

Discrete Mathematics and Applications

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1 Assignment No. 8: Counting

Due: Tuesday, 08 Nov.

Please submit your answer in a neat, readable properly organized format.

2 Elementary counting

1. A multiple choice test has 40 questions. Each question has four possible answer (with only one correct answer).
 - a. In how many ways can a student answer all 40 questions?
 - b. In how many ways can a student answer all question so that exactly 30 questions are answered correctly?
 - c. In how many ways can a student answer the questions if he can leave some answers blank?
2. How many positive integers < 10000 :
 - a. Are divisible by 13.
 - b. Are divisible by 13 or by 15.
 - c. Are divisible by 13 and by 17 but not by both.
 - d. Are relatively prime to 13 and 17.
3. A **palindrome** is a string whose reversal is the same as the priginal string, ABBA is a palindrome. how many bit strings of length n are plaindromes?

2.1 Proofs

1. Prove that among any $n + 1$ positive integers none greater than $2n$ there must be a pair of integers n, m such that n divides m (notation: $n \mid m$).

2. a. Prove that there is an integer $k < 1000$ such that the absolute value of the difference between $k\sqrt{2}$ and its nearest integer is $< \frac{1}{1000}$.
b. * Let x be an irrational number and n a positive integer. Prove that there is an integer $j \leq n$ such that the absolute value of the difference between jx and the nearest integer to jx is less than $\frac{1}{n}$.
3. Prove that there are infinitely many prime numbers of the form $4k + 3$.
4. (**SAGE**): Let $k = 10 \times (\text{ph-num}) + 1$ (ph-num is the last four digits of your phone number). Use SAGE to find an integer m such that $m \times k = 11 \cdots 1$.