Discrete Mathematics and Applications

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Hanoi 2010
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1 Assignment No. 1: proofs

Due: Wednesday, Sep. 15

Please submit your answer in a neat, readable properly organized format. In general, a * in exercises indicates a more challenging problem.

1. Prove that if a,b are real positive numbers then $a^2 + b^2 \geq 2ab$
2. Prove that if a,b,c are real positive numbers then $a^3 + b^3 + c^3 \geq 3abc$
3. Proof:
   (a) $(a-b)^2 + (a-c)^2 + (b-c)^2 \geq 0 \rightarrow a^2 + b^2 + c^2 - (ab + ac + bc) \geq 0.$
   (b) $a(a^2 + b^2 + c^2 - (ab + ac + bc)) = a^3 + ab^2 + ac^2 - a^2b - a^2c - abc \geq 0$
   (c) $b(a^2 + b^2 + c^2 - (ab + ac + bc)) = ba^2 + b^3 + bc^2 - ab^2 - abc - b^2c \geq 0$
   (d) $c(a^2 + b^2 + c^2 - (ab + ac + bc)) = ca^2 + cb^2 + c^3 - abc - ac^2 - bc^2 \geq 0$
   (e) Adding all three inequalities we get:
   (f) $a^3 + b^3 + c^3 - 3abc \geq 0$
   (g) QED.
4. Let $S = \{n | n = a^2 + b^2,\ a, b, n \in N,\ a, b > 0\}$
   Find an integer m such that $\{m, m+1, m+2\} \subset S$
   - Hint: $a^2 + b^2 \not\equiv 3 \text{ mod } 4$.
5. * Prove that there are infinitely many integers m for which $\{m, m+1, m+2\} \subset S$
   - Hint
   - $m = a^2 + b^2 = (a + bi)(a - bi)$
   - Prove that if $m, n \in S\ then\ mn \in S$.
   - Deduce that if $\{m-1, m, m+1\} \subset S\ then\ \{m^2 - 1, m^2, m^2 + 1\} \subset S$