Discrete Mathematics and Applications

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1 Assignment No. 8: counting, permutations, recurrence relations

Due: Friday, 3 Dec.

Please submit your answer in a neat, readable properly organized format.

1.
   - Find the permutation whose number under the Cantor Digits enumeration is your phone number.
   - For your convenience here are the factorials: $6! = 720,\ 7! = 5040,\ 8! = 40320,\ 9! = 362880,\ 10! = 3628800,\ 11! = 39916800,\ 12! = 479001600,\ 13! = 6227020800$
   - Exchange the permutation with a friend. You give him your permutation and he gives you his.
   - Retrieve his permutation number. Show your calculation details.
2. Vua Le decided to mint 25 coins. He needs your help to be able to identify a fake coin using no more than 4 weighings. Your cooperation is required.
3. At the end of execution of the Arrow algorithm, what will be the last permutation? How will the arrows point?
4. How many distinct terms are there in the expansion of:

$$(\sum_{i=1}^{k} x_i)^n$$

   (assume that all equal terms have been collected together. For example, if an expression conatins the terms $3x^2y^5z^3,\ 23x^2y^5z^3$ then after collecting them together we have only one term: $26x^2y^5z^3$.)

5. Let S be a set. Let $M = \{A_1, A_2, \ldots, A_k\}$ be a family of subsets each of size d. A *k-coloring* of S is an assignment of colors to the members of S such that in every set A_i there will be two elements with different colors.

Example: Let $S = \{1, 2, 3, 4\}$ and $A_1 = \{1, 2\}$, $A_2 = \{2, 3\}$, $A_3 = \{3, 4\}$, $A_4 = \{1, 4\}$ then assigning to each number $i \in S$ the color $i \bmod 2$ is a 2-coloring of S.

- Show that the Fano plane is not 2-colorable.
- Show that all collections of 6 triples are 2-colorable.