Discrete Mathematics and Applications

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1 Assignment No. 5: Integers, modular arithmetic, sets

Due: Friday, 15 Oct.

Please submit your answer in a neat, readable properly organized format. This assignment includes some forward preparations simple exercises in number theory.

1. Simple calculation, do by hand or a simple hand held calculator.

   a. Find integers m, n such that $GCD(6743, 273) = 6743m + 273n$

   b. Calculate 2^{-1} mod 751. Verify your answer.

   c. Calculate 200^{-1} mod 751. Verify your answer.

   d. Does $sqrt(2)$ mod 751 exist? If yes, find it and verify your answer.

   e. Let $m = 10\sum_{i=1}^{10} d_i$ where d_i are the digits of your phone number.
   Verify that : $m = (385 \times m \bmod 3 + 231 \times m \bmod 5 + 330 \times m \bmod 7 + 210 \times m \bmod 11) \bmod 1155$
   Explain why this is true. (Some of us know that this is the Chinese Remainder Theorem, I am looking for a direct explanation that will convince a person who does not know the Chienese Remainder Theorem).

2. Clearly, $\sqrt{(49)} \bmod 3869 = 7$. Find other integers $8 \leq x \leq 3868$ such that $x = \sqrt{(49)} \bmod 3869$.

3. Prove that there are infinitely many primes $p = 3 \bmod 4$.

4. For any given positive integer n prove that there are n consecutive composite integers.

2 Primes related sequences

1. Find a formula or rule for the sequence: $1, 2, 3, 2, 5, 2, 7, 2, 2, 2, 11, 2, 13, 2, \ldots$. What are the next two entries?

2. Find a formula or rule for the sequence: $1, 1, 2, 4, 4, 6, 6, 10, 10, 10, 10, 12, 12, 16, 16 \ldots$. What are the next two entries?

A section on sets will be added later.

2.1 Sets

1. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$. Construct a set of 13 quadruples (subsets of size four) such that:
   - each two sets have exactly one number in common.
   - each number appears in exactly four subsets.
   - each pair of numbers appears in one set.

2. You are given a list of 23 teams, each with an odd number of students such that any two teams have an even number of students in common. There are 26 students in the class. Prove that you can add a new team with an odd number of students that has an even number of students with each of the other teams.