33.1. **Model:** Assume the magnetic field is uniform.  
**Visualize:** Please refer to Figure Ex33.1. Since a motional emf was developed, the field must be perpendicular to \( \vec{v} \). The positive charges experienced a magnetic force to the left. By the right-hand rule, the field must be out of the page so that \( \vec{v} \times \vec{B} \) is to the left.  
**Solve:** This is a straightforward use of Equation 33.3. We have  
\[
B = \frac{E}{vl} = \frac{0.050 \text{ V}}{(5.0 \text{ m/s})(0.10 \text{ m})} = 0.10 \text{ T}
\]  
**Assess:** This is reasonable. Laboratory fields are typically up to a few teslas in magnitude.

33.2. **Visualize:**  
\[
\times \vec{B}
\]

To develop a motional emf the magnetic field needs to be perpendicular to both, so let’s say its direction is into the page.  
**Solve:** This is a straightforward use of Equation 33.3. We have  
\[
v = \frac{E}{IB} = \frac{1.0 \text{ V}}{(1.0 \text{ m})(5.0 \times 10^{-3} \text{ T})} = 2.0 \times 10^4 \text{ m/s}
\]  
**Assess:** This is an unreasonable speed for a car. It’s unlikely you’ll ever develop a volt.

33.4. **Model:** Assume the field changes abruptly at the boundary between the two sections.  
**Visualize:** Please refer to Figure Ex33.4. The directions of the fields are opposite, so some flux is positive and some negative. The total flux is the sum of the flux in the two regions.  
**Solve:** The field is constant in each region so we will use Equation 33.10. Take \( \vec{A} \) to be into the page. Then, it is parallel to the field in the left region so the flux is positive, and it is opposite to the field in the right region so the flux is negative. The total flux is  
\[
\Phi = A_l B_l \cos \theta_l + A_r B_r \cos \theta_r
\]
\[
= (0.20 \text{ m})^2 (2.0 \text{ T}) - (0.20 \text{ m})^2 (1.0 \text{ T}) = 0.040 \text{ Wb}
\]  
**Assess:** The flux is positive because the areas are equal and the stronger field is parallel to the normal of the surface.

33.5. **Model:** Consider the solenoid to be long so the field is constant inside and zero outside.  
**Visualize:** Please refer to Figure Ex33.5. The field of a solenoid is along the axis. The flux through the loop is only nonzero inside the solenoid. Since the loop completely surrounds the solenoid, the total flux through the loop will be the same in both the perpendicular and tilted cases.  
**Solve:** The field is constant inside the solenoid so we will use Equation 33.10. Take \( \vec{A} \) to be in the same direction as the field. The magnetic flux is  
\[
\Phi = A_{\text{loop}} \cdot \vec{B}_{\text{loop}} = A_{\text{loop}} \cdot B_{\text{loop}} = \pi r_{\text{loop}}^2 B_{\text{loop}} \cos \theta = \pi (0.010 \text{ m})^2 (20 \text{ T}) = 6.28 \times 10^{-3} \text{ Wb}
\]  
When the loop is tilted the component of \( \vec{B} \) in the direction of \( \vec{A} \) is less, but the effective area of the loop surface through which the magnetic field lines cross is increased by the same factor.

33.6. **Model:** Assume the field strength is uniform over the loop.  
**Visualize:** Please refer to Figure Ex32.6. According to Lenz’s law, the induced current creates an induced field that opposes the change in flux.  
**Solve:** The original field is into the page within the loop and is changing strength. The induced, counterclockwise current produces a field out of the page within the loop that is opposing the change. This implies that the original field must be increasing in strength so the flux into the loop is increasing.
33.9. **Visualize:** Please refer to Figure Ex33.9. The changing current in the solenoid produces a changing flux in the loop. By Lenz’s law there will be an induced current and field to oppose the change in flux.

**Solve:** The current shown produces a field to the right inside the solenoid. So there is flux to the right through the surrounding loop. As the current in the solenoid decreases there is less field and less flux to the right through the loop. There is an induced current in the loop that will oppose the change by creating an induced field and flux to the right. This requires a clockwise current.

33.10. **Model:** Assume the field is uniform.

**Visualize:** Please refer to Figure Ex33.10. If the changing field produces a changing flux in the loop there will be a corresponding induced emf and current.

**Solve:** (a) The induced emf is \( E = \frac{d\Phi}{dt} \) and the induced current is \( I = \frac{E}{R} \). The field \( B \) is changing, but the area \( A \) is not. Take \( \vec{A} \) to be out of the page and parallel to \( \vec{B} \), so \( \Phi = Ab \). Thus,

\[
E = A \frac{dB}{dt} = \pi r^2 \frac{dB}{dt} = \pi (0.050 \text{ m})^2 (0.50 \text{ T/s}) = 3.93 \times 10^{-3} \text{ V}
\]

\[
I = \frac{E}{R} = \frac{3.93 \times 10^{-3} \text{ V}}{0.1 \Omega} = 39.3 \times 10^{-3} \text{ A} = 39.3 \text{ mA}
\]

The field is increasing out of the page. To prevent the increase, the induced field needs to point into the page. Thus, the induced current must flow clockwise.

(b) As in part a, \( E = A(dB/dt) = 3.93 \times 10^{-3} \text{ V} \) and \( I = 39.3 \text{ mA} \). Here the field is into the page and decreasing. To prevent the decrease, the induced field needs to point into the page. Thus the induced current must flow clockwise.

(c) Now \( \vec{A} \) (left or right) is perpendicular to \( \vec{B} \) and so \( \vec{A} \cdot \vec{B} = 0 \) Wb. That is, the field does not penetrate the plane of the loop. If \( \Phi = 0 \) Wb, then \( E = \frac{d\Phi}{dt} = 0 \text{ V/m} \) and \( I = 0 \text{ A} \). There is no induced current.

**Assess:** Note that the induced field opposes the change.

33.11. **Model:** Assume the field strength is changing at a constant rate.

**Visualize:**

```
\[\begin{array}{c}
\times \times \times \times \times \\
\times \times \times \times \times \\
\times \times \times \times \times \\
\times \times \times \times \times \\
\times \times \times \times \times \\
\times \times \times \times \times \\
\end{array}\]
```

The changing field produces a changing flux in the coil and there will be a corresponding induced emf and current.

**Solve:** The induced emf of the coil is

\[
E = N \frac{d\Phi}{dt} = N \left| \frac{d(\vec{A} \cdot \vec{B})}{dt} \right| = NA \left| \frac{dB}{dt} \right| = N \pi r^2 \left| \frac{dB}{dt} \right| = (10^3) \pi (0.01 \text{ m})^2 \left( \frac{0.10 \text{ T}}{10 \times 10^{-3} \text{ s}} \right) = 3.14 \text{ V}
\]

where we’ve used the fact that \( \vec{B} \) is parallel to \( \vec{A} \).

**Assess:** This seems to be a reasonable emf as there are many turns.
33.12. **Model:** Assume the field is uniform.
**Visualize:** Please refer to Figure Ex33.12. The motion of the loop changes the flux through it. This results in an induced emf and current.
**Solve:** The induced emf is \( E = |d\Phi/dt| \) and the induced current is \( I = E/R \). The area \( A \) is changing, but the field \( B \) is not. Take \( \vec{A} \) as being out of the page and parallel to \( \vec{B} \), so \( \Phi = AB \) and \( d\Phi/dt = B(dA/dt) \). The flux is through that portion of the loop where there is a field, that is, \( A = lx \). The emf and current are

\[
E = B \left| \frac{dA}{dt} \right| = B \left| \frac{d(lx)}{dt} \right| = Blv = \left| (0.20 \ T)(0.050 \ m)(50 \ m/s) \right| = 0.50 \ V
\]

\[
I = \frac{E}{R} = \frac{0.50 \ V}{0.10 \ \Omega} = 5.0 \ A
\]

The field is out of the page. As the loop moves the flux increases because more of the loop area has field through it. To prevent the increase, the induced field needs to point into the page. Thus, the induced current flows clockwise.
**Assess:** This seems reasonable since there is rapid motion of the loop.

33.23. **Visualize:** Please refer to Figure P33.23. To calculate the flux we need to consider the orientation of the normal of the surface relative to the magnetic field direction. We will consider the flux through the surface in the two parts corresponding to the two different directions of the surface normals.
**Solve:** The flux is

\[
\Phi = \Phi_{\text{top left}} + \Phi_{\text{top left}} = A_{\text{top}} \cdot \vec{B} + A_{\text{top}} \cdot \vec{B} = A_{\text{top}} B \cos 45^\circ + A_{\text{top}} B \cos 45^\circ
\]

\[
= 2 \times ((0.050 \ m \times 0.10 \ m)(0.050 \ T) \cos 45^\circ = 3.54 \times 10^{-4} \ Wb
\]

33.25. **Model:** Assume the field is uniform in space though it is changing in time.
**Visualize:** The changing magnetic field strength produces a changing flux through the loop, and a corresponding induced emf and current.
**Solve:** (a) Since the field is perpendicular to the plane of the loop, \( \vec{A} \) is parallel to \( \vec{B} \) and \( \Phi = AB \). The emf is

\[
E = \left| \frac{d\Phi}{dt} \right| = A \left| \frac{dB}{dt} \right| = (0.20 \ m)^2 \left| (4 - 4t) T/s \right| = 0.16(1-t)
\]

\[
\Rightarrow I = \frac{E}{R} = \frac{0.16(1-t)}{1.6} \ A
\]

The magnetic field is increasing over the interval \( 0 < t < 1 \) s and is decreasing over the interval \( 1 < t < 2 \) s, so the induced emf and current must have opposite signs in the second half of the time interval. We arbitrarily choose the sign to be positive during the first half.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>( B ) (T)</th>
<th>( E ) (volts)</th>
<th>( I ) (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>0.16</td>
<td>1.6</td>
</tr>
<tr>
<td>0.5</td>
<td>1.50</td>
<td>0.08</td>
<td>0.8</td>
</tr>
<tr>
<td>1.0</td>
<td>2.00</td>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>1.5</td>
<td>1.50</td>
<td>-0.08</td>
<td>-0.8</td>
</tr>
<tr>
<td>2.0</td>
<td>0.00</td>
<td>-0.16</td>
<td>-1.6</td>
</tr>
</tbody>
</table>

(b) To plot the field and current we look at the form of the equations as a function of time. The magnetic field strength is quadratic with a maximum at \( t = 1 \) s and vanishing at \( t = 0 \) s and \( t = 2 \) s. The current equation is linear and decreasing, starting at 1.6 A at \( t = 0 \) s and going through zero at \( t = 1 \) s.

**Assess:** Notice in the graph how \( I = 0 \) A at \( t = 1 \) s, the instant in time when \( B \) is a maximum, that is, when \( dB/dt = 0 \). At this point the flux is (instantaneously) not changing so the corresponding induced emf and current are zero.
**Model:** Assume the field is uniform in space though it is changing in time.

**Visualize:** The changing magnetic field strength produces a changing flux through the coil, and a corresponding induced emf and current.

**Solve:**

(a) 

(b) Since the field is perpendicular to the plane of the coil, $\vec{A}$ is parallel to $\vec{B}$ and $\Phi = AB$. The emf is

$$E(t) = \frac{d\Phi}{dt} = N A \frac{dB}{dt} = 20\pi (0.025 \text{ m})^2 (0.020 + 0.020t) \text{T/s} = 7.85 \times 10^{-4} (1 + t) \text{ V}$$

$$\Rightarrow I(t) = \frac{E(t)}{R} = \frac{7.85 \times 10^{-4} (1 + t) \text{ V}}{0.50 \Omega} = 1.57 \times 10^{-3} (1 + t) \text{ A}$$

(c) Using the expression for $I(t)$ in part (b),

$$I(5 \text{ s}) = 1.57 \times 10^{-3} (1 + 5) \text{ A} = 9.42 \times 10^{-3} \text{ A} \quad I(10 \text{ s}) = 1.73 \times 10^{-2} \text{ A}$$

**Assess:** The field is always increasing and increasing more rapidly with time so the induced current is greater at 10 s than at 5 s.
33.32. Model: Since the solenoid is fairly long compared to its diameter and the loop is located near the center, assume the solenoid field is uniform inside and zero outside.

Visualize: Please refer to Figure P33.32. The solenoid’s magnetic field is perpendicular to the loop and creates a flux through the loop. This flux changes as the solenoid’s current changes, causing an induced emf and corresponding induced current.

Solve: Using Faraday’s law, the induced current is

\[
I_{\text{loop}} = \frac{E_{\text{loop}}}{R} = \frac{1}{R} \left[ \frac{d \Phi}{dt} \right] = \frac{A_{\text{sol}}}{R} \left[ \frac{dB_{\text{sol}}}{dt} \right] = \left( \frac{\pi r_{\text{sol}}^2}{R} \right) \left[ \frac{d}{dt} \left( \frac{\mu_0 NI}{l} \right) \right] = \left( \frac{\pi r_{\text{sol}}^2 \mu_0 N}{RL} \right) \left[ \frac{dI}{dt} \right]
\]

We have used the fact that the field is approximately zero outside the solenoid, so the flux is confined to the area \( A_{\text{sol}} = \pi r_{\text{sol}}^2 \) of the solenoid, not the larger area of the loop itself. The current is constant from 0 s \( \leq t \leq 1 \) s and 2 s \( \leq t \leq 3 \) s, so \( dI/dt = 0 \) A/s and \( I_{\text{loop}} = 0 \) A during these intervals. The current is changing at the rate \( |dI/dt| = 40 \) A/s during the interval 1 s \( \leq t \leq 2 \) s, so the current during this interval is

\[
I_{\text{loop}} = \frac{\pi (0.010 \text{ m})^2 \left( 4\pi \times 10^{-7} \text{ T m/A} \right) (100)}{(0.10 \Omega)(0.10 \text{ m})} (40 \text{ A/s}) = 1.58 \times 10^{-4} \text{ A} = 158 \mu \text{A}
\]

During the first half of this interval, from 1.0 s to 1.5 s, the field is to the right and decreasing. To oppose this decrease, the induced field must point to the right. This requires an induced current coming out of the page at the top of the loop—a positive current with the sign definition given in the problem. During the second half of this interval, from 1.5 s to 2.0 s, the field is to the right and increasing. To oppose this increase, the induced field must point to the right. This also requires a positive induced current coming out of the page at the top of the loop.

Assess: The induced current is proportional to the negative derivative of the solenoid current.
33.35. **Model:** Assume that the magnetic field of coil 1 passes through coil 2 and that we can use the magnetic field of a solenoid for coil 1.

**Visualize:** Please refer to Figure P33.35. The field of coil 1 produces flux in coil 2. The changing current in coil 1 gives a changing flux in coil 2 and a corresponding induced emf and current in coil 2.

**Solve:** From 0 s to 0.1 s and 0.3 s to 0.4 s the current in coil 1 is constant so the current in coil 2 is zero. From 0.1 s to 0 s, the induced current from the induced emf is given by Faraday’s law. The current in coil 2 is

\[ I_2 = \frac{E_2}{R} = \frac{1}{R} N_2 A_2 \frac{dB}{dt} = \frac{1}{R} N_2 \pi r_2^2 \frac{d}{dt} \left( \frac{\mu_0 N_1 I_1}{l_1} \right) = \frac{N_2 \pi r_2^2 \mu_0 N_1}{R l_1} \frac{dI_1}{dt} \]

\[ = \frac{20\pi (0.010 \text{ m})^2 (4\pi \times 10^{-7} \text{ T m/A})(20)}{(2\Omega)(0.020 \text{ m})} = 7.95 \times 10^{-5} \text{ A} = 79 \mu\text{A} \]

We used the facts that the field of coil 1 is constant inside the loops of coil 2 and the flux is confined to the area \( A_2 = \pi r_2^2 \) of coil 2. Also, we used \( l_1 = N_1 d = 20(1.0 \text{ mm}) = 0.020 \text{ m} \) and \( |dB/dt| = 20 \text{ A/s} \). From 0.1 s to 0.2 s the current in coil 1 is initially negative so the field is initially to the right and the flux is decreasing. The induced current will oppose this change and will therefore produce a field to the right. This requires an induced current in coil 2 that comes out of the page at the top of the loops so it is negative. From 0.2 s to 0.3 s the current in coil 1 is positive so the field is to the left and the flux is increasing. The induced current will oppose this change and will therefore produce a field to the right. Again, this is a negative current.

\[ I_2 (\mu\text{A}) \]

\[ \begin{array}{c|c|c}
0 & 0.2 & 0.4 \\
-79 & & \\
\end{array} \]

33.38. **Model:** Assume the field due to the solenoid is uniform inside and vanishes outside.

**Visualize:** The changing current in the solenoid produces a changing field and flux through the coil. This changing flux creates an induced emf in the coil.

**Solve:** The flux is only nonzero within the area of the solenoid, not the entire area of the coil. The induced current in the coil is

\[ I_{\text{coil}} = \frac{E_{\text{coil}}}{R} = \frac{1}{R} N_{\text{coil}} A_{\text{coil}} \frac{d\Phi}{dt} = \frac{1}{R} N_{\text{coil}} \pi r_{\text{coil}}^2 \frac{d}{dt} \left( \frac{\mu_0 N_i I_i}{l_i} \right) = \frac{N_{\text{coil}} \pi r_{\text{coil}}^2 \mu_0 N_i}{R l_i} \frac{dI_i}{dt} \]

\[ = \frac{N_{\text{coil}} \pi r_{\text{coil}}^2 \mu_0 N_i}{R l_i} \left( \frac{2\pi f I_0}{2\pi f I_0} \right) \cos \left( 2\pi f t \right) \]

The maximum coil current occurs when the cosine is +1. The maximum current in the solenoid is

\[ I_0 = \frac{I_{\text{coil, max}} R l_i}{N_{\text{coil}} \pi r_{\text{coil}}^2 \mu_0 N_i (2\pi f)} = \frac{(0.20 \text{ A})(0.40\Omega)(0.20 \text{ m})}{40\pi (0.015 \text{ m})^2 (4\pi \times 10^{-7} \text{ T m/A})(200) 2\pi (60 \text{ Hz})} = 5.97 \text{ A} \]

**Assess:** This is a large current, but the induced current is large as well.
33.40. **Model:** Assume the field changes abruptly at the boundary and is uniform.

**Visualize:** Please refer to Figure P33.40. As the loop enters the field region the amount of flux will change as more area has field penetrating it. This change in flux will create an induced emf and corresponding current. While the loop is moving at constant speed, the rate of change of the area is not constant because of the orientation of the loop. The loop is moving along the x-axis.

**Solve:** (a) If the edge of the loop enters the field region at \( t = 0 \) s, then the leading corner has moved a distance \( x = v_0 t \) at time \( t \). The area of the loop with flux through it is

\[
A = 2\left(\frac{1}{2}\right)yx = x^2 = (v_0 t)^2
\]

where we have used the fact that \( y = x \) since the sides of the loop are oriented at 45° to the horizontal. Take the surface normal of the loop to be into the page so that \( \Phi = \vec{A} \cdot \vec{B} = BA \). The current in the loop is

\[
I = \frac{E}{R} = \frac{1}{R} \left| \frac{d\Phi}{dt} \right| = \frac{1}{R} B \left| \frac{dA}{dt} \right| = \frac{1}{R} B \left| \frac{d(v_0 t)^2}{dt} \right| = \frac{1}{R} B (2v_0^2 t) = \left( \frac{2(0.80 \text{ T})(10 \text{ m/s})^2}{0.10 \Omega} \right) t = \left( 1.6 \times 10^3 \text{ A} \right) t
\]

The current is increasing at a constant rate. This expression is good until the loop is halfway into the field region. The time for the loop to be halfway is found as follows:

\[
\frac{10 \text{ cm}}{\sqrt{2}} = v_0 t = (10 \text{ m/s}) t \Rightarrow t = 7.07 \times 10^{-3} \text{ s} = 7.07 \text{ ms}
\]

At this time the current is 11.3 A. While the second half of the loop is moving into the field, the flux continues to increase, but at a slower rate. Therefore, the current will decrease at the same rate as it increased before, until the loop is completely in the field at \( t = 14.1 \text{ ms} \). After that the flux will not change and the current will be zero.

(b) The maximum current of 11.3 A occurs when the flux is changing the fastest and this occurs when the loop is halfway into the region of the field.
**33.45. Model:** Assume that the magnetic field is uniform in the region of the loop.

**Visualize:** Please refer to Figure P33.45. The rotating semicircle will change the area of the loop and therefore the flux through the loop. This changing flux will produce an induced emf and corresponding current in the bulb.

**Solve:** (a) The spinning semicircle has a normal to the surface that changes in time, so while the magnetic field is constant, the area is changing. The flux through in the lower portion of the circuit does not change and will not contribute to the emf. Only the flux in the part of the loop containing the rotating semicircle will change. The flux associated with the semicircle is

\[
\Phi = \overrightarrow{A} \cdot \overrightarrow{B} = BA = BA \cos \theta = BA \cos(2\pi ft)
\]

where \( \theta = 2\pi ft \) is the angle between the normal of the rotating semicircle and the magnetic field and \( A \) is the area of the semicircle. The induced current from the induced emf is given by Faraday’s law. We have

\[
I = \frac{E}{R} = \frac{1}{R} \frac{d\Phi}{dt} = \frac{1}{R} \frac{d}{dt} BA \cos(2\pi ft) = \frac{B}{R} \frac{\pi r^2}{2} f \sin(2\pi ft)
\]

\[
= \frac{2(0.20 \text{ T})\pi^2 (0.050 \text{ m})^2}{2(1.0 \Omega)} f \sin(2\pi ft) = 4.93 \times 10^{-3} f \sin(2\pi ft) \text{ A}
\]

where the frequency \( f \) is in Hz.

(b) We can now solve for the frequency necessary to achieve a certain current. From our study of DC circuits we know how power relates to resistance:

\[
P = I^2 R \Rightarrow I = \sqrt{P/R} = \sqrt{4.0 \text{ W}/1.0 \Omega} = 2.0 \text{ A}
\]

The maximum of the sine function is +1, so the maximum current is

\[
I_{\text{max}} = 4.93 \times 10^{-3} f \text{ A s} = 2.0 \text{ A} \Rightarrow f = \frac{2.0 \text{ A}}{4.93 \times 10^{-3} \text{ A s}} = 405 \text{ Hz}
\]

**Assess:** This is not a reasonable frequency to obtain by hand.

**33.46. Model:** Assume the magnetic field is uniform over region where the bar is sliding and that friction between the bar and the rails is zero.

**Visualize:** Please refer to Figure P33.46. The battery will produce a current in the rails and bar and the bar will experience a force. With the battery connected as shown in the figure, the current in the bar will be down and by the right-hand rule the force on the bar will be to the right. The motion of the bar will change the flux through the loop and there will be an induced emf that opposes the change.

**Solve:** (a) As the bar speeds up the induced emf will get larger until finally it equals the battery emf. At that point, the current will go to zero and the bar will continue to move at a constant velocity. We have

\[
E = Blv_{\text{term}} = E_{\text{bat}} \Rightarrow v_{\text{term}} = \frac{E_{\text{bat}}}{Bl}
\]

(b) The terminal velocity is

\[
v_{\text{term}} = \frac{1.0 \text{ V}}{(0.50 \text{ T})(0.060 \text{ m})} = 33.3 \text{ m/s}
\]

**Assess:** This is pretty fast, about 70 mph.
33.49.  **Model:** Assume the magnetic field is uniform over the loop.

**Visualize:** Please refer to Figure 33.49. As the wire falls, the flux into the page will increase. This will induce a current to oppose the increase, so the induced current will flow counterclockwise. As this current passes through the slide wire, it experiences an upward magnetic force. So there is an upward force—a retarding force—on the wire as it falls in the field. As the wire speeds up the retarding force will become larger until it balances the weight.

**Solve:** (a) The force on the current-carrying slide wire is \( F_m = iB \). The induced current is
\[
I = \frac{E}{R} = \frac{1}{R} \frac{\mathrm{d} \Phi}{\mathrm{d} t} = \frac{1}{R} \frac{\mathrm{d}}{\mathrm{d} t} \int AB = \frac{B}{R} \frac{\mathrm{d}}{\mathrm{d} t} l = \frac{Blv}{R}
\]
Consequently, the retarding magnetic force is
\[
F_m = \frac{I^2 B^2}{R} = \left( \frac{I^2 B^2}{R} \right) v
\]
The important point is that \( F_m \) is proportional to the speed \( v \). As the wire begins to fall and its speed increases, so does the retarding force. Within a very short time, \( F_m \) will increase in size to where it matches the weight \( w = mg \). At that point, there is no net force on the loop, so it will continue to fall at a constant speed. The condition that the magnetic force equals the weight is
\[
\left( \frac{I^2 B^2}{R} \right) v_{\text{term}} = mg \Rightarrow v_{\text{term}} = \frac{mgR}{I^2 B^2}
\]
(b) The terminal speed is
\[
v_{\text{term}} = \frac{(0.010 \text{ kg}) \left( 9.80 \text{ m/s}^2 \right) \left( 0.10 \Omega \right)}{(0.20 \text{ m})^2 \left( 0.50 \text{ T} \right)^2} = 0.98 \text{ m/s}
\]

33.57.  **Model:** Assume the magnetic field is constant throughout the atmosphere.

**Visualize:** The energy density depends on the field strength in a particular region of space. The total energy is the energy density times the volume.

**Solve:** (a) The atmosphere is a spherical shell between the surface of the earth at \( R_e \) and the top of the atmosphere at \( R_e + h \) where \( h \) is the thickness of the atmosphere. We have
\[
\mu_0 u_b = \frac{1}{2} B^2 = \frac{\left( 50 \times 10^{-6} \text{ T} \right)^2}{2 \left( 4\pi \times 10^{-7} \text{ T m/A} \right)} = 9.95 \times 10^{-4} \text{ J/m}^3
\]
\[
V_{\text{atm}} = \frac{4}{3} \pi \left[ (R_e + h)^3 - R_e^3 \right] = \frac{4}{3} \pi \left[ \left( 6.37 \times 10^6 \text{ m} + 20 \times 10^3 \text{ m} \right)^3 - \left( 6.37 \times 10^6 \text{ m} \right)^3 \right] = 1.02 \times 10^{19} \text{ m}^3
\]
\[
\Rightarrow U_{\text{tot}} = u_b V_{\text{atm}} = \left( 9.95 \times 10^{-4} \text{ J/m}^3 \right) \left( 1.02 \times 10^{19} \text{ m}^3 \right) = 1.0 \times 10^{16} \text{ J}
\]
(b) The ratio is
\[
\frac{U_{\text{mag}}}{U_{\text{tot}}} = \frac{1.0 \times 10^{16} \text{ J}}{4.0 \times 10^{16} \text{ J}} = 0.25 \times 10^2 = 0.25 \%
\]

**Assess:** Not enough to do much good.
### 33.58. Model: Assume the solenoid is long enough that we can approximate the field as being constant inside and zero outside.

**Visualize:** The energy density depends on the field strength in a particular region of space.

**Solve:** (a) We can use the given field strength to find the energy density and then determine the volume of a cylinder. We have

\[
\frac{u_h}{\mu_0} = \frac{1}{2} B^2 = \frac{(5.0 \text{ T})^2}{2(4\pi \times 10^{-7} \text{ T m/A})} = 9.95 \times 10^6 \text{ J/m}^3
\]

\[
V_{\text{sol}} = \pi r_{\text{sol}}^2 l = \pi (0.20 \text{ m})^2 (1.0 \text{ m}) = 0.126 \text{ m}^3 \Rightarrow U_{\text{sol}} = u_h V_{\text{sol}} = \left(9.95 \times 10^6 \text{ J/m}^3\right)(0.126 \text{ m}^3) = 1.25 \times 10^5 \text{ J}
\]

(b) Using the magnetic field of the solenoid, the number of turns is calculated as follows:

\[
B = \frac{\mu_0 NI}{l} \Rightarrow N = \frac{B l}{\mu_0} = \frac{1.0 \text{ m}(5.0 \text{ T})}{(4\pi \times 10^{-7} \text{ T m/A})(100 \text{ A})} = 4.0 \times 10^4
\]

**Assess:** This is a reasonable number for a solenoid that is a meter long.

### 33.65. Visualize: The current through the inductor is changing with time which leads to a changing potential difference across the inductor.

**Solve:** (a) To find the potential difference we differentiate the current. We have

\[
\Delta V_L = -L \frac{dI}{dt} = -L \frac{d}{dt}\left(I_0 e^{-t/\tau}\right) = \left(\frac{LI_0}{\tau}\right)e^{-t/\tau}
\]

(b) For \(t = 1 \text{ ms}\), the potential difference is

\[
\Delta V_L = \left(20 \times 10^{-3} \text{ H}\right)(50 \times 10^{-3} \text{ A}) e^{-1000\text{ms}/1000\text{ms}} = (1.0 \text{ V})e^{-1} = 0.37 \text{ V}
\]

Similar calculations give 1.0 V, 0.13 V, and 0.05 V for \(t = 0 \text{ ms}, 2 \text{ ms}, \text{ and } 3 \text{ ms}\).
**33.81. Model:** Assume the conductor is long enough to use the formula for the magnetic field of an “infinite” wire.

**Visualize:** Please refer to Figure CP33.81. The current in the inner conductor creates a field that circles it in the space between the inner and outer conductor. Magnetic flux exists in the space.

**Solve:** (a) We will consider a rectangle between the conductors of length \( l \) that is perpendicular to the magnetic field produced by the inner wire. Take \( \vec{A} \) parallel to \( \vec{B} \) so the flux through a small piece of the rectangle of width \( dr \) is \( d\Phi = \vec{B} \cdot d\vec{A} = Bl dr \). The total flux is

\[
\Phi = \int d\Phi = \int_{r_i}^{r_o} \left( \frac{\mu_0 I}{2\pi r} \right) dr = \frac{\mu_0 I}{2\pi} \ln \left( \frac{r_o}{r_i} \right)
\]

where we have used the formula for the field of an infinite wire. We want the inductance per unit length, so we take the inductance and divide by the length. We have

\[
L = \frac{\Phi}{l} \Rightarrow \tilde{L} = \frac{\Phi}{2\pi l} = \frac{\mu_0 I}{2\pi} \ln \left( \frac{r_o}{r_i} \right)
\]

(b) The inductance per unit length is

\[
\tilde{L} = \frac{\left(4\pi \times 10^{-7} \text{ T m/A}\right)}{2\pi} \ln \left( \frac{3.0}{0.50} \right) = 3.6 \times 10^{-7} \text{ H/m} = 0.36 \mu\text{H/m}
\]