Discrete Mathematics and Applications

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Hanoi 2010
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1 Assignment No. 6: sequences, counting, binomial coefficients

Due: Friday, 5 Nov.

Please submit your answer in a neat, readable properly organized format.

1. In a group of 50 persons, 29 speak Vietnamese, 22 speak English and 17 speak French. It is also known that 12 speak both English and French, 9 speak English and Vietnamese and 11 speak both French and Vietnamese. 6 persons speak all three languages. How many persons do not speak any of the three languages?

2. Find the segment with the largest weight in the following array:

   $9, -8, -8, 9, -4, 0, -1, -5, 5, 6, 2, 8, 6, 6, 7, 0, -10, 10, 8, 2, -2, 8, -8, 1, 11, 1, -7, -11, -7, -6,$
   $2, 2, 4, -9, -4, 4, -3, -3, 2, 2, 9, -9, 5, 13, 12, 3, 11, 0, 7, 11, -9, -4, -8, 5, 11, 5, -6, 12, 8, 4$

3. Find a_{11}, a_{12} and the general term a_n for the sequence whose first 10 terms are: $2, 9, 22, 41, 66, 97, 134, 177, 226, 281, 342, 409$.

4. How many bit strings of length n contain exactly two occurences of the substring 10?

5. Show that a sequence of numbres $a_1, a_2, \ldots, a_{n^2+1}$ contains a monotonic subsequence of length $\geq n + 1$.

2 Binomial Coefficients

1. What is the coefficient of x^{12} in the expansion of $(2x^2 + \frac{1}{2x^2})^{20}$.

2. Use a counting argument to prove that $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$.

3. Prove that $\binom{2n}{n} \geq \frac{4^n}{2n}$.