Discrete Mathematics and Applications

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1 Assignment No. 7: counting, permutations, recurrence relations

Due: Friday, 12 Nov.

Please submit your answer in a neat, readable properly organized format.

1. A necklace is composed of 20 beads. The beads are blue, green and red. How many distinct necklaces can be produced?
2. How many distinct necklaces can be produced if no three consecutive beads are of the same color?
3. An integer is **squarefree** if it is not divisible by the square of an integer. For example, 35 is squarfree while 50 is not. Find the number of squaerfree integers less than 200.
4. Find a_{11}, a_{12} and the general term a_n for the sequence whose first 10 terms are: $2, 9, 22, 41, 66, 97, 134, 177, 226, 281, 342, 409$.
5. How many bit strings of length n contain exactly two occurrences of the substring 10?
   Hint: build a few sequences of various lengths. How many times the content of such sequences changes? (from 0 to 1 or from 1 to 0).
6. **Show that a sequence of numbres $a_1, a_2, \ldots, a_{n^2+1}$ contains a monotonic subsequence of length $\geq n+1$.**

2 Binomial Coefficients

1. What is the coefficient of x^{12} in the expansion of $(2x^2 + \frac{1}{2x^2})^{20}$.
2. Use a counting argument to prove that $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$.

3. Prove that $\binom{2n}{n} \geq \frac{4^n}{2n}$.