Discrete Mathematics and Applications

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1 Assignment No. 2: logic

Due: Wednesday, Sep. 22

Please submit your answer in a neat, readable properly organized format.

1. a. Construct a truth table for a Boolean expression $F(p, q, r)$ such that $F(p, q, r) = T$ only when either one or all three variables equal T.
   b. Use logic gates to implement this table.
   c. Use only NAND gates to implement this table.

2. a. Given the disjunction $(x + y + s + t + w)$. Construct an equivalent 3-SAT instance.
   b. Prove that any disjunction $(x_1 + \ldots + x_n)$ is equivalent to a 3-SAT instance.

3. Prove that any 3-SAT instance with 7 clauses is satisfiable.

   - Hint
   - Every 3-sat clause $(x + y + z)$ has 8 differnt possible assignment to x, y, z, $(x + y + z)$ is FALSE for only one assignment and TRUE for the other 7.

4. Find a compound proposition logically equivalent to $p \rightarrow q$ using only the NAND operator.

5. The police suspects that four persons hacked into the university computer system. The four persons made the following statement to the police:
   Anh: Phuong did it.
   Phuong: Ben did it.
   Tuan: I did not do I
   Ben: Phuong lied when he said that I did it.

a. Suppose the police know that exactly one person is lying, who did it?
b. Suppose the police know that exactly one person is telling the truth, who did it?
Justify your answer.

2 SAGE exercises

1. Let n be the sum of the digits of your cell phone number. Let m be the number obtained from n by appending the digit 7 at the end. For example, if n 46 then m = 467. Find an integer q such that $mq = 11....1$.

2. Create two sets:
   $A = \{n | n \in N, n = a^2 + b^2, \ a, b \in N, \ n \leq 10000\}$
   $B = \{n | n \in N, n \bmod 4 = 1, \ n \leq 10000\}$
   Let $C = A \cap B$.
   Compare the sets B and C. Can you draw a conclusion?