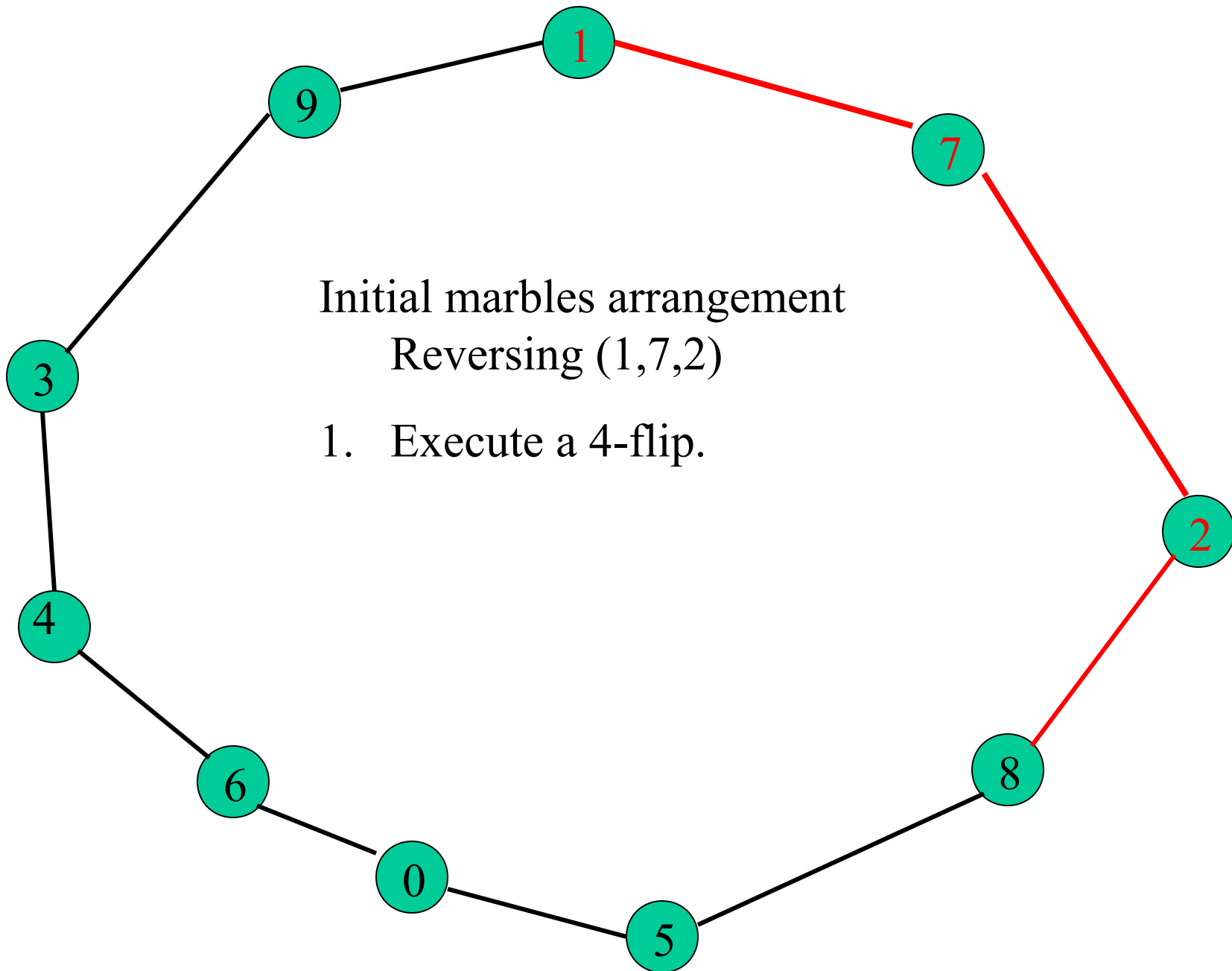


Claim: we can reverse 3 consecutive marbles.

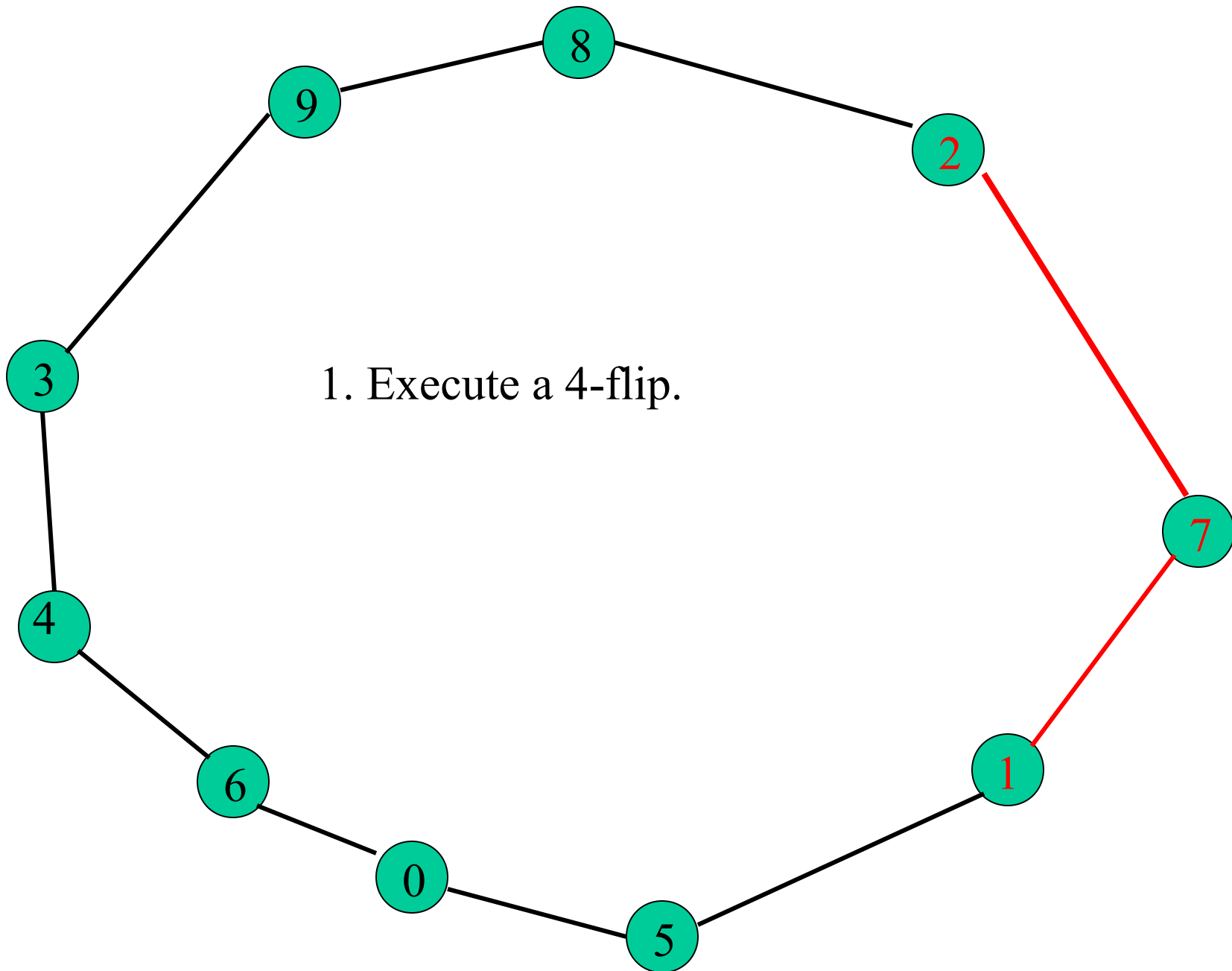
1. Reversing 3 consecutive marbles (wxy):
2. ...w x y z a b c d... \rightarrow ...z y x w a b c d... \rightarrow
3. ...z a w x y b c d... \rightarrow ...z a b y x w c d ... \rightarrow
4. ...z a b c w x y d... \rightarrow ...z a b c d y x w...
5. After an odd number of 4-flips the triple wxy is reversed to yxw.
6. If we continue with pairs of 4-flips ...yxwkm \rightarrow ...kwxym \rightarrow ...kmyxw \rightarrow ... \implies
7. Since we have an even number of marbles, we shall reach: ...y x w z a b c d...

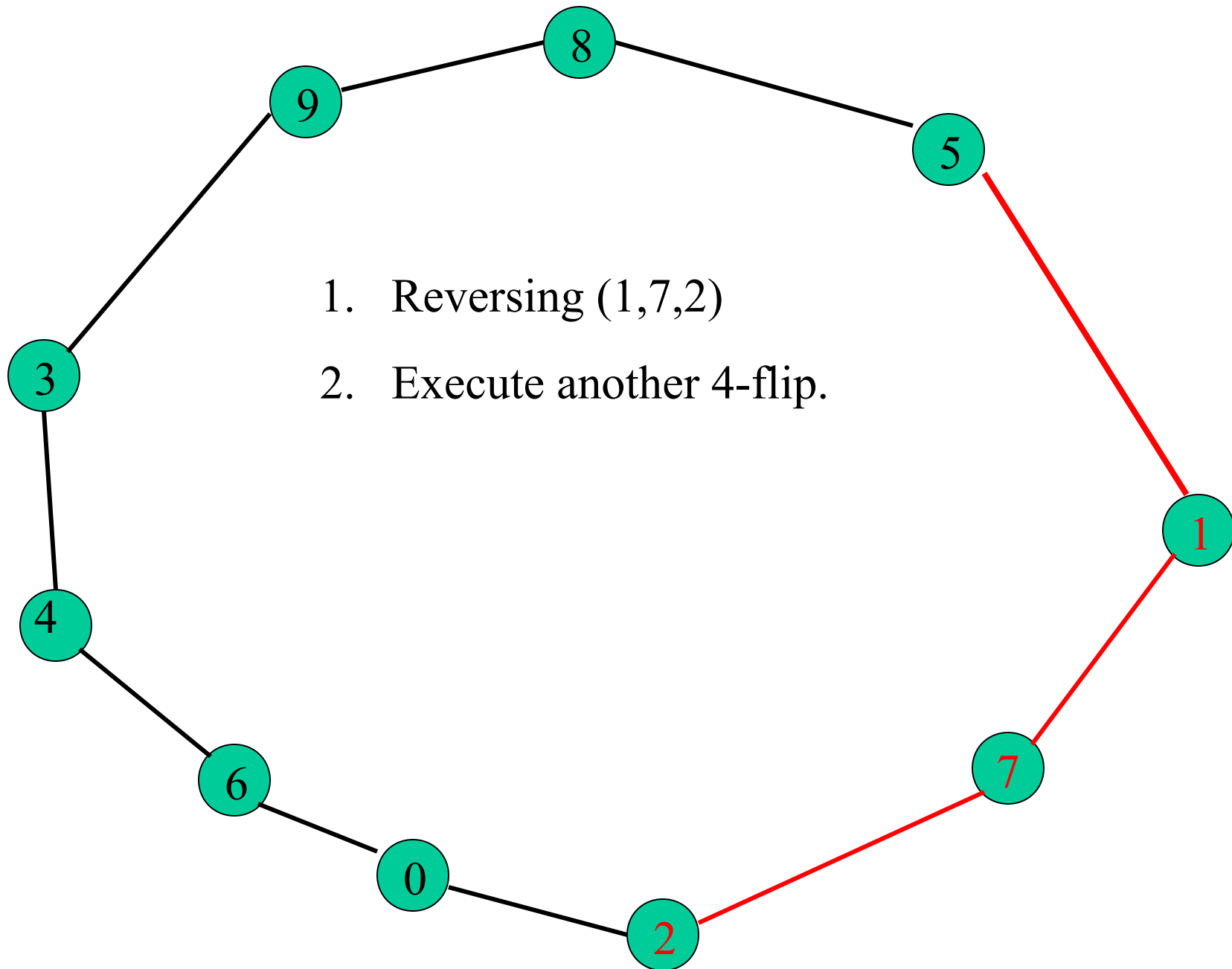


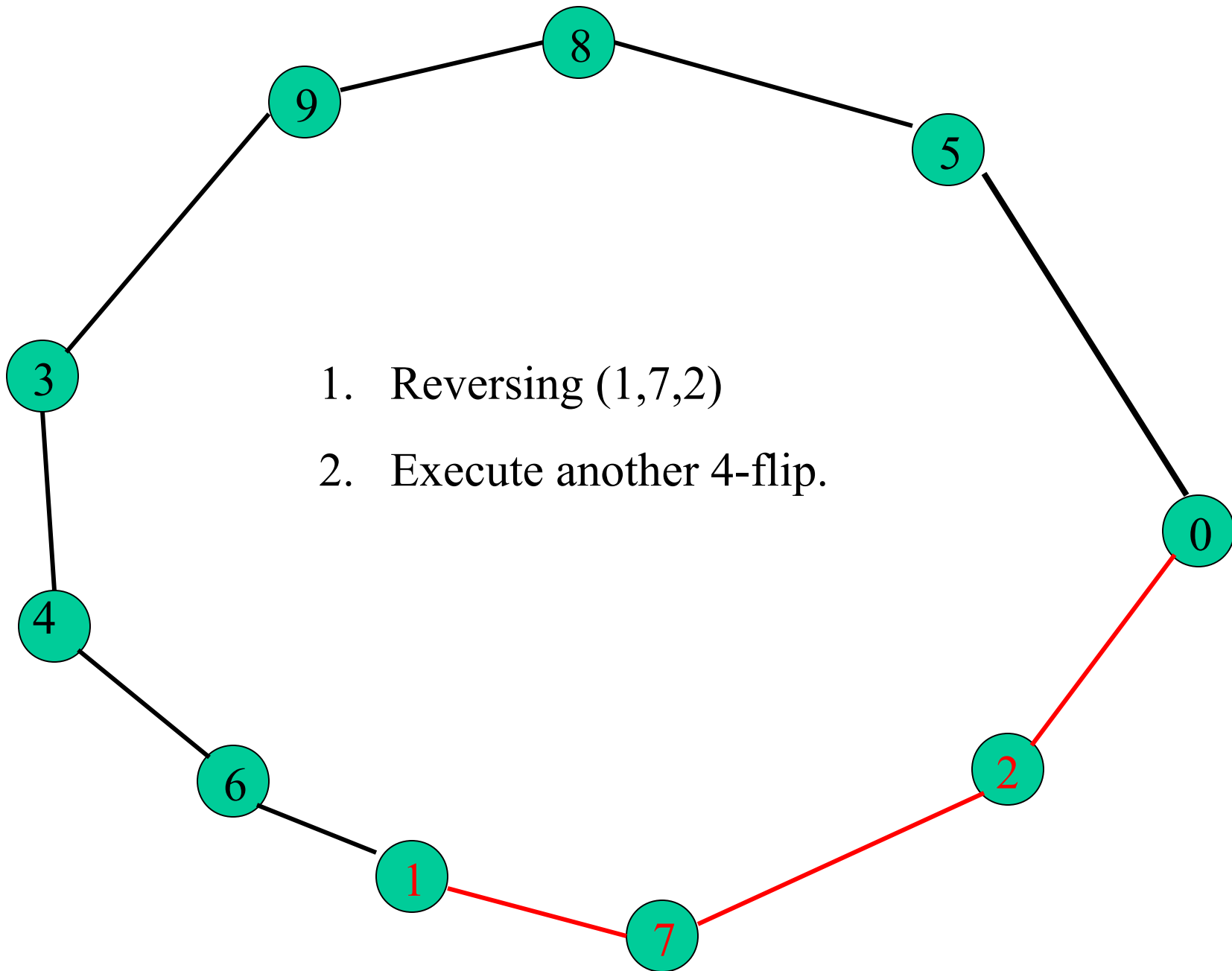
Initial marbles arrangement

Reversing (1,7,2)

1. Execute a 4-flip.

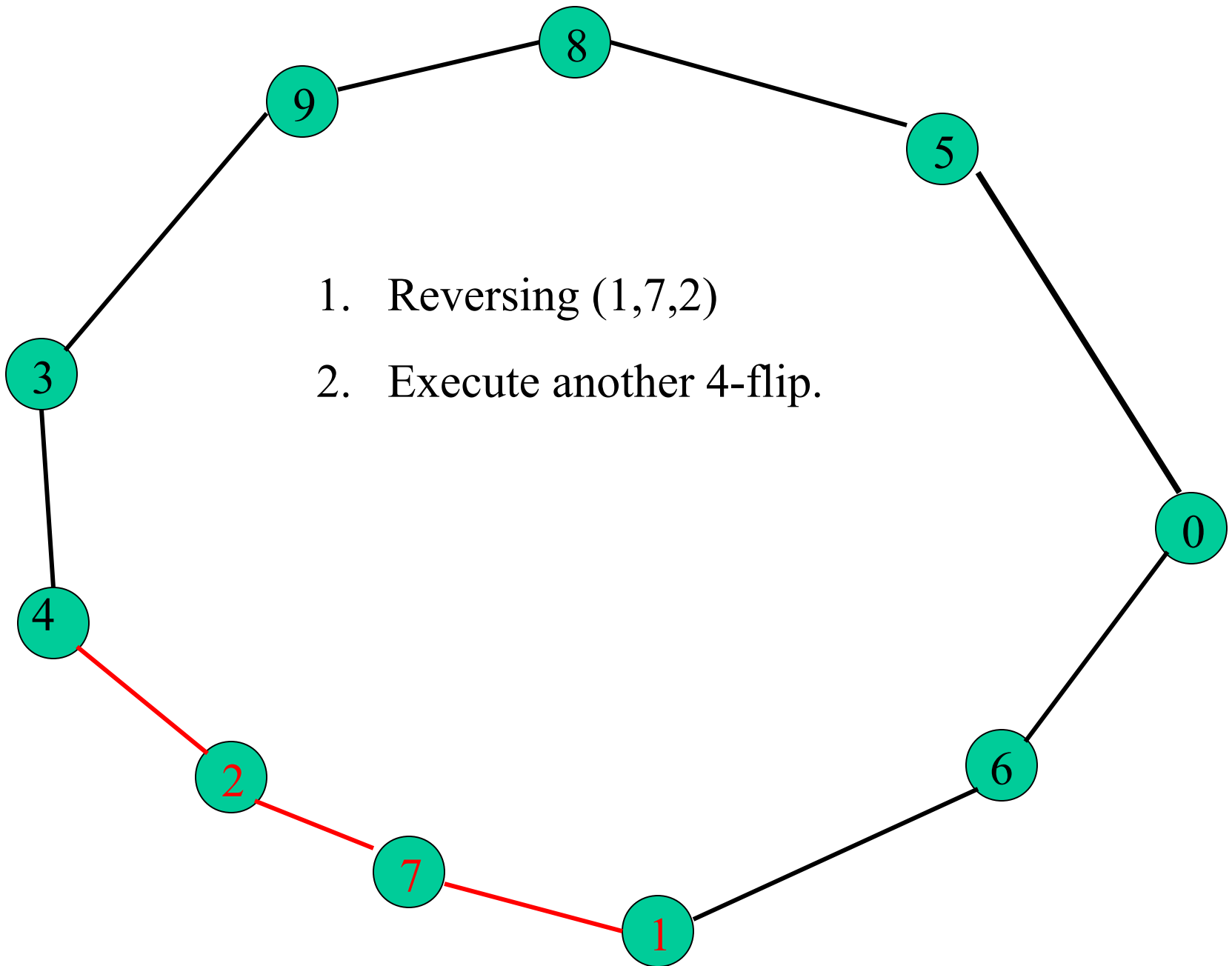


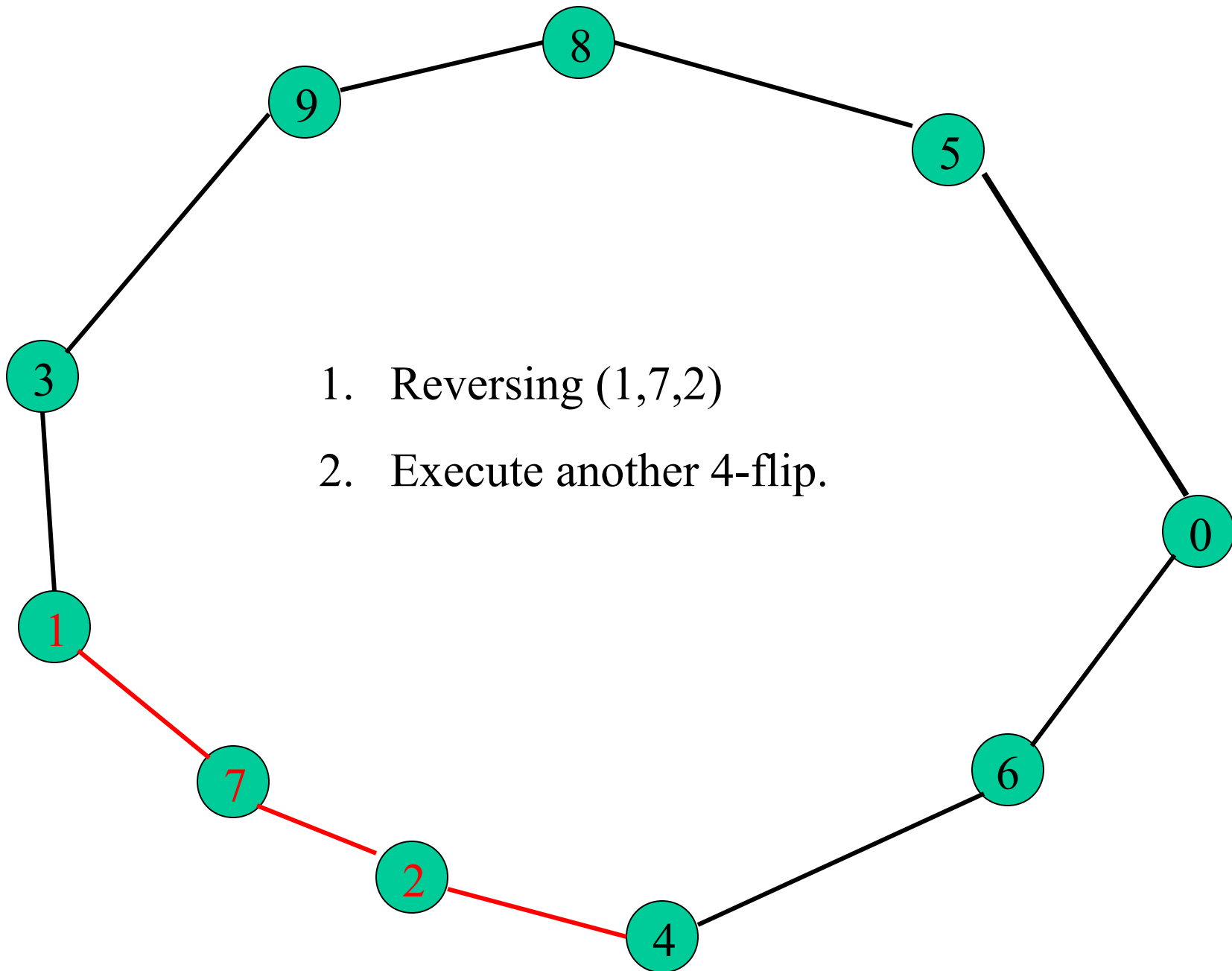




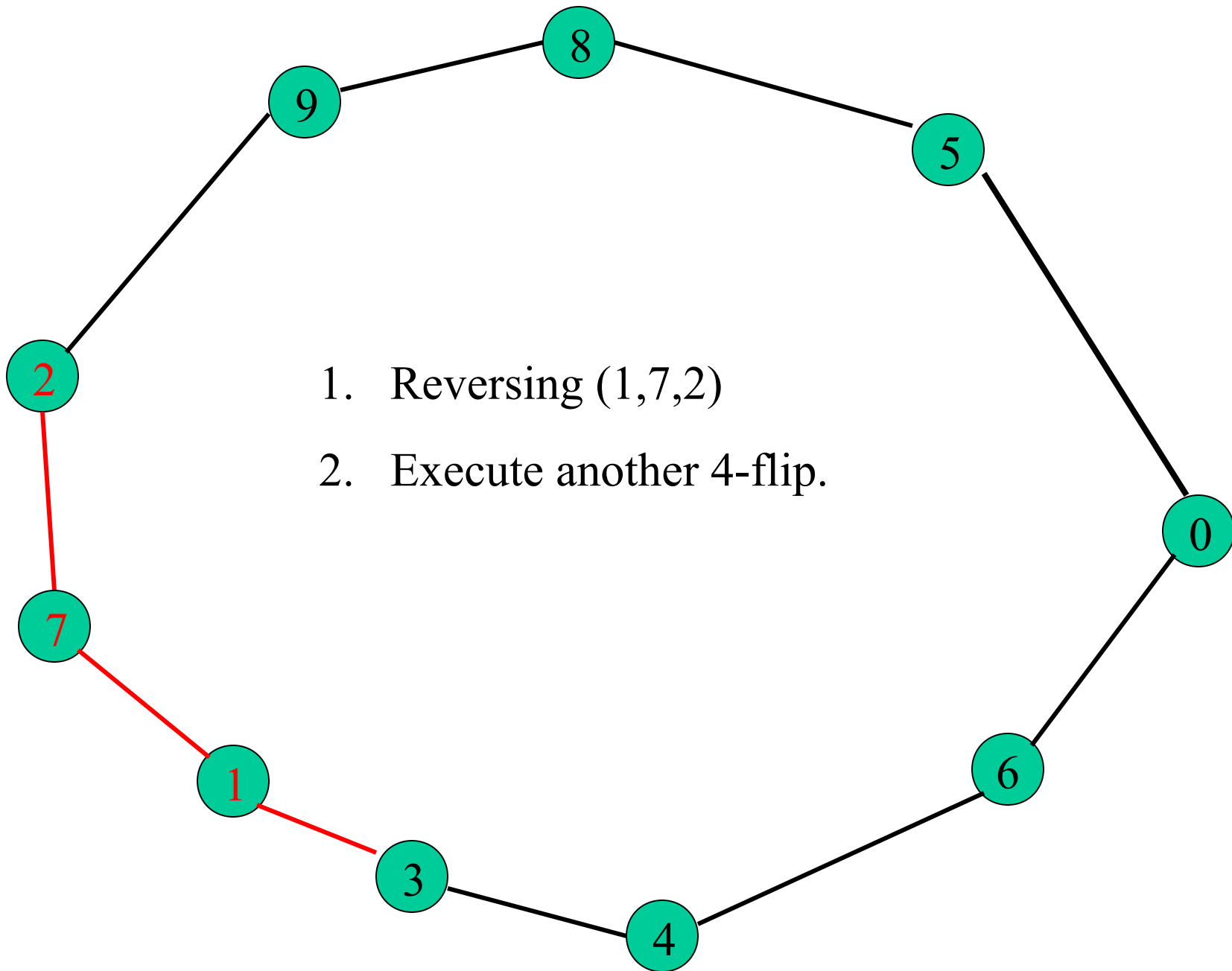
1. Reversing (1,7,2)
2. Execute another 4-flip.

1. Reversing (1,7,2)
2. Execute another 4-flip.

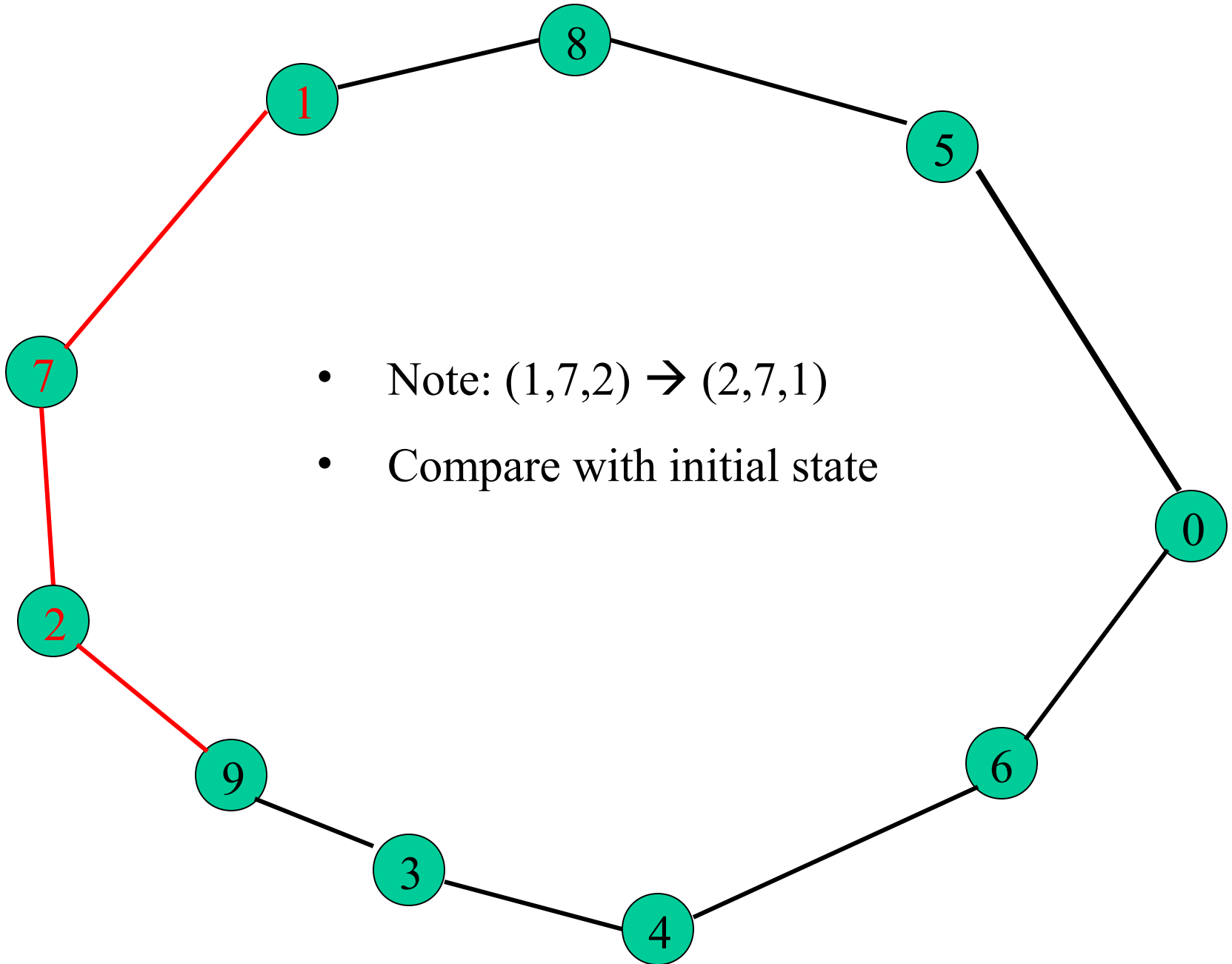




1. Reversing (1,7,2)
2. Execute another 4-flip.



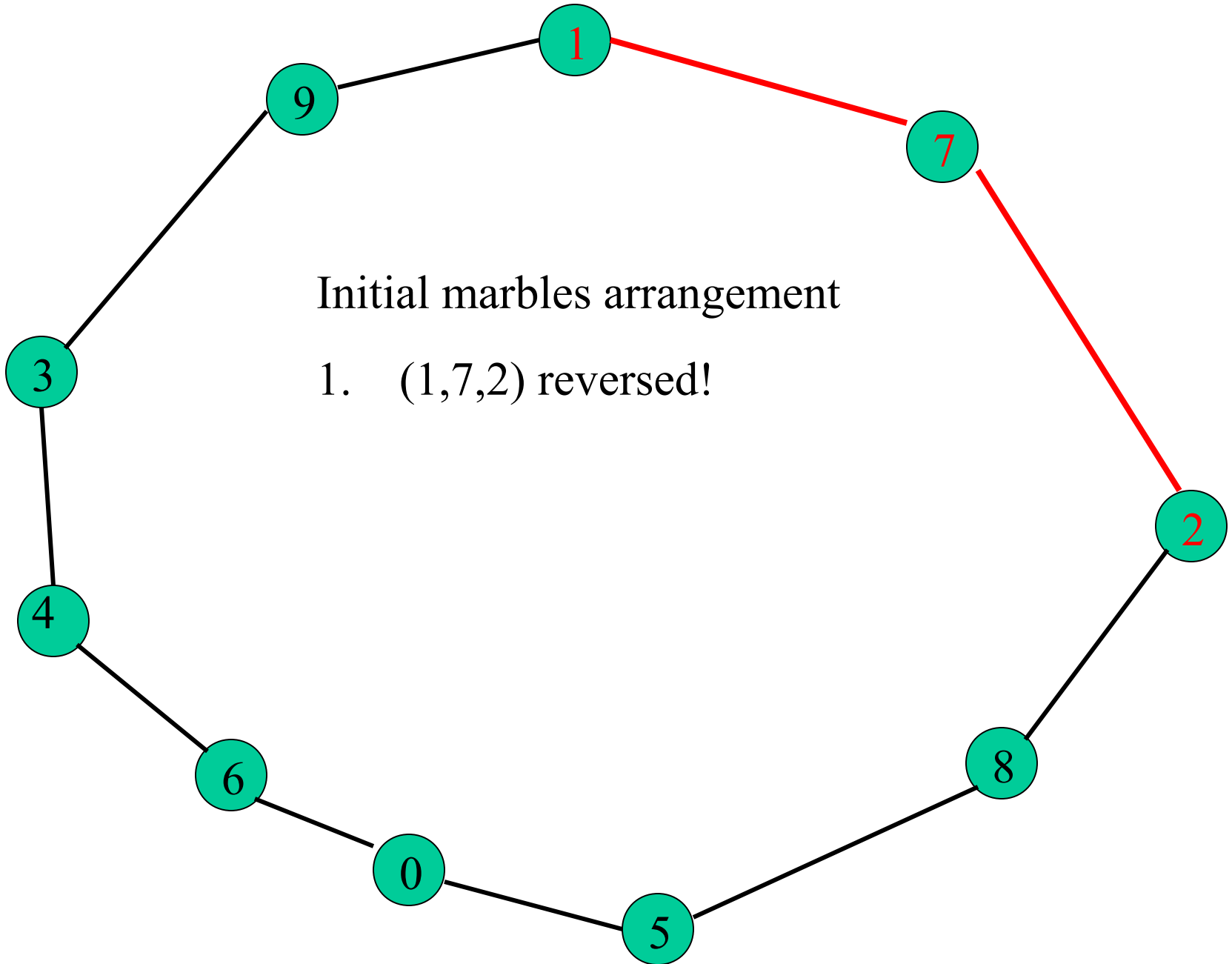
1. Reversing (1,7,2)
2. Execute another 4-flip.



- Note: $(1,7,2) \rightarrow (2,7,1)$
- Compare with initial state

Initial marbles arrangement

1. (1,7,2) reversed!



Flipping two adjacent marbles.

- ...e f g a **b x y z** ... \rightarrow ...e f g **a z y x** b... \rightarrow ...e f g x y z **a b**...
- After each pair of flips “ab” moves 3 positions “clockwise” while all other marbles remain in their original relative position.
- Stop when 1, 3 or 5 marbles are left between b and x.
- This will happen since there is an odd number of marbles (25 in our case) left to the right of b.
- Call “triple-exchange” 1, 3 or 5 times alternating start between a and b.
- Example:
 - ...a **b e f** g x y z... \rightarrow ...**a f e** b g x y... \rightarrow ...e f **a b** g x y \rightarrow ...e f g **b a** x y...
- a b is now b a.
- Final solution: exchange any pair of adjacent marbles that are out of order.

finish...

- This means that any two consecutive marbles can be interchanged.
- But these exchanges generate all permutations.