Wave Solutions Handout

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June 24, 2009

1 Plane Waves

When modelling wave behavior a useful and accurate tool is the sinusoidal function. Without delving into the details of the wave equation, we can write the solution for a plane wave travelling in the positive x-direction by:

$$\psi(x,t) = A\sin(kx - \omega t) \tag{1}$$

where

$$k = \frac{2\pi}{\lambda} \tag{2}$$
$$\omega = 2\pi f, \tag{3}$$

which are labelled the wave number and angular frequency respectively. One should typically interpret ψ as the amplitude of the wave at a given position and time. This of course varies depending on the nature of the wave.

The wave number and freqency are related to the wave speed via,

$$v = f\lambda, \tag{4}$$
$$= \frac{\omega}{k}. \tag{5}$$

For a wave travelling in the negative x direction we remove the minus sign from the argument of the sine function:

$$\psi(x,t) = A\sin(kx + \omega t). \tag{6}$$

1.1 Standing Waves

When two waves travelling in opposite directions collide, they may produce a standing wave. Using superposition (adding of waves) we take two waves of the same amplitude and frequency and add them:

$$\Psi = \psi_1 + \psi_2 = A\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right]. \tag{7}$$

Using the trig identity $\sin(a \pm b) = \sin(a)\cos(b) \pm \sin(b)\cos(a)$ we get:

$$\Psi = 2A\sin(kx)\cos(\omega t), \tag{8}$$

which is a standing wave.

2 Spherical Waves

Often waves are created and travel outward radially from the source. In this case a viable solution is given by,

$$\psi(r,t) = \frac{A}{r}\sin(kr - \omega t), \tag{9}$$

where the coordinate r is the radial distance from the source.

2.1 Power from a Source

The intensity of a wave (in units of Watts per square meter) is written as:

$$I = \frac{P}{A} \tag{10}$$

where P is a constant determined by the source of the waves.