{j=1}^{n} \frac{r_j}{\alpha + r_j}$$
$$- \sum_{i=1}^{n} \frac{(r_i^3 + 2\alpha r_i^2)(\alpha + \beta_i + r_i) k_i}{(\alpha + \beta_i)(\alpha + r_i)^2} \prod_{j=1}^{i-1} \frac{r_j}{\alpha + r_j}. \tag{5}$$

To find the values of r_i^* and β_i^* (and therefore p_i^*), we initially assume that the values of β_i are fixed and use the variable transformation $g_i = \alpha/(\alpha + r_i)$. For given β_i, we will denote the expected client's profit by $E[\Pi_C^{\mathbb{I}}(\beta_1, \ldots, \beta_n)]$. Using the g_i variables, the expected client's profit can be stated as

$$E[\Pi_C^{\mathbb{I}}(\beta_1, \ldots, \beta_n)]$$
$$= Q \prod_{j=1}^{n} (1 - g_j) - \sum_{i=1}^{n} \left(\frac{\alpha k_i}{\alpha + \beta_i} \right) (1 - g_i)^2$$
$$\cdot \left(\frac{1}{g_i} + 1 \right) \left(\frac{\alpha}{g_i} + \beta_i \right) \prod_{j=1}^{i-1} (1 - g_j). \tag{6}$$

Since (5) is not jointly concave in r_i^* and β_i^*, we define a search procedure with order $O(n)$ to find the equilibrium values of g_i^* and r_i^* and subsequently find β_i^*. We denote this search procedure as Algorithm SIP ("search for incentive payments") to find the unique equilibrium values of g_i that we denote as $\hat{g}_i$. This algorithm is described and its correctness proved in Appendix A of the online supplement.

To find the equilibrium values of $\beta_i \geq 0$, we note that Equation (6) indicates that the client's expected profit increases monotonically with increasing values of β_i and, in fact, approaches an asymptote as $\beta_i \to \infty$. Support for this statement is given in Proposition 3.

Proposition 3. *If $O_i = K_i = 0\ \forall\, i$, for unique equilibrium values $(\hat{g}_1, \ldots, \hat{g}_n)$ defined for given $\{\beta_1, \ldots, \beta_n\}$, the expected client's profit increases with increasing values of $\beta_i \geq 0$.*

Similar to Proposition 2, we can prove that the expected client's profit is always positive for equilibrium values of $r_i^* > 0$; that is, the client would always participate in this game when using contract $\mathbb{I}$. In §3, we explore the client's trade-offs between increasing values of β_i and expected payoff, as well as positive values of $O_i > 0$ and $K_i > 0$.

The following proposition characterizes the project makespan's dependence on the incentive parameters β_i. We utilize the notation r_i^{**} to denote the subcontractor i's best-response function *induced* by the prices given by Algorithm SIP.

Proposition 4. *If $K_i = 0$, a subcontractor's equilibrium work rate r_i^{**} increases in β_i. Furthermore, r_j^{**} increases in β_i for all predecessor subcontractors $j < i$.*

Proposition 4 indicates that greater values of β_i will result in a greater work rate for subcontractor i (and smaller expected duration). In addition, the work rate for all subcontractors that precede the ith subcontractor will also increase, thereby further reducing the project makespan. These observations result in two significant implications for the client. First, increasing the value of β_n will influence all subcontractors, whereas a comparable increase in β_{n-1}, for example, will only influence $n - 1$ subcontractors. Second, the project makespan is decreasing in each incentive parameter β_i. The latter implication may be important if the client faces a due date and associated penalty cost (this is further discussed in §5).

2.3. Variations of the Basic Incentive Payment Contract $\mathbb{I}$

2.3.1. Fixed Payment Contracts: Incentive Payment Contracts with $\beta_i = 0$. When $\beta_i = 0$, the client pays subcontractor i an amount p_i regardless of t_i (the realized duration of stage i). We denote this as contract $\mathbb{F}$. The expected NPV for subcontractor i for contract $\mathbb{F}$ can be defined as follows:

$$E[\pi_i^{\mathbb{F}}] = E\left[p_i e^{-\alpha\sum_{j=1}^{i} t_j} - e^{-\alpha\sum_{j=1}^{i-1} t_j}\int_0^{t_i}(K_i+k_i r_i^2)e^{-\alpha t}\,dt\right]$$
$$= \left(\frac{p_i r_i}{\alpha+r_i} - \frac{(K_i+k_i r_i^2)}{\alpha+r_i}\right)\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}. \qquad (7)$$

The expected profit defined by (7) is strictly concave in the work rate, r_i; as a result, we can find a closed-form solution for a subcontractor's equilibrium work rate when contract $\mathbb{F}$ is used. This result is given in Proposition 5.

Proposition 5. *Given contract $\mathbb{F}$, the unique equilibrium work rate, r_i^*, for each subcontractor $i = 1, \ldots, n$ is given by $r_i^* = \sqrt{\alpha^2 + (p_i\alpha + K_i)/k_i} - \alpha$. Subcontractor i will participate in Contract $\mathbb{F}$ if and only if $(p_i \prod_{k=1}^{i} r_k^*/(\alpha + r_k^*) - (K_i + k_i r_i^{*2})/(\alpha + r_i^*))(\prod_{j=1}^{i-1} r_j^*/(\alpha + r_j^*)) \geq O_i$.*

Assuming that the subcontractor participates in the project (that is, the resulting profits exceed her opportunity cost), the results from Proposition 5 indicate that each subcontractor acts independently when setting their own work rate (and expected task duration) under contract $\mathbb{F}$. This observation, however, does not apply to their expected profits defined by (7), which indicates that subcontractor i's expected profit $E[\pi_i^{\mathbb{F}}]$ increases (decreases) as the work rate r_j of its predecessors (that is, for $j < i$) increase (decrease). In contrast, the work rates of successor subcontractors (for $j > i$) have no impact on the profits of subcontractor i. Under contract $\mathbb{F}$, the outcome of each individual subcontractor varies with the performance of preceding subcontractors, although such performance is outside of any individual subcontractor's control.

2.3.2. Delayed Payment Contracts. In contracts $\mathbb{I}$ or $\mathbb{F}$, the client may specify that the subcontractors are paid at the completion of the project (*i.e.*, at time $T = \sum_{j=1}^{n} t_j$). These delayed payment contracts are analogous to the contracts used by the Boeing Company for the suppliers of the Boeing 787 Dreamliner and a possible factor behind the development delays of that airliner (Greising and Johnsson 2007). We denote these contracts by $\mathbb{I}_{\mathbb{D}}$ or $\mathbb{F}_{\mathbb{D}}$, respectively.

In contract $\mathbb{I}_{\mathbb{D}}$, the client sets p_i and $\beta_i > 0$ at time 0 and pays $p_i e^{-\beta_i t_i}$ when the project is completed. Subcontractor i's expected profit in this case is given by

$$E[\pi_i^{\mathbb{I}_{\mathbb{D}}}] = E\left[(p_i e^{-\beta_i t_i})e^{-\alpha T} - e^{-\alpha\sum_{j=1}^{i-1} t_j}\int_0^{t_i}(K_i + k_i r_i^2)e^{-\alpha t}\,dt\right]. \qquad (8)$$

The expected payoffs at equilibrium for the client and subcontractors under contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$ are equivalent (as well as the expected makespan), even for nonexponentially distributed task durations; this can be shown by rewriting subcontractor i's expected payoff under contract $\mathbb{I}$ as defined by Equation (1) as

$$E[\pi_i^{\mathbb{I}}] = E[e^{-\alpha\sum_{j=1}^{i-1} t_j}] \times E\left[(p_i e^{-\beta_i t_i})e^{-\alpha t_i} - \int_0^{t_i}(K_i + k_i r_i^2)e^{-\alpha t}\,dt\right].$$

Under contract $\mathbb{I}_{\mathbb{D}}$, subcontractor i's payoff as defined by (8) can be rewritten as

$$E[\pi_i^{\mathbb{I}_{\mathbb{D}}}] = E[e^{-\alpha\sum_{j=1}^{i-1} t_j}] \times E\left[(e^{-\alpha\sum_{j=i+1}^{n} t_j} p_i e^{-\beta_i t_i})e^{-\alpha t_i} - \int_0^{t_i}(K_i + k_i r_i^2)e^{-\alpha t}\,dt\right].$$

If we let $p_i' = p_i \; E[e^{-\alpha\sum_{j=i+1}^{n} t_j}]$, then the expected discounted client's profit for the delayed payment contract $\mathbb{I}_{\mathbb{D}}$ can be rewritten as $E[\Pi_C^{\mathbb{I}_{\mathbb{D}}}] = E[Qe^{-\alpha T} - \sum_{i=1}^{n}(p_i' e^{-\beta_i t_i})e^{-\alpha\sum_{j=1}^{i} t_j}]$ that is equivalent to the expected client's profit under contract $\mathbb{I}$ as defined by (3).

Likewise, the unique equilibrium work rates for all subcontractors are equal under the two contracts as stated and proved in Proposition 6. This result does not require the task durations to be exponentially distributed and only requires the nonnegative random variables to be statistically independent.

Proposition 6. *For each subcontractor, the unique equilibrium work rate under contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$ are equal; that is, $r_j^{\mathbb{I}*} = r_j^{\mathbb{I}_{\mathbb{D}}*}$ for $j = 1, \ldots, n$. Furthermore, the expected client's profit, expected subcontractors' profit, and the expected makespan are equal under contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$ given independent task durations.*

Proposition 6 also holds under positive opportunity costs. If subcontractors willingly participate in the project, opportunity costs are a nonbinding constraint, and the analysis of Proposition 6 is applicable. If subcontractors are not willing to participate, an "adjustment" is necessary, which is discussed in detail in §3.2. However, since the functional forms of the subcontractors' profits are essentially the same, the adjustments will be identical under both contracts, and Proposition 6 remains applicable.

Note that the equivalence of delayed and nondelayed contracts only holds for sequential projects; Kwon et al. (2010b) showed that there is no equivalence in projects where tasks are performed in parallel. The intuition behind this observation is differing motivations: (1) under a delayed parallel contract, a subcontractor has a motivation to slow down if other subcontractors are slow, whereas (2) there is no such motivation under a delayed sequential project or any nondelayed (sequential or parallel) project.

2.3.3. Hybrid Contracts: Incentive Payment Contracts with Minimum Guaranteed Payments. To reduce subcontractor risk, a client may offer a subcontractor an incentive payment contract with a guaranteed amount $\gamma_i > 0$ that is paid regardless of the stage duration t_i. We denote this (combination of a fixed and incentive payment contract) as a hybrid contract $\mathbb{I}_{\mathbb{H}}$. To avoid trivial cases, we assume that $\gamma_i < p_i$. In a hybrid contract $\mathbb{I}_{\mathbb{H}}$, a subcontractor is paid an amount equal to $\max\{p_i e^{-\beta_i t_i}, \gamma_i\}$ following the realization of t_i.

We assume that the amount γ_i is negotiated at time $t = 0$ with the values of p_i and β_i. A subcontractor under incentive contract $\mathbb{I}$ would have to complete their stage no later than time θ_i to earn at least an amount γ_i, where $\theta_i = -\beta_i^{-1}\ln(\gamma_i/p_i)$. In this hybrid contract $\mathbb{I}_{\mathbb{H}}$, the expected discounted profit for subcontractor i is

$$E[\pi_i^{\mathbb{I}_{\mathbb{H}}}] = E\bigg[e^{-\alpha\sum_{j=1}^{i} t_j}\max(p_i e^{-\beta_i t_i}, \gamma_i) - e^{-\alpha\sum_{j=1}^{i-1} t_j}\int_0^{t_i}(K_i + k_i r_i^2)e^{-\alpha t}\,dt\bigg]$$
$$= \left\{\frac{p_i r_i[1 - e^{-(\alpha+\beta_i+r_i)\theta_i}]}{\alpha+\beta_i+r_i} + \frac{\gamma_i r_i e^{-(\alpha+r_i)\theta_i}}{\alpha+r_i} - \frac{K_i + k_i r_i^2}{\alpha+r_i}\right\}\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}.$$

The definition of $E[\pi_i^{\mathbb{I}_{\mathbb{H}}}]$ indicates that the hybrid contract approaches a pure incentive payment contract as $\gamma_i \to 0$ and a fixed price contract as $\gamma_i \to p_i$. Whereas we find analytical results intractable, numerical experiments indicate that the expected subcontractor profits and expected project makespan increase monotonically with γ_i at equilibrium, whereas the expected client profit decreases. Overall, the hybrid contract falls between the pure incentive payment contract $\mathbb{I}$ and the fixed price contract $\mathbb{F}$.

2.3.4. Dynamic Incentive Payment Contracts. In a dynamic contract, each subcontractor waits until preceding subcontractors have completed their respective stages before negotiating the terms of their contract with the client. (In previous discussions, we assumed that static contracts were used where negotiations between the client and subcontractors occurred at time zero.) When using an incentive payment contract $\mathbb{I}$, however, the optimal values of p_i and β_i are the same in both the dynamic and static contracts for all subcontractors and, therefore, the expected client's profit is the same as well.

To understand the equivalence between static and dynamic contracts, assume that $(m-1)$ stages have been completed at time $t = S$, where S is the realized sum of the random durations of the $(m-1)$ preceding stages (that subcontractor m observes). Subcontractor m's expected payoff at time $t = S$ then becomes

$$E[\pi_m^{\mathbb{I}}] = E\bigg[(p_m e^{-\beta_m t_m})e^{-\alpha t_m} - \int_0^{t_m}(K_m + k_m r_m^2)e^{-\alpha t}\,dt\bigg]e^{-\alpha S}$$
$$= \left(\frac{p_m r_m}{\alpha + r_m + \beta_m} - \frac{(K_m + k_m r_m^2)}{\alpha + r_m}\right)e^{-\alpha S}.$$

Since $e^{-\alpha S}$ is a constant, previously completed stages do not affect subcontractor m's first-order condition, resulting in the same work rate that maximizes (1) at time $t = 0$. Intuitively, the dynamic contract changes the time at which subcontractor m makes a decision (time S versus time zero), but the decision itself is the same since the subcontractor can only *directly* influence her discounted profit over the interval $(\sum_{j=1}^{m-1} t_j, \sum_{j=1}^{m} t_j]$. Her expected profit, discounted to time zero, depends only on the decisions of predecessor subcontractors that are outside of her control in both the static and dynamic contracts. The subsequent analysis for the client, which only requires the best-response functions of the subcontractors, is identical. Thus, static and dynamic incentive payment contracts are equivalent.

3. Implications of Incentive Payment Contracts

We initially investigate the impact of positive fixed costs $K_i > 0$ in §3.1, positive opportunity costs $O_i > 0$ in §3.2, and profit-makespan trade-offs in §3.3.

3.1. Positive Fixed Costs ($K_i > 0$)

Whereas our analytical results do not extend to cases with positive $K_i > 0$, we investigated the impact of positive fixed costs numerically (including subcontractor participation constraints). In this section we assume zero opportunity costs and the following parameters for $n = 3$ subcontractors:

$$Q = \$1{,}000,$$
$$\alpha = 0.1,$$
$$k_1 = k_2 = k_3 = 200.$$

When $K_i > 0$, Proposition 3 no longer necessarily holds, although we observe that the client's equilibrium profit is unimodal in each β_i for most K_i. Furthermore, we find that the client's profit is maximized at finite values of β_i for the above parameters when $K_i > 5$. If $K_i < 5$, the client's equilibrium profit is still strictly increasing in the β_i parameters. These results are summarized in Table 1, which provides the equilibrium client profit and maximizing values of β_i^* as a function of K, where $K_i = K$ for all i. We found similar results for varying values of k_i, as well as the

Table 1 Numerical Example (Contract $\mathbb{I}$) with $n = 3$ Subcontractors

K	$E[\Pi_C^{\mathbb{I}*}(\beta)]$	β_1^*	β_2^*	β_3^*
0	338.5	Inf	Inf	Inf
5	351.1	10.9	Inf	Inf
10	332.3	1.8	3.8	7.5
15	312.7	0.9	1.9	3.4
20	293.5	0.6	1.3	2.2

cases when task durations follow both the normal and gamma distributions.

Comparing contracts $\mathbb{F}$ and $\mathbb{I}$, Proposition 3 shows that an incentive payment contract (contract $\mathbb{I}$) dominates a fixed price contract (contract $\mathbb{F}$) with respect to the client's expected profit; Table 1 indicates this result continues to hold when $K_i > 0$. With respect to a subcontractor's expected profit, however, the reverse appears to hold; that is, a subcontractor's expected profit is greater with a fixed price contract than an incentive payment contract given the same payments p_i to each subcontractor.

3.2. Subcontractor Opportunity Costs Considered

When subcontractors have positive opportunity costs O_i, they may not participate if offered the prices from Algorithm SIP (described in Appendix A of the online supplement). In this case, we propose a procedure that appropriately modifies the SIP prices to guarantee subcontractor participation, while sacrificing a minimum amount of the client's profit. We discuss contracts $\mathbb{F}$ and $\mathbb{I}$ in detail; other contracts can be analyzed in an analogous fashion.

Using contract $\mathbb{F}$, subcontractor i's expected profit, defined by (7) and evaluated at the equilibrium work rate r_i^* of Proposition 5, can be written as

$$\left(p_i - \frac{K_i}{r_i^*} - k_i r_i^*\right)\left(\frac{r_i^*}{\alpha + r_i^*}\right)\prod_{j=1}^{i-1}\frac{r_j^*}{\alpha + r_j^*}.$$

This expression is strictly increasing in p_i (easily seen by calculating its derivative and noting that $\partial r_i^*/\partial p_i > 0$); when the price is zero, the subcontractor profit is also zero. Conversely, the opportunity costs are decreasing in p_i assuming $O_i = a_i + b_i r_i^{-1}$. Thus, there exists a unique price $\underline{p}_i(O_i)$ where the subcontractor's profit is exactly equal to its opportunity cost O_i. This is the minimum price that must be offered to subcontractor i to induce participation; our heuristic adjustment is basically to offer the subcontractor a price equal to $\max\{p_i^{\mathrm{SIP}}, \underline{p}_i(O_i)\}$ where p_i^{SIP} is the price determined by Algorithm SIP.

A similar case exists under contract $\mathbb{I}$ when $O_i > 0$, although the adjustment procedure is more involved. Basically, if the subcontractor's expected profit is too low when $\beta = 0$ to entice her to participate in the project, prices are adjusted upward; if the expected profit exceeds O_i, the value of β_i is increased (to a finite value) to reduce the subcontractor's expected profit to O_i. The adjusted SIP algorithms for both contracts $\mathbb{F}$ and $\mathbb{I}$ are fully described in Appendix B of the online supplement.

To further illustrate the impact of opportunity costs, we modified the example in Table 1 to include positive subcontractor opportunity costs that are presented in Table 2. For simplicity, we assume that all subcontractors have the same positive opportunity cost structure: $O_i = a + b/r_i$ for $i = 1, \ldots, 3$. We consider a set of values for the parameter $a \in \{0, 2, 4, 6\}$ and let $b = 1$. Recall that cost values for the three subcontractors are $k_1 = k_2 = k_3 = 200$, and we let $K_1 = K_2 = K_3 = 3$ (the common fixed cost value of 3 is selected to preserve the monotonicity of Proposition 3).

In case 1, the opportunity costs, at equilibrium, are smaller than the corresponding subcontractor profits under contract $\mathbb{F}$. Therefore, each subcontractor is offered contract $\mathbb{I}$, and, per the opportunity-cost adjustment (contract $\mathbb{I}$), the β_i parameters are adjusted upward for each subcontractor to lower the subcontractor profits to their opportunity costs, while simultaneously increasing the client's profit. In case 2, the opportunity cost is larger than subcontractor 1's profit under contract $\mathbb{F}$, but lower than subcontractors 2 and 3's profit. Contract $\mathbb{F}$ is offered to subcontractor 1, with the p_1^{SIP} price appropriately adjusted upward, per the opportunity-cost adjustment (contract $\mathbb{F}$), to guarantee participation. In contrast, contract $\mathbb{I}$ is offered to subcontractors 2 and 3, and the β_i parameters are adjusted upward for these subcontractors to lower their profits to their opportunity costs, while simultaneously increasing the client's profit. Case 3 is similar to case 2 with the difference that contract $\mathbb{I}$ is only offered to subcontractor 3. In case 4, the opportunity costs at equilibrium are larger than the corresponding subcontractor profits under contract $\mathbb{F}$. Therefore, each subcontractor is offered contract $\mathbb{F}$, and the p_i^{SIP} prices are adjusted upward for each subcontractor to raise the subcontractor profits to the level of their opportunity costs, thereby guaranteeing their participation.

The numerical example in Table 2 leads to several general insights. For simplicity we fix the b parameter and vary the a parameter. In strong economic climates (suggested by high values of the parameter a), contract $\mathbb{I}$ is not feasible and contract $\mathbb{F}$, with

Table 2 Numerical Example (Contract $\mathbb{I}$) with Opportunity Costs

Case	O_i	Client's profit	Makespan	System profit
1	$0 + r_i^{-1}$	349.0	6.1	359.6
2	$2 + r_i^{-1}$	270.5	9.6	300.9
3	$4 + r_i^{-1}$	182.0	12.1	239.5
4	$6 + r_i^{-1}$	139.2	17.4	191.2

prices adjusted upward, must be used. In weak economic environments (suggested by low values of the parameter a), contract $\mathbb{I}$ dominates contract $\mathbb{F}$, and the opportunity costs lead to finite subcontractor-dependent incentive parameters β_i. In the intermediate economic climates, our numerical results suggest that both contracts $\mathbb{F}$ and $\mathbb{I}$ can be utilized. Therefore, it appears that contract $\mathbb{I}$ is more appropriate in weaker economic environments. Our analysis indicates that the client benefits as the economy weakens: the client's profit is decreasing in the parameter a, and the project makespan is increasing in a. However, the entire system's profit (client and all subcontractors) decreases as the parameter a increases. This observation can be understood intuitively if we consider the incentive parameters β_i as proxies for the level of coordination in the project: as the parameters are increased, the subcontractor incentives become more aligned with that of the project (and client), resulting in more system profit. However, opportunity costs restrict the values of β_i and consequently limit the level of coordination. Therefore, whereas the subcontractors' profits increase because of higher opportunity costs, the client's profit decreases by an amount that is (much) more than all subcontractor increases combined. Similar behaviors are observed by varying the parameter b.

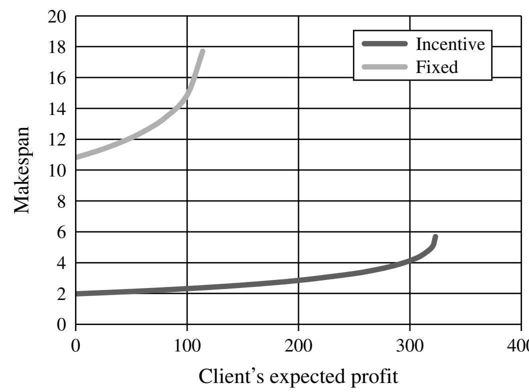
3.3. Expected Profit/Makespan Trade-Offs

As indicated in the previous discussion, there is a trade-off between the client and subcontractor profits, as well as project makespan. The concept of opportunity cost allows us to compare the fixed price contract $\mathbb{F}$ with the incentive payment contract $\mathbb{I}$ when each subcontractor earns an amount exactly equal to their respective opportunity cost (in this example, we set $K = 0$ and $O_i = 24$ for all $i = 1, 2, 3$). Specifically, we modified our algorithm to find the equilibrium solution when the expected makespan must be less than or equal to a given parameter, X. By solving our model for varying values of X, we derived the results given in Figure 1, which indicate that the incentive payment contract is clearly superior to a fixed price contract in this example. Furthermore, our analysis showed that the two curves (and contracts) retain the shapes indicated in Figure 1 but converge as the opportunity cost O increases (since the optimal values of β_i converge to zero).

4. Using an Incentive Payment Contract to Define an Optimal I/D Contract

Given an equilibrium solution to the (exponential) form of contract $\mathbb{I}$, we can use this solution to generate piecewise linear contracts that incorporate a deadline, penalties for late completion, and rewards for

Figure 1 **Expected Client's Profit vs. Makespan for $O = 24$ Example**



early completion. These linear contracts are generally known as I/D contracts in practice. By using linear approximations of contract $\mathbb{I}$, we are able to derive an I/D contract that is more likely to be adopted in practice, yet retain many of the benefits of the nonlinear incentive payment contract.

Recall that the duration of stage i is a nonnegative random variable with pdf $f_i(t) = r_i e^{-r_i t}$ and cdf $F_i(t) = 1 - e^{-r_i t}$. Contract $\mathbb{I}$'s payment function is $\rho_i(t_i) = p_i e^{-\beta_i t_i}$ where the equilibrium parameters p_i and β_i are found using Algorithm SIP (modified appropriately if subcontractor opportunity costs are positive) and t_i is the realized duration of stage i. To convert the incentive payment contract to a form that is more likely to be adopted in practice, we want to use our results to define an incentive contract with the general form:

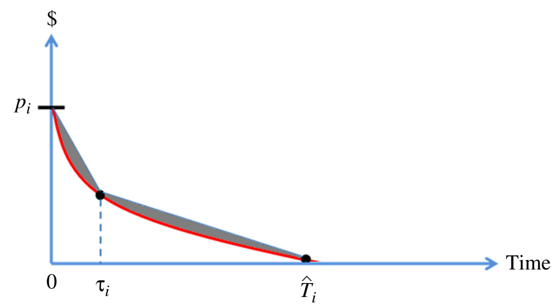
$$\text{payment} + \text{bonus} \times (\text{due date} - t_i)^+ \\ - \text{penalty} \times (t_i - \text{due date})^+.$$

To derive such a contract, we initially define a maximum possible duration $\hat{T}_i$ for each ith subcontractor that could occur with high probability. To determine a value for $\hat{T}_i$, we let $0 < \omega < 1$ denote the probability that the duration of stage i is within time $\hat{T}_i$. Given a value of ω (e.g., 0.95), we let $\hat{T}_i = F_i^{-1}(\omega) = -\ln(1-\omega)/r_i$.

Next, let τ_i denote the due date for the ith subcontractor, which we derive from the equilibrium form of Contract $\mathbb{I}$: The piecewise linear approximation to $\rho_i(t_i) = p_i e^{-\beta_i t_i}$ will share three points with the original function, namely $t_i \in \{0, \tau_i, \hat{T}_i\}$. Therefore, the linear approximation is a global over-estimator of $\rho_i(t_i) = p_i e^{-\beta_i t_i}$, as indicated in Figure 2.

The quality of the approximation can be measured by comparing the areas under the two payment curves; since the approximation is an over-estimator, we can simply subtract the area under $\rho_i(t_i) = p_i e^{-\beta_i t_i}$ (from 0 to $\hat{T}_i$) from that of the linear approximation (this is the L_1 function space norm of the nonnegative

Figure 2 (Color online) Piecewise Linear Approximation of Incentive Payment Contract Solution



difference). This results in the following nonnegative expression for the difference in areas:

$$A_i(\tau_i) = \tau_i p_i + (\hat{T}_i - 2\tau_i)\rho_i(\tau_i) + \tau_i \rho_i(\hat{T}_i).$$

To find τ_i, we want to minimize $A_i(\tau_i)$. We can show that the function $A_i(\tau_i)$ is convex and using the first order condition, the minimizing value of τ_i satisfies $\tau_i = -\beta_i^{-1}\ln[(1+e^{-\beta_i \hat{T}_i})/(2-2\tau_i\beta_i+\hat{T}_i\beta_i)]$. There is no closed form solution for τ_i; however, given that $A_i(\tau_i)$ is convex, we can use a gradient search procedure to find τ_i easily.

The base payment is defined as $\rho_i(\tau_i)$, the bonus is then defined as $(p_i - \rho_i(\tau_i))/\tau_i$ (the absolute value of the first segment's slope), and the penalty is defined as $(\rho_i(\tau_i) - \rho_i(\hat{T}_i))/(\hat{T}_i - \tau_i)$ (the absolute value of the second segment's slope). Therefore, the piecewise linear approximation to contract $\mathbb{I}$, at equilibrium, is defined as

$$\begin{aligned} &\textit{Payment to ith subcontractor}\\ &\quad = \rho_i(\tau_i) + (\tau_i - t_i)^+\left[\frac{p_i - \rho_i(\tau_i)}{\tau_i}\right]\\ &\quad\quad - (t_i - \tau_i)^+\left[\frac{\rho_i(\tau_i) - \rho_i(\hat{T}_i)}{\hat{T}_i - \tau_i}\right]. \end{aligned}$$

We can extend this approach to $n+1$ segments, with n "deadlines," which would allow multiple levels of penalties and rewards. Finding these values requires the solution of a convex optimization problem that can be solved efficiently using standard algorithms and programs.

5. Conclusions and Extensions

We proposed and analyzed an "incentive payment" contract for a stochastic project that consists of $n \geq 1$ serial stages, where each stage is completed by an independent subcontractor. In the basic form of the "incentive payment" contract, the client pays each subcontractor an amount $p_i e^{-\beta_i t_i}$ at the conclusion of a subcontractor's stage or the entire project. The parameters $p_i > 0$ and $\beta_i \geq 0$ are revealed to each subcontractor at the beginning of the project.

The primary contribution of this paper is to analytically demonstrate the superiority of an incentive contract over a fixed price contract from the perspective of a client who wants to maximize his expected discounted profit in serial stochastic projects. We also showed how a client can calculate optimal parameters for these incentive contracts. Our analysis revealed several other significant implications as well. For example, we showed that the two incentive payment contracts (contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$) are equivalent with respect to expected profit for the client and subcontractors, as well as the expected makespan (we showed that this result also holds for the fixed price contracts $\mathbb{F}$ and $\mathbb{F}_{\mathbb{D}}$). However, we showed that there are significant differences between the incentive payment contracts (contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$) and the fixed payment contracts (contracts $\mathbb{F}$ and $\mathbb{F}_{\mathbb{D}}$). The client will always have a greater expected profit with an incentive payment contract, although subcontractors will have a greater expected profit with a fixed price contract. We also showed that the expected makespan is always less with an incentive type contract than with a fixed price contract. Unfortunately, the client is not always able to utilize an incentive payment contract; if subcontractors have large opportunity costs, then only contract $\mathbb{F}$ (with appropriate adjustments) can be utilized; if subcontractors have small opportunity costs, then contract $\mathbb{I}$ can be used, but the choice of parameters (i.e., β_i) is restricted. We also showed how an incentive payment contract can be applied in practice by deriving a piecewise linear approximation. In its simplest form, this approximation allows for a deadline, a penalty rate for late completion, and a reward rate for early completion.

Our results remain applicable if subcontractors and the client discount cash flows at different rates. Specifically, Proposition 1 still holds when the discount rate is replaced by subcontractor-dependent discount rates α_i. We found a generalization of the analysis of the client's equilibrium profit intractable, but we have confirmed via computational studies that our main findings still hold. Our results also remain applicable if the subcontractors discount their respective opportunity costs, O_i, from time $\sum_{j=1}^{i} t_j$ (assuming contract $\mathbb{I}$) based on the expected duration of preceding stages calculated using Equation (2).

We also studied the effect of changing subcontractors' risk preferences. Whereas we were unable to derive analytical results, our numerical studies indicated that when subcontractors are risk averse, the equilibrium work rates are strictly less than those of risk-neutral subcontractors. On the other hand, we observed that risk-taking subcontractors' equilibrium work rates are strictly greater than the risk-neutral counterparts.

There are a number of important extensions that should be considered in future work. First, our analysis has not directly considered indirect/overhead costs that vary with the duration of a project; examples include security costs and most costs relating to the management of the project. Indirect/overhead costs are typically allocated to projects as a linear function of the makespan of the project; these costs could be included by modifying the client's expected profit defined by (5); i.e., the client now receives an amount $[Q - C_o \sum_{i=1}^{n} t_i]$ at the completion of the project where C_o denotes the overhead cost per time period. Whereas our analytical results would likely change, the problem of maximizing expected client's profits could be solved numerically. The resultant incentive payment contract could then be converted to an I/D contract as described in the previous section. We are currently investigating this problem in more detail. Second, I/D contracts frequently include quality and/or scope incentives as well as budget and schedule incentives. Whereas these incentives are not part of this work, we are currently investigating how quality and/or scope goals can be included in an incentive payment contract and resultant I/D contract.

We are currently investigating two extensions that we feel are critically important. First, this work is the first (to our knowledge) that analytically compares incentive and fixed price contracts. We are currently working on extensions that compare our incentive payment contract with "cost plus" contracts that are also widely used in practice. Second, we are extending our results to projects that are characterized by general network topologies. Preliminary numerical results suggest that many of the results reported in this paper continue to hold in this case.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2015.0528.

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