30 Potential and Field

30.1 Connecting Potential and Field

1. The top graph shows the $x$-component of $\vec{E}$ as a function of $x$. On the axes below the graph, draw the graph of $V$ versus $x$ in this same region of space. Let $V = 0$ V at $x = 0$ m. Include an appropriate vertical scale. (Hint: Integration is the area under the curve.)

$$\Delta V = - \int_{0}^{2} E_x \, dx$$

The electric field is the negative of the slope of the $V$ vs. $x$ graph.

![Graph showing electric field and potential graphs]

2. a. Suppose that $\vec{E} = \vec{0}$ V/m throughout some region of space. Can you conclude that $V = 0$ V in this region? Explain.

No. But you can conclude that $\Delta V = 0$ V—that the potential is not changing in that region of space.

b. Suppose that $V = 0$ V throughout some region of space. Can you conclude that $\vec{E} = \vec{0}$ V/m in this region? Explain.

Yes. As long as $V = $ constant in the region of space then $\vec{E} = \vec{0} \frac{V}{m}$.

$$E = - \frac{dV}{ds}$$
30.2 Finding the Electric Field from the Potential

3. The top graph on the right shows the electric potential as a function of x. On the axes below the graph, draw the graph of $E_x$ versus $x$ in this same region of space.

If $V$ is constant in a region then $\Delta V = 0$ and therefore $E = 0$ also in that region.

4. For each contour map:
   i. Estimate the electric fields $E_a$ and $E_b$ at points a and b. Don’t forget that $E$ is a vector. Show how you made your estimate.
   ii. Draw electric field vectors on top of the contour map.

   a. $0 \text{ V} \quad 10 \text{ V} \quad 20 \text{ V} \quad 30 \text{ V} \quad 40 \text{ V}$
      
      $\overrightarrow{E_a}$
      $\overrightarrow{E_b}$
      
      $0 \text{ m} \quad 1 \text{ m} \quad 2 \text{ m} \quad 3 \text{ m} \quad 4 \text{ m}$

      
      $-\frac{\Delta V}{\Delta s} = \frac{10 \text{ V}}{1 \text{ m}} = 10 \frac{\text{ V}}{\text{ m}}$
      $E_a = 10 \frac{\text{ V}}{\text{ m}}$
      $E_b = \frac{V}{m}$

   b. $0 \text{ V} \quad 10 \text{ V} \quad 20 \text{ V} \quad 30 \text{ V} \quad 40 \text{ V}$
      
      $\overrightarrow{E_a}$
      $\overrightarrow{E_b}$
      
      $0 \text{ m} \quad 1 \text{ m} \quad 2 \text{ m} \quad 3 \text{ m} \quad 4 \text{ m}$

      
      $\frac{10 \text{ V} - 0 \text{ V}}{2 \text{ m} - 0 \text{ m}} = 5 \frac{\text{ V}}{\text{ m}}$
      $E_a = 5 \frac{\text{ V}}{\text{ m}}$
      $E_b = \frac{10 \text{ V} - 20 \text{ V}}{4 \text{ m} - 3 \text{ m}} = 20 \frac{\text{ V}}{\text{ m}}$

The minus sign in the equation means that the electric field vector points “downhill” on the contour map.
5. The top graph shows \( E_x \) versus \( x \) for an electric field that is parallel to the \( x \)-axis.
   a. Draw the graph of \( V \) versus \( x \) in this region of space. Let \( V = 0 \) \( \text{V} \) at \( x = 0 \) \( \text{m} \). Add an appropriate scale on the vertical axis. (Hint: Integration is the area under the curve.)
   b. Draw a contour map above the \( x \)-axis on the right. Space your equipotential lines every 20 volts and label each equipotential line.
   c. Draw electric field vectors on top of the contour map.

   ![Contour Map](image)

6. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.

   ![Electric Field Vectors](image)

7. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.
8. Rank in order, from largest to smallest, the electric field strengths $E_1$, $E_2$, $E_3$, and $E_4$ at the four labeled points.

Order: $E_3 > E_1 > E_2 = E_4$

Explanation:
The electric field strength is largest in the region where the potential lines are closest together.

9. For each of the figures below, is this a physically possible potential map if there are no free charges in this region of space? If so, draw an electric field line diagram on top of the potential map. If not, why not?

Potential lines will never cross because no single point can have two different potential values.

10. The figure shows an electric field diagram. Dotted lines 1 and 2 are two surfaces in space, not physical objects.

a. Is the electric potential at point a higher than, lower than, or equal to the electric potential at point b? Explain.

The electric field vector points toward decreasing potential. Therefore $V_a > V_b$.

b. Rank in order, from largest to smallest, the potential differences $\Delta V_{ab}$, $\Delta V_{cd}$, and $\Delta V_{ef}$.

Order: $\Delta V_{ab} > \Delta V_{cd} > \Delta V_{ef}$

Explanation:

$\Delta V = -E \Delta S$

$\Delta S_{ab} < \Delta S_{cd} < \Delta S_{ef}$  The minus sign reverses the inequality direction.

c. Is surface 1 an equipotential surface? What about surface 2? Explain why or why not.

Yes, surface 1 is an equipotential surface because it is perpendicular to the electric field vectors. Surface 2 is not perpendicular so it is not an equipotential surface.
30.3 A Conductor in Electrostatic Equilibrium

11. The figure shows a negatively charged electroscope. The gold leaf stands away from the rigid metal post. Is the electric potential of the leaf higher than, lower than, or equal to the potential of the post? Explain. \( E_{\text{equal}} \)

When a conductor is in electrostatic equilibrium, the entire conductor is at the same potential.

12. Two metal spheres are connected by a metal wire that has a switch in the middle. Initially the switch is open. Sphere 1, with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward, sphere 1 has charge \( Q_1 \), is at potential \( V_1 \), and the electric field strength at its surface is \( E_1 \). The values for sphere 2 are \( Q_2 \), \( V_2 \), and \( E_2 \).

a. Is \( V_1 \) larger than, smaller than, or equal to \( V_2 \)? Explain.

\[ V_1 = V_2 \]

Both spheres and the wire become one conductor all at the same potential.

b. Is \( Q_1 \) larger than, smaller than, or equal to \( Q_2 \)? Explain.

\[ \frac{1}{4\pi \varepsilon_0} \frac{Q_1}{r_1} = \frac{1}{4\pi \varepsilon_0} \frac{Q_2}{r_2} \]

\[ \frac{Q_1}{r_1} = \frac{Q_2}{r_2} \]

\[ r_1 > r_2 \Rightarrow Q_1 > Q_2 \]

13. The figure shows a hollow metal shell. A negatively charged rod touches the top of the sphere, transferring charge to the shell. Then the rod is removed.

a. Show on the figure the equilibrium distribution of charge.

b. Draw the electric field diagram.

Note: Charges should have been shown as negative!
14. The figure shows two flat metal electrodes that are held at potentials of 100 V and 0 V.
   a. Sketch a reasonable approximation of the 20 V, 40 V, 60 V, and 80 V equipotential lines.
   b. Draw enough electric field lines to indicate the shape of the electric field.
30.4 Sources of Potential

30.5 Connecting Potential and Current

15. The figure shows two 3 V batteries with metal wires attached to each end. Points a and c are inside the wire. Point b is inside the battery. For each figure:

- What are the potential differences $\Delta V_{12}$, $\Delta V_{23}$, $\Delta V_{34}$, and $\Delta V_{14}$?
- Does the electric field at a, b, and c point left, right, up, or down? Or is $\vec{E} = 0$?

\[ \Delta V_{12} = 0 \text{ V} \quad \Delta V_{23} = 3 \text{ V} \]
\[ \Delta V_{34} = 0 \text{ V} \quad \Delta V_{14} = 3 \text{ V} \]
\[ \vec{E}_a \text{ ZERO } \vec{E}_b \text{ Down } \vec{E}_c \text{ ZERO } \]

16. A continuous metal wire connects the two ends of a 3 V battery with a rectangular loop. The negative terminal of the battery has been chosen as the point where $V = 0$ V.

a. Locate and label the approximate points along the wire where $V = 3$ V, $V = 2$ V, and $V = 1$ V.

b. Points a and c are inside the wire. Point b is inside the battery. Does the electric field at a, b, and c point left, right, up, or down? Or is $\vec{E} = 0$?

\[ \vec{E}_a \text{ Right } \vec{E}_b \text{ Down } \vec{E}_c \text{ Left } \]

c. Estimate the value of $\Delta V_{14}$. Explain how you did so.

\[ \Delta V_{14} \approx 0.5 \text{ V} \text{ Because } E_{\text{wire}} = \frac{\Delta V_{\text{wire}}}{L}, \text{ and } E_{\text{wire}} \text{ is a constant, the change in potential is linearly related to the position along the wire.} \]

d. In moving through the wire from point 2 to point 3, does the potential increase, decrease, or not change? If the potential changes, by how much does it change?

The potential decreases and the decreases is linear with the distance traveled around the wire. \[ \Delta V_{23} = -3 \text{ V} \]
e. In moving through the battery from point 2 to point 3, does the potential increase, decrease, or not change? If the potential changes, by how much does it change?

The potential decreases by 3 V.

f. In moving all the way around the loop in a clockwise direction, starting from point 2 and ending at point 2, is the net change in the potential positive, negative, or zero?

Zero. \( \Delta V_{23} = -3 \, V \) and \( \Delta V_{32} = +3 \, V \)

17. a. Which direction—clockwise or counterclockwise—does an electron travel through the wire? Explain.

Counterclockwise. The electron will be repelled by the negative electrode of the battery and attracted to the positive electrode.

b. Does an electron’s electric potential energy increase, decrease, or stay the same as it moves through the wire? Explain.

Decreases. A charge always moves in the direction of decreasing potential energy as it gains kinetic energy.

c. If you answered “decrease” in part b, where does the energy go? If you answered “increase” in part b, where does the energy come from?

The energy goes into the increase in the kinetic energy of the electron and therefore into an increase in its speed.

d. Which way—up or down—does an electron move through the battery? Explain.

Down. The chemical reactions in the electrolytes separate the charges by moving positive charges up and negative charges down.

e. Does an electron’s electric potential energy increase, decrease, or stay the same as it moves through the battery? Explain.

Increase. The chemical reaction does work to separate the charges. As the electron is pushed toward the negative electrode, its potential energy increases.

f. If you answered “decrease” in part e, where does the energy go? If you answered “increase” in part e, where does the energy come from?

The energy comes from the work done on the charges by the chemical reactions in the electrolytes.
18. The wires below are all made of the same material. Rank in order, from largest to smallest, the resistances $R_1$ to $R_5$ of these wires.

Order: $R_4 > R_1 = R_5 > R_3 > R_2$

Explanation:

$R = \frac{\rho L}{\pi R^2}$

$R_1 = \frac{\rho L}{\pi (2R)^2} = \frac{1}{4} R_1$

$R_2 = \frac{\rho (2L)}{(\pi)(2R)^2} = \frac{1}{2} R_1$

$R_3 = \frac{\rho (2L)}{(\pi)(2R)^2} = \frac{1}{2} R_1$

$R_4 = \frac{\rho (2L)}{\pi R^2} = 2R_1$

$R_5 = \frac{\rho (4L)}{(\pi)(2R)^2} = R_1$

19. A wire is connected to the terminals of a 6 V battery. What is the potential difference $\Delta V_{\text{ends}}$ between the ends of the wire, and what is the current $I$ through the wire, if the wire has the following resistances:

a. $R = 1 \, \Omega$  
   $\Delta V_{\text{ends}} = 6 \, V$  
   $I = 6 \, A$

b. $R = 2 \, \Omega$  
   $\Delta V_{\text{ends}} = 6 \, V$  
   $I = 3 \, A$

c. $R = 3 \, \Omega$  
   $\Delta V_{\text{ends}} = 6 \, V$  
   $I = 2 \, A$

d. $R = 6 \, \Omega$  
   $\Delta V_{\text{ends}} = 6 \, V$  
   $I = 1 \, A$

20. The two circuits use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents $I_1, I_2, I_3,$ and $I_4$ at points 1 to 4.

Order: $I_1 = I_2 > I_3 = I_4$

Explanation:

Conservation of current requires $I_1 = I_2$ and $I_3 = I_4$.

However, since the wire on the right is longer then its resistance is greater ($R = \frac{\rho L}{A}$) and therefore the current is smaller ($I = \frac{V}{R}$).
21. The two circuits use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents $I_1$ to $I_7$ at points 1 to 7.

Order: $I_1 = I_2 = I_3 = I_5 = I_6 > I_4 = I_7$

Explanation:

Conservation of current requires $I_1 = I_2 = I_3$.

The current in each wire is $I = \Delta V/R$. The wires are identical and have the same resistance. The batteries have the same potential difference. Therefore $I_5 = I_6 = I_7$.

From conservation of current, the current at point 4 must go partially to point 5 and partially to point 6. Therefore $I_4 < I_5$.

22. Which, if any, of these statements are true? (More than one may be true.)

i. A battery supplies the energy to a circuit.

ii. A battery is a source of potential difference. The potential difference between the terminals of the battery is always the same.

iii. A battery is a source of current. The current leaving the battery is always the same.

Explain your choice or choices.

i. True. The chemical reactions in the electrolytes separate the positive and negative charges. This creates a potential difference. The charges flowing in the circuit have energy due to this potential difference.

ii. It is true that a battery is a source of potential difference. But the potential difference is always the same ONLY for an ideal battery. In a real battery there are energy losses so the terminal voltage is not always the same.

iii. False. The current leaving the battery depends upon the resistance in the circuit.
30.6 Capacitance and Capacitors

30.7 The Energy Stored in a Capacitor

23. A parallel-plate capacitor with plate separation $d$ is connected to a battery that has potential difference $\Delta V_{\text{bat}}$. Without breaking any of the connections, insulating handles are used to increase the plate separation to $2d$.

a. Does the potential difference $\Delta V_C$ change as the separation increases? If so, by what factor? If not, why not?

No, it does not change because the upper plate is still connected to the positive electrode of the battery and the bottom plate to the negative electrode.

b. Does the capacitance change? If so, by what factor? If not, why not?

Yes. $C = \frac{\varepsilon A}{d}$. So when the plate separation is doubled, the capacitance decreases by a factor of 2.

c. Does the capacitor charge $Q$ change? If so, by what factor? If not, why not?

Yes. It also decreases by a factor of 2.

$C = \frac{Q}{\Delta V_C}$

d. As the plates are being pulled apart, does current flow cw, ccw, or not at all? Explain.

The charge flows ccw. Positive charge is added to the negative plate to decrease the net charge on the capacitor.

24. For the capacitor shown, the potential difference $\Delta V_{ab}$ between points a and b is

a. $6 \text{ V}$

b. $6\sin30^\circ \text{ V}$

c. $6\sin30^\circ \text{ V}$

d. $6\tan30^\circ \text{ V}$

e. $6\cos30^\circ \text{ V}$

f. $6\cos30^\circ \text{ V}$

Explain your choice.

The entire top plate is at the same potential as the positive electrode of the battery ($6 \text{ V}$) and the bottom plate is at the same potential as the negative electrode ($0 \text{ V}$). Each capacitor plate is an equipotential surface.
25. Rank in order, from largest to smallest, the potential differences \((\Delta V_c)_1\) to \((\Delta V_c)_4\) of these four capacitors.

\[
\begin{array}{c|c|c|c|c}
& 1 & 2 & 3 & 4 \\
\hline \Delta V_c & \frac{Q}{C} & \frac{Q}{2C} & \frac{2Q}{C} & \frac{2Q}{2C} \\
\hline C & + & - \quad -Q & + & - \quad -2Q & + & - \quad -3Q & + & - \quad -2Q \\
\end{array}
\]

Order: \((\Delta V_c)_3 > (\Delta V_c)_1 = (\Delta V_c)_4 > (\Delta V_c)_2\)

Explanation:
\[
(\Delta V_c)_1 = \frac{Q}{C}, \quad (\Delta V_c)_2 = \frac{Q}{2C} = \frac{1}{2}(\Delta V_c)_1, \quad (\Delta V_c)_3 = \frac{2Q}{C} = 2(\Delta V_c)_1, \quad (\Delta V_c)_4 = \frac{2Q}{2C} = (\Delta V_c)_1.
\]

26. Light bulbs can be used to indicate current flow in a circuit. The brightness of a bulb is proportional to the amount of current passing through it. The figure shows a battery, a switch, two light bulbs, and a capacitor that is initially uncharged.

a. Immediately after the switch is closed, are either or both bulbs glowing? Explain.

Both bulbs are glowing because current is flowing in both wires as the capacitor is being charged.

b. If both bulbs are glowing, which is brighter? Or are they equally bright? Explain.

They are equally bright because the currents are the same. The same amount of positive charge is flowing off the bottom plate and onto the upper plate in the same time.

c. For any bulb (A or B or both) that lights up immediately after the switch is closed, does its brightness increase with time, decrease with time, or remain unchanged? Explain.

Its brightness decreases with time. As the charge on the plates increases, the potential differences along the wire decrease and so does the current in the wire.
27. Each capacitor in the circuits below has capacitance $C$. What is the equivalent capacitance of the group of capacitors?

a. \[ C_{eq} = \frac{C}{2} \]

b. \[ C_{eq} = 2C \]

c. \[ C_{eq} = \frac{C}{3} \]

d. \[ C_{eq} = 3C \]

e. \[ C_{eq} = C \]

f. \[ C_{eq} = \frac{2C}{5} \]

28. Rank in order, from largest to smallest, the equivalent capacitances $(C_{eq})_1$ to $(C_{eq})_4$ of these four groups of capacitors.

Order: \[(C_{eq})_2 > (C_{eq})_4 > (C_{eq})_3 > (C_{eq})_1\]

Explanation:

\[(C_{eq})_1 = \left( \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \right)^{-1} = \frac{1}{3} C \quad (C_{eq})_2 = C + C + C = 3C \]

\[(C_{eq})_3 = \left( \frac{1}{2C} + \frac{1}{C} \right)^{-1} = \frac{2}{3} C \quad (C_{eq})_4 = \frac{C}{2} + C = \frac{3}{2} C \]
29. Rank in order, from largest to smallest, the energies \((U_C)_1\) to \((U_C)_4\) stored in each of these capacitors.

\[
\begin{array}{cccc}
C & \frac{1}{2}C & \frac{1}{2}C & \frac{1}{2}C \\
\frac{\Delta V}{2} & \frac{\Delta V}{2} & \frac{\Delta V}{2} & \frac{\Delta V}{2} \\
1 & 2 & 3 & 4
\end{array}
\]

Order: \((U_C)_2 > (U_C)_1 > (U_C)_3 > (U_C)_4\)

Explanation:

\[
\begin{align*}
U_1 &= \frac{1}{2} C \left( \frac{\Delta V}{2} \right)^2 \\
U_2 &= \frac{1}{2} \left( \frac{1}{2} C \right) \left( 2 \Delta V \right)^2 = 2 U_1 \\
U_3 &= \frac{1}{2} \left( 2C \right) \left( \frac{1}{2} \Delta V \right)^2 = \frac{1}{2} U_1 \\
U_4 &= \frac{1}{2} \left( 4C \right) \left( \frac{1}{3} \Delta V \right)^2 = \frac{4}{9} U_1
\end{align*}
\]