Discrete Mathematics and Applications

Moshe Rosenfeld

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moishe@u.washington.edu

1 A selection of practice counting problems

1. Let n be a positive integer. How many positive integer solution does the equation $x_1 + x_2 + \ldots + x_r = n$ has ?

2. In how many different ways can you write 50 as a sum of 4 positive integers. (Note: 10+10+15+15 is the same solution as 10+15+15+10).

3. In how many ways can 2048 be written as the product of 4 integers?

4. In each of the 6-digit numbers: 333333, 225522, 118818, 707099 each digit in the number appears at least twice. Find the number of such 6-digit natural numbers.

5. A survey shows that 90% of high-schoolers in Hanoi like at least one of the following activities: going to the movies, playing sports, or reading. It is known that 45% like the movies, 48% like sports, and 35% like reading. Also, it is known that 12% like both the movies and reading, 20% like only the movies, and 15% only reading. What percent of high-schoolers like all three activities?

6. We have 25 kinds of candies. In how many ways can we distribute candies to 15 children provided that no child gets two candies of the same kind?

7. How many digits does 200! have? (Hint: use Stirling's formula to estimate it and compare the result with SAGE or MATLAB).

8. Prove that:
$$n! \leq en(\frac{n}{e})^n$$
(hint: it can be done by induction).

9. Prove that
$$n^{\frac{n}{2}} \leq n! \leq (\frac{n+1}{2})^n$$
(hint: you can use the Arithmetic-Geometric mean inequality)

10. Prove that
$$\binom{n}{k} \leq (\frac{en}{k})^k$$
.

11. Which permutations have Cantor numbers $n! - 1$ and $n! + 1$ respectively?

12. There are $\binom{n}{k}$ combinations. Which combination is in the "middle" in the lexicographic order?