11.5. Solve:  
(a) \[ W = \vec{F} \cdot \Delta \vec{r} = (6.0\hat{i} - 3.0\hat{j}) \cdot (2.0\hat{i}) \text{ N} \cdot \text{m} = (12.0\hat{i} + 3.0\hat{j} \cdot 12.0\hat{i}) \text{ J} = 12.0 \text{ J}. \]
(b) \[ W = \vec{F} \cdot \Delta \vec{r} = (6.0\hat{i} - 3.0\hat{j}) \cdot (2.0\hat{j}) \text{ N} \cdot \text{m} = (12.0\hat{i} \cdot \hat{j} - 6.0\hat{j} \cdot \hat{j}) \text{ J} = -6.0 \text{ J}. \]

11.9. Model: Model the piano as a particle and use \( W = \vec{F} \cdot \Delta \vec{r} \), where \( W \) is the work done by the force \( \vec{F} \) through the displacement \( \Delta \vec{r} \).

Visualize:

Solve: For the force \( \vec{w} \):
\[ W = \vec{F} \cdot \Delta \vec{r} = \vec{w} \cdot \Delta \vec{r} = (\vec{w}(\Delta \vec{r})) \cos 0^\circ = (2500 \text{ N})(5.0 \text{ m})(1) = 12,500 \text{ J} \]

For the tension \( T_1 \):
\[ W = T_1 \cdot \Delta \vec{r} = (T_1)(\Delta \vec{r}) \cos 150^\circ = (1830 \text{ N})(5.0 \text{ N})(-0.8660) = -7920 \text{ J} \]

For the tension \( T_2 \):
\[ W = T_2 \cdot \Delta \vec{r} = (T_2)(\Delta \vec{r}) \cos 135^\circ = (1295 \text{ N})(5.0 \text{ m})(-0.7071) = -4580 \text{ J} \]

Assess: Note that the displacement \( \Delta \vec{r} \) in all the above cases is directed downwards along \( -\hat{j} \).

11.15. Model: Use the work-kinetic energy theorem.

Visualize: Please refer to Figure Ex11.15.

Solve: The work-kinetic energy theorem is
\[ \Delta K = W = \int_{x_i}^{x_f} F_x \ dx = \text{area of the } F_x \text{-versus}-x \text{ graph between } x_i \text{ and } x_f \]
\[ = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}(F_{\text{max}})(2 \text{ m}) \]

Using \( m = 0.500 \text{ kg} \), \( v_i = 6.0 \text{ m/s} \), and \( v_f = 2.0 \text{ m/s} \), the above equation yields \( F_{\text{max}} = 8.0 \text{ N} \).

Assess: Problems in which the force is not a constant can not be solved using constant-acceleration kinematic equations.
11.17. **Model:** Use the definition $F_x = -\frac{dU}{dx}$.

**Visualize:** Please refer to Figure Ex11.17.

**Solve:** $F_x$ is the negative of the slope of the potential energy graph at position $x$.

$$F_x = -\left(\frac{dU}{dx}\right)$$

Between $x = 0$ m and $x = 2$ m, the slope is

$$\text{slope} = \frac{(U_t - U_i)}{(x_t - x_i)} = \frac{(60 \text{ J} - 0 \text{ J})}{(2 \text{ m} - 0 \text{ m})} = 30 \text{ N}$$

and between $x = 2$ m and $x = 5$ m, the slope is

$$\text{slope} = \frac{(U_t - U_i)}{(x_t - x_i)} = \frac{(0 \text{ J} - 60 \text{ J})}{(5 \text{ m} - 2 \text{ m})} = -20 \text{ J}$$

Thus, $F_x = -30$ N at $x = 1$ m and $F_x = 20$ N at $x = 3$ m.

11.23.

**Visualize:**

**Solve:** (a) $K_i = K_o = \frac{1}{2}mv_0^2 = 0 \text{ J}$ \quad $U_i = U_{g0} = mgy_0 = (20 \text{ kg})(9.8 \text{ m/s}^2)(3 \text{ m}) = 588 \text{ J}$

$$W_{ext} = 0 \text{ J} \quad K_f = K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(20 \text{ kg})(2.0 \text{ m/s})^2 = 40 \text{ J} \quad U_f = U_{g1} = mgy_1 = 0 \text{ J}$$

At the top of the slide, the child has gravitational potential energy of 588 J. This energy is transformed into thermal energy of the child’s pants and the slide and the kinetic energy of the child. This energy transfer and transformation is shown on the energy bar chart.

(b) The change in the thermal energy of the slide and of the child’s pants is $588 \text{ J} - 40 \text{ J} = 548 \text{ J}$. 

The change in the thermal energy of the slide and of the child’s pants is $588 \text{ J} - 40 \text{ J} = 548 \text{ J}$. 
11.25. Visualize:

Note that the conservation of energy equation
\[ K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th} \]
requires that \( W_{ext} \) be equal to +400 J.

11.32. Solve: (a) A kilowatt hour is a kilowatt multiplied by 3600 seconds. It has the dimensions of energy.
(b) One kilowatt hour is energy
\[ 1 \text{ kwh} = (1000 \text{ J/s})(3600 \text{ s}) = 3.6 \times 10^6 \text{ J} \]
Thus
\[ 500 \text{ kwh} = (500 \text{ kwh}) \left( \frac{3.6 \times 10^6 \text{ J}}{1 \text{ kwh}} \right) = 1.8 \times 10^9 \text{ J} \]


Visualize:

Solve: (a) The work done by gravity on the elevator is
\[ W_g = -\Delta U = mg y_0 - mg y_f = -mg(y_f - y_0) = -(1000 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m}) = -9.8 \times 10^4 \text{ J} \]
(b) The work done by the tension in the cable on the elevator is
\[ W_T = T(\Delta y) \cos \theta = T(y_f - y_0) = T(10 \text{ m}) \]
To find \( T \) we write Newton’s second law for the elevator:
\[ \sum F_y = T - w = ma_y \Rightarrow T = w + ma_y = m(g + a_y) = (1000 \text{ kg})(9.8 \text{ m/s}^2 + 1.0 \text{ m/s}^2) = 1.08 \times 10^5 \text{ N} \Rightarrow W_T = (1.08 \times 10^5 \text{ N})(10 \text{ m}) = 1.08 \times 10^7 \text{ J} \\
\]
(c) The work-kinetic energy theorem is
\[ W_{net} = W_g + W_T = \Delta K = K_f - K_i = K_f - \frac{1}{2}mv_0^2 \Rightarrow K_f = W_g + W_T + \frac{1}{2}mv_0^2 \Rightarrow K_f = (-9.8 \times 10^4 \text{ J}) + (1.08 \times 10^5 \text{ J}) + \frac{1}{2}(1000 \text{ kg})(0 \text{ m/s})^2 = 1.0 \times 10^5 \text{ J} \]
(d) \[ K_f = \frac{1}{2}mv_f^2 \Rightarrow 1.0 \times 10^4 \text{ J} = \frac{1}{2}(1000 \text{ kg})v_f^2 \Rightarrow v_f = 4.47 \text{ m/s} \]
11.47. **Model:** We will use the spring, the package, and the ramp as the system. We will model the package as a particle.

**Visualize:** We place the origin of our coordinate system on the end of the spring when it is compressed and is in contact with the package to be shot.

**Model:** (a) The energy conservation equation is

\[
\frac{1}{2} m v_1^2 + mg y_1 + \frac{1}{2} k (x_c - x_1)^2 + \Delta E_{th} = \frac{1}{2} m v_0^2 + mg y_0 + \frac{1}{2} k (\Delta x)^2 + W_{ext}
\]

Using \( y_1 = 1 \text{ m}, \Delta E_{th} = 0 \text{ J} \) (note the frictionless ramp), \( v_0 = 0 \text{ m/s}, y_0 = 0 \text{ m}, \Delta x = 30 \text{ cm}, \text{ and } W_{ext} = 0 \text{ J} \), we get

\[
\frac{1}{2} (2.0 \text{ kg}) v_1^2 + mg(1 \text{ m}) + 0 \text{ J} + 0 \text{ J} + 0 \text{ J} + \frac{1}{2} k (0.30 \text{ m})^2 + 0 \text{ J}
\]

\[
\Rightarrow v_1 = 1.70 \text{ m/s}
\]

(b) How high can the package go after crossing the sticky spot? If the package can reach \( y_1 \geq 1.0 \text{ m} \) before stopping \( (v_1 = 0) \), then it makes it. But if \( y_1 < 1.0 \text{ m} \) when \( v_1 = 0 \), it does not. The friction of the sticky spot generates thermal energy

\[
\Delta E_{th} = -W_{diss} = -\int f \cdot dx = -(-\mu_k mg) \Delta x = (0.30)(2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 2.94 \text{ J}
\]

The energy conservation equation is now

\[
\frac{1}{2} m v_1^2 + mg y_1 + \Delta E_{th} = \frac{1}{2} k (\Delta x)^2
\]

If we set \( v_1 = 0 \text{ m/s} \) to find the highest point the package can reach, we get

\[
y_1 = \left( \frac{1}{2} k (\Delta x)^2 - \Delta E_{th} \right)/mg = \left( \frac{1}{2} (500 \text{ N/m})(0.30 \text{ m})^2 - 2.94 \text{ J} \right)/(2.0 \text{ kg})(9.8 \text{ m/s}^2) = 0.998 \text{ m}
\]

The package does not make it. It just barely misses.
**11.48. Model:** Model the two blocks as particles. The two blocks make our system.

**Visualize:**

![Diagram of two blocks](image)

We place the origin of our coordinate system at the location of the 3.0 kg block.

**Solve:** (a) The conservation of energy equation is 

\[ K_1 + U_{g1} + \Delta E_{in} = K_2 + U_{g2} + W_{ext} \]

Using \( \Delta E_{in} = 0 \) J and \( W_{ext} = 0 \) J we get

\[ \frac{1}{2} m_2(v_i)_2^2 + \frac{1}{2} m_3(v_i)_3^2 + m_2g(y_i) = \frac{1}{2} m_2(v_f)_2^2 + \frac{1}{2} m_3(v_f)_3^2 + m_2g(y_f) \]

Noting that \( (v_i)_2 = (v_f)_3 = v_i \) and \( (v_i)_2 = (v_f)_3 = 0 \) m/s, this becomes

\[
\frac{1}{2}(m_2 + m_3)v_i^2 = -m_2g(y_i - y_f)
\]

\[
v_i = \sqrt{\frac{2m_2g(y_i - y_f)}{m_2 + m_3}} = \sqrt{\frac{2(2.0 \text{ kg})(9.8 \text{ m/s}^2)(1.50 \text{ m})}{(2.0 \text{ kg} + 3.0 \text{ kg})}} = 3.43 \text{ m/s}
\]

(b) We will use the same energy conservation equation. However, this time

\[ \Delta E_{th} = (f_i)(\Delta x) = (\mu_c n)(\Delta x) = \mu_c m_2g(y_f - y_i) = (0.15)(3.0 \text{ kg})(9.8 \text{ m/s}^2)(1.50 \text{ m}) = 6.615 \text{ J} \]

The energy conservation equation is now

\[
\frac{1}{2} m_2v_i^2 + \frac{1}{2} m_3v_i^2 + m_2gy_i + 6.615 \text{ J} = \frac{1}{2} m_2(v_f)_2^2 + \frac{1}{2} m_3(v_f)_3^2 + m_2gy_i + 0 \text{ J}
\]

\[
\frac{1}{2}(m_2 + m_3)v_i^2 + 6.615 \text{ J} = m_2g(y_i - y_f) \Rightarrow v_i = \sqrt{\frac{2}{m_2 + m_3}} [m_2g(y_i - y_f) - 6.615 \text{ J}]
\]

\[
= \sqrt{\frac{2}{5.0 \text{ kg}}} [(2.0 \text{ kg})(9.8 \text{ m/s}^2)(1.50 \text{ m}) - 6.615 \text{ J}] = 3.02 \text{ m/s}
\]

**Assess:** A reduced speed when friction is present compared to when there is no friction is reasonable.
11.53. Model: Assume an ideal spring, so Hooke’s law is obeyed. Treat the box as a particle and apply the energy conservation law. Box, spring, and the ground make our system, and we also use the model of kinetic friction.

Visualize: We place the origin of the coordinate system on the ground directly below the box’s starting position.

Solve: (a) The energy conservation equation is

$$K_i + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\frac{1}{2}mv_i^2 + mgy_i + 0J + 0J = \frac{1}{2}mv_0^2 + mgy_0 + 0J + 0J \Rightarrow \frac{1}{2}mv_i^2 + 0J = 0J + mgy_0$$

$$\Rightarrow v_i = \sqrt{2gy_0} = \sqrt{2(9.8 \text{ m/s}^2)(5 \text{ m})} = 9.90 \text{ m/s}$$

(b) The work of friction creates thermal energy. The energy conservation equation for this part of the problem is

$$K_2 + U_{g2} + U_{s2} + \Delta E_{th} = K_1 + U_{g1} + U_{s1} + W_{ext}$$

$$\frac{1}{2}mv_2^2 + \mu mg(x_2 - x_1) = \frac{1}{2}mv_1^2 + \mu mg(x_2 - x_1)$$

$$\Rightarrow v_2 = \sqrt{v_1^2 - 2\mu g(x_2 - x_1)} = \sqrt{(9.90 \text{ m/s}^2)^2 - 2(0.25)(9.8 \text{ m/s}^2)(2.0 \text{ m})} = 9.39 \text{ m/s}$$

(c) To find how much the spring is compressed, we apply the energy conservation once again:

$$K_3 + U_{g3} + U_{s3} + \Delta E_{th} = K_2 + U_{g2} + U_{s2} + W_{ext}$$

$$0J + 0J + \frac{1}{2}k(x_3 - x_2)^2 + 0J = \frac{1}{2}mv_2^2 + 0J + 0J$$

Using $v_2 = 9.39 \text{ m/s}$, $k = 500 \text{ N/m}$ and $m = 5.0 \text{ kg}$, the above equation yields $(x_3 - x_2) = \Delta x = 93.9 \text{ cm}$.

(d) The initial energy $mgy_0 = (5.0 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m}) = 245 \text{ J}$. The energy transformed to thermal energy during each passage is

$$f_k(x_2 - x_1) = \mu mg(x_2 - x_1) = (0.25)(5.0 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m}) = 24.5 \text{ J}$$

The number of passages is equal to $245 \text{ J}/24.5 \text{ J} = 10$. 
11.55. **Model:** Treat the block as a particle, use the model of kinetic friction, and apply the energy conservation law. The block and the incline comprise our system. 

**Visualize:** We place the origin of the coordinate system directly below the block’s starting position at the same level as the horizontal surface. On the horizontal surface the model of kinetic friction applies.

**Solve:**

(a) For the first incline, the conservation of energy equation gives

\[
K_i + U_{g1} + \Delta E_{an} = K_f + U_{g2} + W_{ext}
\]

\[
\frac{1}{2}mv_1^2 + 0 J + 0 J = 0 J + mgy_0 + 0 J \Rightarrow v_1 = \sqrt{2gy_0} = \sqrt{2gh}
\]

(b) The work of friction creates thermal energy. Applying once again the conservation of energy equation, we have

\[
K_i + U_{g1} + \Delta E_{th} = K_f + U_{g2} + W_{ext}
\]

\[
\frac{1}{2}mv_1^2 + mgy_1 + f_k(x_2 - x_1) = \frac{1}{2}mv_2^2 + mgy_1 + W_{ext}
\]

Using \(v_3 = 0 \text{ m/s}, y_1 = 0 \text{ m}, W_{ext} = 0 \text{ J}, f_k = \mu_k mg, \ \text{and} \ (x_2 - x_1) = L\), we get

\[
mgy_1 + \frac{1}{2}m(2gh) = y_3 = h - \mu_k L
\]

**Assess:** For \(\mu_k = 0\), \(y_3 = h\) which is predicted by the law of the conservation of energy.

11.58. **Solve:**

(a) Because \(\sin(cx)\) is dimensionless, \(F_0\) must have units of force in newtons.

(b) The product \(cx\) is an angle because we are taking the sine of it. An angle has no real physical units. If \(x\) has units of m and the product \(cx\) is unitless, then \(c\) has to have units of \(m^{-1}\).

(c) At \(x_0 = 0 \text{ m}\), the force is \(F_0 \sin(0) = 0 \text{ N}\). The particle is able to leave \(x_0 = 0 \text{ m}\) only because it has an initial velocity.

(d) The force is a maximum when \(\sin(cx) = 1\). This occurs when \(cx = \pi/2\), or for \(x_{max} = \pi/2c\).

(e) The graph is the first quarter of a sine curve.

(f) We can find the velocity \(v_i\) at \(x_i = x_{max}\) from the work-kinetic energy theorem:

\[
\Delta K = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_0^2 = W \Rightarrow v_i = \sqrt{v_0^2 + \frac{2W}{m}}
\]

This is a variable force. As the particle moves from \(x_i = 0 \text{ m}\) to \(x_i = x_{max} = \pi/2c\), the work done on it is

\[
W = \int_{x_i}^{x_{max}} F(x) \, dx = F_0 \int_{0}^{\sqrt{2}\pi/2c} \sin(cx) \, dx = -\frac{F_0}{c} \cos(cx) \bigg|_{0}^{\sqrt{2}\pi/2c} = -\frac{F_0}{c} \left( \cos \left( \frac{\pi}{2} \right) - \cos 0 \right) = \frac{F_0}{c}
\]

Thus, the particle’s speed at \(x_i = x_{max} = \pi/2c\) is \(v_i = \sqrt{v_0^2 + 2F_0/mc}\).

11.62. **Solve:** Using the conversion 746 W = 1 hp, we have a power of 1492 J/s. This means \(W = P t = (1492 \text{ J/s})(1 \text{ hr}) = 5.3712 \times 10^6 \text{ J}\) is the total work done by the electric motor in one hour. Furthermore,

\[
W_{\text{motor}} = -W_g = U_{g2} - U_{g1} = mg(y_f - y_i) = m(10 \text{ m})
\]

\[
m = \frac{W_{\text{motor}}}{g(10 \text{ m})} = \frac{5.3712 \times 10^6 \text{ J}}{(9.8 \text{ m/s}^2)(10 \text{ m})} = 5.481 \times 10^4 \text{ kg} = 5.481 \times 10^4 \text{ kg} \times \frac{1 \text{ liter}}{1 \text{ kg}} = 54,800 \text{ liters}
\]