

Chapter 6

1. What is the probability that a card chosen from an ordinary deck of 52 cards is an ace?
Ans: $4/52$.
2. What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?
Ans: $50/100$.
3. What is the probability that a randomly selected day of the year (366 days) is in May?
Ans: $31/366$.
4. What is the probability that the sum of the numbers on two dice is even when they are rolled?
Ans: $18/36$.
5. What is the probability that a fair coin lands Heads 6 times in a row?
Ans: $1/2^6$.
6. What is the probability that a fair coin lands Heads 4 times out of 5 flips?
Ans: $C(5,4)/2^5 = 5/32$.
7. Three coins are tossed.
 - (a) List the elements in the sample space.
 - (b) Find the probability that exactly two heads show.Ans: (a) HHH, HHT, HTH, THH, HTT, THT, TTH, TTT. (b) $3/8$.
8. Suppose you pick two cards, one at a time, at random from an ordinary deck of 52 cards. Find the probability that
 - (a) $p(\text{both cards are diamonds})$.
 - (b) $p(\text{the cards form a pair})$.Ans: (a) $(13/52)(12/51)$. (b) $3/51$.
9. Suppose you and a friend each choose at random an integer between 1 and 8. For example, some possibilities are (3,7), (7,3), (4,4), (8,1), where your number is written first and your friend's number second. Find
 - (a) $p(\text{you pick 5 and your friend picks 8})$.
 - (b) $p(\text{sum of the two numbers picked is } < 4)$.
 - (c) $p(\text{both numbers match})$.
 - (d) $p(\text{the sum of the two numbers is a prime})$.
 - (e) $p(\text{your number is greater than your friend's number})$.Ans: (a) $1/64$. (b) $3/64$. (c) $8/64$. (d) $23/64$. (e) $28/64$.
10. Prove or disprove: $p(E \cup F) = p(E) + p(F)$ for all events E and F .
Ans: False. Choose one card at random from a deck of 52 cards. Let E = choose a diamond, F = choose a king. Then $p(E \cup F) = 16/52$ while $p(E) + p(F) = 17/52$.

11. Find and correct the error in the solution to the following problem:

Problem: You flip two coins and want to find the probability that both coins show heads.

Solution: There are three possible outcomes: 2 heads, 2 tails, or 1 head and 1 tail. Since a “success” is one of these three outcomes, $p(\text{both heads}) = 1/3$.

Ans: The probabilities of the three outcomes are not equal. Using $\{HH, HT, TH, TT\}$ as the sample space, the correct answer, $1/4$, is obtained.

12. Let A be the set of all strings of decimal digits of length 5. For example 00312 and 19483 are two strings in A . You pick a string from A at random. What is the probability that

(a) the string begins with 7575.

(b) the string has no 4 in it.

Ans: (a) $10/10^5$. (b) $9^5/10^5$.

Use the following to answer questions 13-16:

In the questions below suppose you have 40 different books (20 math books, 15 history books, and 5 geography books).

13. You pick one book at random. What is the probability that the book is a history book?

Ans: $15/40$.

14. You pick one book at random. What is the probability that the book is not a geography book?

Ans: $35/40$.

15. You pick two books at random, one at a time. What is the probability that both books are history books?

Ans: $(15 \cdot 14)/(40 \cdot 39)$.

16. You pick two books at random, one at a time. What is the probability that the two books are from different disciplines?

Ans: $1 - \frac{20 \cdot 19 + 15 \cdot 14 + 5 \cdot 4}{40 \cdot 39}$.

Use the following to answer questions 17-19:

In the questions below suppose you have a class with 30 students — 10 freshmen, 12 sophomores, and 8 juniors.

17. You pick one student at random. What is the probability that the student is not a junior?

Ans: $22/30$.

18. You pick two students at random, one at a time. What is the probability that both are freshmen?

Ans: $(10 \cdot 9)/(30 \cdot 29)$.

19. You pick two students at random, one at a time. What is the probability that the second student is a freshman, given that the first is a freshman?

Ans: $9/29$.

20. In a certain lottery game, three distinct numbers between 10 and 25 (inclusive) are chosen as the winning numbers. What is the probability that the winning numbers are all composite numbers.

Ans: $\binom{11}{3} / \binom{16}{3}$.

21. In a certain lottery game you choose a set of six numbers out of 54 numbers. Find the probability that none of your numbers match the six winning numbers.

Ans: $\binom{48}{6} / \binom{54}{6}$.

Use the following to answer questions 22-27:

In the questions below you pick a bit string from the set of all bit strings of length ten.

22. What is the probability that the bit string has exactly two 1s?

Ans: $\binom{10}{2} / 2^{10}$.

23. What is the probability that the bit string has exactly two 1s, given that the string begins with a 1?

Ans: $9/2^{10}$.

24. What is the probability that the bit string begins and ends with 0?

Ans: $2^8/2^{10}$.

25. What is the probability that the bit string has more 0s than 1s?

Ans: $(\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}) / 2^{10}$.

26. What is the probability that the bit string has the sum of its digits equal to seven?

Ans: $\binom{10}{7} / 2^{10}$.

27. What is the probability that the bit string begins with 111?

Ans: $2^7/2^{10}$.

28. A group of ten women and ten men are in a room. If five of the 20 are selected at random and put in a row for a picture, what is the probability that the five are of the same sex?

$$\text{Ans: } \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16} + \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16} \dots$$

29. A group of ten women and ten men are in a room. A committee of four is chosen at random. Find the probability that the committee consists only of women?

$$\text{Ans: } \binom{10}{4} / \binom{20}{4}.$$

30. You pick a word at random from the set of all words of length six of letters of the alphabet with no repeated letters. What is the probability that the word has exactly one vowel?

$$\text{Ans: } 5 \cdot 6 \cdot P(21,5) / P(26,6).$$

31. You pick a word at random from the set of all words of length six of letters of the alphabet with no repeated letters. What is the probability that the word begins and ends with a vowel?

$$\text{Ans: } 5 \cdot 4 \cdot P(24,4) / P(26,6).$$

32. A red and a green die are rolled. What is the probability of getting a sum of six, given that the number on the red die is even.

$$\text{Ans: } 1/9.$$

33. A red and a green die are rolled. What is the probability of getting a sum of six, given that the number on the green die is odd?

$$\text{Ans: } 1/6.$$

Use the following to answer questions 34-39:

In the questions below an experiment consists of picking at random a bit string of length five. Consider the following events:

E_1 : the bit string chosen begins with 1;

E_2 : the bit string chosen ends with 1;

E_3 : the bit string chosen has exactly three 1s.

34. Find $p(E_1 | E_3)$.

$$\text{Ans: } 6/10.$$

35. Find $p(E_3 | E_2)$.

$$\text{Ans: } 6/16.$$

36. Find $p(E_2 | E_3)$.

$$\text{Ans: } 6/10.$$

37. Find $p(E_3 | E_1 \cap E_2)$.

Ans: $3/8$.

38. Determine whether E_1 and E_2 are independent.

Ans: Yes.

39. Determine whether E_2 and E_3 are independent.

Ans: No.

Use the following to answer questions 40-42:

In the questions below you flip an unfair coin, where $p(\text{heads}) = 3/4$ and $p(\text{tails}) = 1/4$, ten times.

40. Find $p(\text{exactly 9 heads})$.

$$\text{Ans: } \binom{10}{9} \left(\frac{3}{4}\right)^9 \frac{1}{4}.$$

41. Find $p(\text{exactly 7 heads})$.

$$\text{Ans: } \binom{10}{7} \left(\frac{3}{4}\right)^7 \left(\frac{1}{4}\right)^3.$$

42. Find $p(\text{at least 7 heads})$.

$$\text{Ans: } \binom{10}{7} \left(\frac{3}{4}\right)^7 \left(\frac{1}{4}\right)^3 + \binom{10}{8} \left(\frac{3}{4}\right)^8 \left(\frac{1}{4}\right)^2 + \binom{10}{9} \left(\frac{3}{4}\right)^9 \frac{1}{4} + \binom{10}{10} \left(\frac{3}{4}\right)^{10}.$$

43. Urn 1 contains 2 blue tokens and 8 red tokens; urn 2 contains 12 blue tokens and 3 red tokens. You pick an urn at random and draw out a token at random from that urn. Given that the token is blue, what is the probability that the token came from urn 1?

$$\text{Ans: } \left(\frac{2}{10} \cdot \frac{1}{2}\right) / \left(\frac{2}{10} \cdot \frac{1}{2} + \frac{12}{15} \cdot \frac{1}{2}\right) = 1/5.$$

44. Urn 1 contains 2 blue tokens and 8 red tokens; urn 2 contains 12 blue tokens and 3 red tokens. You roll a die to determine which urn to choose: if you roll a 1 or 2 you choose urn 1; if you roll a 3, 4, 5, or 6 you choose urn 2. Once the urn is chosen, you draw out a token at random from that urn. Given that the token is blue, what is the probability that the token came from urn 1?

$$\text{Ans: } \left(\frac{2}{10} \cdot \frac{1}{3}\right) / \left(\frac{2}{10} \cdot \frac{1}{3} + \frac{12}{15} \cdot \frac{2}{3}\right) = 1/9.$$

Use the following to answer questions 45-47:

In the questions below a bowl has eight ping pong balls numbered 1,2,2,3,4,5,5,5. You pick a ball at random.

45. Find p (the number on the ball drawn is ≥ 3).
Ans: $5/8$.
46. Find p (the number on the ball drawn is even).
Ans: $3/8$.
47. Find $E(X)$, where X = the number on the ball you draw.
Ans: $27/8$.
48. A die has the numbers 1,2,2,3,3,3 on its six sides. If the die is rolled, what is the expected value and variance of the number showing?
Ans: $7/3$, $5/9$.
49. A pair of dice, each with the numbers 1,2,2,3,3,3 on its six sides are rolled.
(a) What is the expected value of the sum of the numbers showing?
(b) What is the expected value of the product of the numbers showing in part (a)?
Ans: (a) $14/3$. (b) $49/9$.
50. You have seven cards, numbered 3 through 9, and you pick one at random. If you pick a card with a prime number, you get 1 point; if you pick a card with a composite number, you lose 1 point. Find the expected value of the number of points you get.
Ans: $-1/7$.
51. You flip a coin. If it lands heads, you lose 1 point. If it lands tails, you flip the coin again, and lose 1 point if it lands heads and get 3 points if it lands tails. What is the expected value of the number of points you get when you play this game.
Ans: 0.
52. Each of 26 cards has a different letter of the alphabet on it. You pick one card at random. A vowel is worth 3 points and a consonant is worth 0 points. Let X = the value of the card picked. Find $E(X)$, $V(X)$, and the standard deviation of X .
Ans: $15/26$, 1.38, 1.18.
53. You have two decks of 26 cards. Each card in each of the two decks has a different letter of the alphabet on it. You pick at random one card from each of the two decks. A vowel is worth 3 points and a consonant is worth 0 points. Let X = the sum of the values of the two cards picked. Find $E(X)$, $V(X)$, and the standard deviation of X .
Ans: $2 \cdot \frac{15}{26}$, $2 \cdot 1.38$, 1.66.