33 Electromagnetic Induction

33.1 Induced Currents

33.2 Motional EMF

1. The figures below show one or more metal wires sliding on fixed metal rails in a magnetic field. For each, determine if the induced current flows clockwise, flows counterclockwise, or is zero.

   a. [Diagram: CW]

   b. [Diagram: CCW]

   c. [Diagram: Zero]

   d. [Diagram: CCW]

   e. [Diagram: Zero]

   f. [Diagram: Zero]

2. A loop of copper wire is being pulled from between two magnetic poles.

   a. Show on the figure the current induced in the loop. Explain your reasoning.

      The side of the loop between the poles is in a magnetic field that points up. This length of the loop moves to the right so the force on the charges in the wire is out of the page.

   b. Does either side of the loop experience a magnetic force? If so, draw a vector arrow or arrows on the figure to show any forces. Yes.

   c. Label the magnetic poles of the induced current in the loop. Do this on the figure.

   d. Are the magnetic poles you labeled in part c attracted to or repelled by the permanent magnet? Attracted.
e. Is your answer to part d consistent with your force vectors in part b? Yes.

3. You want to insert a loop of copper wire between two permanent magnets. Is there an attractive magnetic force that tends to pull the loop in, like a magnet pulls on a paper clip? Or do you need to push the loop in against a repulsive force? Give a step-by-step analysis to support your answer. Push against repulsive force. The motion of the loop in the magnetic field induces a current in the loop in the direction shown. The magnetic poles of the induced current loop are also shown in the diagram. The magnetic force on the loop is to the right.

4. A vertical, rectangular loop of copper wire is half in and half out of a horizontal magnetic field. (The field is zero beneath the dotted line.) The loop is released and starts to fall.

a. Add arrows to the figure to show the direction of the induced current in the loop.

b. Is there a net magnetic force on the loop? If so, in which direction? Explain.

Yes. The net force is up since there is only an upward force on the top side of the loop and no forces on the remaining three sides. (The bottom edge does not move through a magnetic field.)
5. Two very thin sheets of copper are pulled through a magnetic field. Do eddy currents flow in the sheet? If so, show them on the figures, with arrows to indicate the direction of flow. If not, why not?

a.  

b.  

No eddy currents because the force is perpendicular to the plane.

6. The figure shows an edge view of a copper sheet being pulled between two magnetic poles.

a. Add a dot or an $\times$ to each of the circles to indicate the direction in which eddy currents are flowing in and out of the page.

b. Label the magnetic poles of any induced current loops.

c. Do the magnetic poles you labeled in part b experience magnetic forces? If so, add force vectors to the figure to show the directions. If not, why not?

Yes. The eddy current on the left experiences a repulsive force exerted by the magnetic poles. The eddy current on the right experiences an attractive force.

d. Is there a net magnetic force on the copper sheet? If so, in which direction?

Yes. To the left.
7. An insulating rod pushes a copper loop back and forth. The left edge of the loop, which is always in the magnetic field, oscillates between \( x = -L \) and \( x = +L \), as shown in the top graph. The right edge of the loop, which includes a lightbulb, is always outside the magnetic field.

a. Draw the velocity graph for the loop. Make sure it aligns with the position graph above it.

b. Draw a graph of the induced current in the loop as a function of time. Let a clockwise current be a positive number and a counterclockwise current be a negative number.

c. Draw a graph of the brightness of the lightbulb as a function of time.

Note: There are no numbers on the vertical scale. The shape of each graph is the important result.
33.3 Magnetic Flux

8. The figure shows five loops in a magnetic field. The numbers indicate the lengths of the sides and the strength of the field. Rank in order, from largest to smallest, the magnetic fluxes \( \Phi_1, \ldots, \Phi_5 \). Some may be equal.

Order: \( \Phi_1 = \Phi_4 > \Phi_2 = \Phi_3 = \Phi_5 \)

Explanation:
\[
\Phi = A_{\text{eff}} B \\
\Phi_1 = 2 \cdot 2 \cdot 1 = 4 \\
\Phi_2 = 1 \cdot 1 \cdot 2 = 2 \\
\Phi_3 = 1 \cdot 2 \cdot 1 = 2 \\
\Phi_4 = 2 \cdot 1 \cdot 2 = 4 \\
\Phi_5 = 1 \cdot 1 \cdot 2 = 2
\]

9. The figure shows four circular loops that are perpendicular to the page. The radius of loops 3 and 4 is twice that of loops 1 and 2. The magnetic field is the same for each. Rank in order, from largest to smallest, the magnetic fluxes \( \Phi_1, \ldots, \Phi_4 \). Some may be equal.

Order: \( \Phi_3 > \Phi_1 > \Phi_2 > \Phi_4 \)

Explanation:
\[
\Phi = B \cdot A \cdot \cos \theta = B (\pi r^2) \cos \theta \\
\Phi_1 = B \pi (1)^2 \cos 0^\circ = \pi B \\
\Phi_2 = B \pi (1)^2 \cos 45^\circ = 0.707 \Phi_1 \\
\Phi_3 = B \pi (4) \cos 45^\circ = 2.83 \Phi_1 \\
\Phi_4 = B \pi (4) \cos 90^\circ = 0
\]
10. A circular loop rotates at constant speed about an axle through the center of the loop. The figure shows an edge view and defines the angle \( \phi \), which increases from 0\(^\circ\) to 360\(^\circ\) as the loop rotates.

a. At what angle or angles is the magnetic flux a maximum?

\[ 0^\circ \text{ and } 180^\circ \]

b. At what angle or angles is the magnetic flux a minimum?

\[ 90^\circ \text{ and } 270^\circ \]

c. At what angle or angles is the magnetic flux changing most rapidly? Explain your choice.

At 90\(^\circ\) and 270\(^\circ\). The flux is \( \Phi = ABC \cos \phi \)
and \( \cos \phi \) changes most rapidly (has the steepest slope) at 90\(^\circ\) and 270\(^\circ\).

11. A magnetic field is perpendicular to a loop. The graph shows how the magnetic field changes as a function of time, with positive values for \( B \) indicating a field into the page and negative values a field out of the page. Several points on the graph are labeled.

a. At which lettered point or points is the flux through the loop a maximum?

\[ b \text{ and } d \]

b. At which lettered point or points is the flux through the loop a minimum?

\[ c \]

c. At which point or points is the flux changing most rapidly?

\[ c \]

d. At which point or points is the flux changing least rapidly?

\[ b \text{ and } d \]
33.4 Lenz's Law

33.5 Faraday's Law

12. Does the loop of wire have a clockwise current, a counterclockwise current, or no current under the following circumstances? Explain.
   a. The magnetic field points out of the page and its strength is increasing.  
   
   **Clockwise.** The induced magnetic field is into the page and by the right hand rule the induced current is clockwise. 
   b. The magnetic field points out of the page and its strength is constant.  
   
   **No current.** A changing magnetic flux induces a current. 
   c. The magnetic field points out of the page and its strength is decreasing.  
   
   **Counterclockwise.** The induced $\mathbf{B}$ is out of the page.

13. Two loops of wire are stacked vertically, one above the other. Does the upper loop have a clockwise current, a counterclockwise current, or no current at the following times? Explain.
   a. Before the switch is closed.  
   
   **No current.** There is not magnetic flux that is changing.
   b. Immediately after the switch is closed.  
   
   **Clockwise.** The lower loop has a new current (looking down) and its flux is increasing. An induced $\mathbf{B}$ will be down.
   c. Long after the switch is closed.  
   
   **No current because the magnetic flux is no longer changing.**
   d. Immediately after the switch is reopened.  
   
   **Counterclockwise.** The original $\mathbf{B}$ is pointing up but decreasing so $\mathbf{B}$ and also points up.
14. A loop of wire is perpendicular to a magnetic field. The magnetic field strength as a function of time is given by the top graph. Draw a graph of the current in the loop as a function of time. Let a positive current represent a current that comes out of the top of the loop and enters the bottom of the loop. There are no numbers for the vertical axis, but your graph should have the correct shape and proportions.

15. A loop of wire is horizontal. A bar magnet is pushed toward the loop from below, along the axis of the loop.
   a. What is the current direction in the loop? Explain.
      
      Clockwise as seen from above. The $\vec{B}_{\text{original}}$ is up and increasing so $\vec{B}_{\text{induced}}$ must be down.

   b. Is there a magnetic force on the loop? If so, in which direction? Explain. Yes. The force is up.
      
      Hint: A current loop is a magnetic dipole.
      The current loop is a magnetic dipole with N pole down and south pole up. The loop experiences a repulsive force.

   c. Is there a force on the magnet? If so, in which direction?
      Yes. Down.

16. A bar magnet is pushed toward a loop of wire, as shown. Is there a current in the loop? If so, in which direction? If not, why not?

   No. The net magnetic field through the loop is zero.
17. A bar magnet is dropped, south pole down, through the center of a loop of wire. The center of the magnet passes the plane of the loop at time $t_c$.

- Sketch a graph of the magnetic flux through the loop as a function of time.
- Sketch a graph of the current in the loop as a function of time. Let a clockwise current be a positive number and a counterclockwise current be a negative number.

18. a. As the magnet is inserted into the coil, does current flow right to left or left to right through the current meter? Or is the current zero? Explain. Right to left. $\vec{B}_{\text{original}}$ is right to left. $\vec{I}_{\text{original}}$ is increasing. Therefore $\vec{B}_{\text{induced}}$ is left to right leading to the current direction.

b. As the magnet is held at rest inside the coil, does current flow right to left or left to right through the current meter? Or is the current zero? Explain. The current is zero. The $\vec{B}_{\text{original}}$ being constant means that the change in flux is zero.

c. As the magnet is withdrawn from the coil, does current flow right to left or left to right through the current meter? Or is the current zero? Explain. Left to right. Now, although $\vec{B}_{\text{original}}$ is still right to left, it is decreasing through the loops so $\vec{B}_{\text{induced}}$ is also right to left.

d. If the magnet is inserted into the coil more rapidly than in part a, does the size of the current increase, decrease, or remain the same? Explain. Increases. 

$$\mathcal{E} = \left| \frac{d \Phi}{dt} \right|$$ so that if the flux through the coil increases more rapidly then $\mathcal{E}$ is larger and therefore $I = \frac{\mathcal{E}}{R}$ is also larger.
19. a. Just after the switch on the left coil is closed, does current flow right to left or left to right through the current meter of the right coil? Or is the current zero? Explain.

The original magnetic field is left to right through both coils and right when the switch closes the flux is increasing. So $\vec{B}_{\text{induced}}$ is right to left through the right coil.

b. Long after the switch on the left coil is closed, does current flow right to left or left to right through the current meter of the right coil? Or is the current zero? Explain.

The current is zero because the $\vec{B}_{\text{original}}$ and the flux through the second coil are not changing.

c. Just after the switch on the left coil is reopened, does current flow right to left or left to right through the current meter of the right coil? Or is the current zero? Explain.

The original magnetic field is still left to right but now it is decreasing. $\vec{B}_{\text{induced}}$ is also left to right.

20. A solenoid is perpendicular to the page, and its field strength is increasing. Three circular wire loops of equal radii are shown. Rank in order, from largest to smallest, the size of the induced emf in the three rings.

Order: $E_1 = E_2 > E_3$

Explanation:

The magnetic field is inside the solenoid only. No field exists in Loop 3 so $E_3 = 0$. Loop 1 and Loop 2 have the same $B$ field over the same area changing at the same rate.
21. A conducting loop around a magnetic field contains two lightbulbs, A and B. The wires connecting the bulbs are ideal, with no resistance. The magnetic field is increasing rapidly.
   a. Do the bulbs glow? Why or why not?

   Yes. The flux through the loop changes with time leading to an induced emf and induced current in the wire.

   b. If they glow, which bulb is brighter? Or are they equally bright? Explain.

   They are equally bright because the same induced current flows through each bulb. (Assuming the bulbs are identical.)

22. A conducting loop around a magnetic field contains three lightbulbs, A, B, and C. The wires connecting the bulbs are ideal, with no resistance. The magnetic field is increasing rapidly. Rank in order, from brightest to least bright, the brightness of the three bulbs.

   Order: \( b_A = b_B = b_C \) all equally bright.

   Explanation:

   The same induced current flows through each bulb according to conservation of current.
23. A metal wire is resting on a U-shaped conducting rail. The rail is fixed in position, but the wire is free to move.

a. If the magnetic field is increasing in strength, does the wire:
   i. Remain in place?  vi. Move out of the plane of the page, breaking contact with the rail?
   ii. Move to the right?  vii. Rotate clockwise?
   iii. Move to the left?  viii. Rotate counterclockwise?
   iv. Move up on the page?  ix. Some combination of these? If so, which?

Explain your choice.

The wire moves to the left. \( \vec{B}_{\text{original}} \) is into the page and increasing so \( \vec{B}_{\text{induced}} \) is out of the page. This leads to a counterclockwise induced current. By the right hand rule, with a current going up through the sliding wire and \( \vec{B}_{\text{net}} \) into the page, the force exerted by the field on the wire is left.

b. If the magnetic field is decreasing in strength, which of the above happens?

Now \( \vec{B}_{\text{induced}} \) is into the page and the induced current is clockwise. The magnetic force on the wire is to the right. The field extends to the right beyond the rail so the wire will break contact. At that point the force on the wire will drop to zero.

### 33.6 Induced Field and Electromagnetic Waves

### 33.7 Induced Current: Three Applications

No exercises.
33.8 Inductors

24. The figure shows the current through an inductor. A positive current is defined as a current going from top to bottom. At the time corresponding to each of the labeled points, does the potential across the inductor (going from top to bottom) increase, decrease, or stay the same?

a. decreases
e. increases
b. same
f. same
c. increases
g. decreases
d. increases

25. a. Can you tell which of these inductors has the larger current flowing through it? If so, which one? If not, why not?

No. \( \Delta V = -L \frac{dI}{dt} \) with \( \Delta V \) and \( L \) both known we can only find \( \frac{dI}{dt} \), not \( I \).

b. Can you tell through which inductor the current is changing most rapidly? If so, which one? If not, why not?

Yes. Through the 1 H inductor.

c. If the current enters the inductor from the bottom, can you tell if the current is increasing, decreasing, or staying the same? If so, which one and what is your reasoning? If not, why not?

Yes you can tell. The current is decreasing.

If \( \frac{dI}{dt} < 0 \) then \( \Delta V > 0 \) and the input side is more negative. The potential increases in the direction of the current.

26. Rank in order, from most positive to most negative, the inductor's potential difference \((\Delta V_L)_a, (\Delta V_L)_b, \ldots, (\Delta V_L)_f\) at the six labeled points. \( \Delta V_L \) is the change in going from the top of the inductor to the bottom. Some may be equal.

Note that 0 V > -2 V.

Order: \( (\Delta V_L)_a > (\Delta V_L)_b > (\Delta V_L)_c > (\Delta V_L)_d > (\Delta V_L)_e > (\Delta V_L)_f \)

Explanation:

When the current decreases the potential increases and \( \Delta V \) is positive. This occurs for points a and b and they have equal slopes. No current change at c and f means \( \Delta V = 0 \).

When the current increases the potential decreases and \( \Delta V < 0 \). Slope of d is less than slope of e but they are negative numbers.
33.9 LC Circuits

27. An LC circuit oscillates at a frequency of 2000 Hz. What will the frequency be if the inductance is quadrupled?

\[ f_2 = \frac{2\pi}{2\pi} \sqrt{\frac{1}{4LC}} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} = \frac{1}{2} \sqrt{\frac{1}{4L_1C}} = \frac{1}{2} f_1 \]

28. The capacitor in an LC circuit has maximum charge at \( t = 1 \mu s \). The current through the inductor next reaches a maximum at \( t = 3 \mu s \).

a. When will the inductor current reach a maximum in the opposite direction?

\[ t = 7 \mu s \]

The capacitor and inductor are 90° out of phase. So \( \frac{1}{4} \) of the period is equal to 2\( \mu s \). We need to add \( \frac{1}{2} T = 4 \mu s \) to the 3\( \mu s \).

b. What is the circuit’s period of oscillation?

\[ \frac{1}{4} T = 2 \mu s \]

\[ T = 8 \mu s \]

29. Three LC circuits are made with the same capacitor but different inductors. The figure shows the inductor current as a function of time. Rank in order, from largest to smallest, the three inductances \( L_1, L_2, \) and \( L_3 \). Some may be equal.

Order: \( L_2 > L_1 = L_3 \)

Explanation:

From the graph \( T_3 = T_1 > T_2 \) \( \omega = 2\pi f = \frac{2\pi}{T} \)

so that \( \omega_3 = \omega_1 > \omega_2 \)

But \( \omega = \sqrt{\frac{1}{LC}} \) so \( L = \frac{1}{\omega^2 C} \)

Therefore \( L_2 > L_1 = L_3 \)
33.10 LR Circuits

30. Rank in order, from largest to smallest, the three time constants $\tau_1$, $\tau_2$, and $\tau_3$ for these three circuits.

Order: $\tau_3 > \tau_1 > \tau_2$

Explanation:
$$\tau_1 = \frac{L}{R} \quad \tau_2 = \frac{L}{2R} = \frac{1}{2} \tau_1 \quad \tau_3 = \frac{L}{(R/2)} = 2 \tau_1$$

31. Three LR circuits are made with the same resistor but different inductors. The figure shows the inductor current as a function of time. Rank in order, from largest to smallest, the three inductances $L_1$, $L_2$, and $L_3$.

Order: $L_3 > L_2 > L_1$

Explanation: $I = I_0 e^{-\frac{t}{\tau LR}}$

As $L$ increases, $I$ decreases less rapidly.

32. a. What is the battery current immediately after the switch closes? Explain. Zero.

The potential across the resistor is zero and there is a zero initial current because the current in the inductor can't change instantaneously.

b. What is the battery current after the switch has been closed a long time? Explain.

$$I = \frac{E}{R} = \frac{10 \, V}{5 \, \Omega} = 2 \, A$$

The current has reached its final steady state and $\frac{dI}{dt} = 0$. 